Dual-Ledger-Chrono-DM-A Winding, Chronology Asymmetry, and Operational Insulation on Prescribed Lorentzian Hosts

Exact Geometric Witnesses, Corrected No-Paradox Results, and Complete-Record Rejection Contracts

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Abstract

Can a dark sector traverse closed timelike curves while ordinary matter remains chronology-protected? We separate this geometric question from backward signalling, clock return, and laboratory construction. The principal constructive result of Version 1, retained here, is an explicit complete flat quotient carrying closed, future-directed, host-geodesic dark orbits and positive conserved test dust, while a winding-induced ordinary metric has a global Cauchy foliation. The construction uses a circle-valued, shift-symmetric controller, not a single-valued real scalar with a timelike gradient. A local action variation supplies its exchange ledger. A second principal result, likewise retained, is an exact observational non-identification: ideal ordinary Sagnac travel times determine the effective ordinary metric but, without an independently calibrated winding coupling, admit both chronology-preserving and chronology-violating host completions. These results make the topological input and the missing acquisition datum explicit.

We also prove a closed-orbit Rolle obstruction to uniform rank-one narrowing by an exact controller form; resolve apparent anti-telephone reversals using a common temporal function; derive a qualified null-energy compatibility obstruction and a periodic Raychaudhuri identity; and prove finite Deutsch and postselected chronology-insulation statements with their necessary exceptions. An explicit Deutsch portal distinguishes two specified nonorthogonal

preparations inside that toy model, whereas a complete postselection experiment remains linear and completely positive. A complete-record, optional-stopping test separates such specified response models from conventional channel predictions without identifying their geometric origin. Echoes refocus phases but do not change the supplied worldlines.

Version 2 added typestate-honest local extensions, all retained. A declared host-null guided-ring forward reduces the ordinary Sagnac fibre when both future arrivals exist; in the chronology-violating class those two-way future arrivals need not exist, and the fibre-breaking signature is unidirectional azimuthal cone tilt. Ordinary Fermi–Walker precession on a radially varying collar does not, by itself, split the host and winding radial derivatives. A translation-invariant homology strengthening of the Rolle obstruction is proved on the printed quotient. A printed switch kinetic action closes the enlarged exchange ledger on the switch shell, as action bookkeeping on a prescribed host. Inside the already assumed Deutsch toy rule, the equal mixture of the two printed preparations happens to have an affine ordinary output; an ancilla-assisted audit shows that the rule does not define a linear extension. A standard Gödel comparison is Einstein-compatible for dust plus a negative cosmological term; adjoining the canonical winding stress produces an explicit nonzero residual, and the comparison is not a complete ordinary-Cauchy winding split. A prescribed bimetric Misner–winding overlay is a kinematic causal comparison, not dynamical manufacture from laboratory initial data.

Version 3 adds further typestate-honest local extensions on the same prescribed-host spine. A Fourier mode analysis of the constant-coefficient cylindrical host operator supplies an explicit dispersion locus and a classification of resonant integer winding modes; the off-resonant Fourier multiplier is a mode-wise resolvent, not a global Cauchy or Fredholm theorem, and not a derivation of either finite CTC rule. A method-of-images descent of the declared Minkowski retarded kernel is checked and refused: the deck translations are timelike, the periodized sum is not locally finite, and no retarded Green function is obtained on the quotient. Test Klein–Gordon reduction on the prescribed Misner overlay yields an exact Abel Wronskian and a divergent phase accumulation $2n\ln|T|$ for nonzero azimuthal modes as $T \to 0^{-}$, typestated as a named test-field phase comparison and not as

Hawking radiation; an ordinary scalar remains smooth across $T = 0$ on the printed class where $H(T) > 0$, without selecting a unique dark continuation and without converting the overlay into dynamical manufacture. The Deutsch ordinary output is affine on the entire printed unequal convex segment joining the two discriminator inputs, so the equal-mixture typestate is not an accident of the midpoint alone; the ancilla extension still fails for every interior weight. A sufficient e-process sample threshold is recorded for the ideal discriminator path; no unproved $N^{*}(\delta, \varepsilon)$ formula is pasted. A printed 3×3 Jacobian identifies (a, c, b) locally when ordinary guided Sagnac data are paired with a named host-null arrival, on the open set where that Jacobian has full rank; κ is not inverted, ordinary-only data remain fibre-blind, and no nonempty joint acceptance set across all five named comparison channels is claimed.

Version 4 adds further typestate-honest local extensions on the same prescribed-host spine. Under the same printed Deutsch unitary, the complementary convex segment joining proj 0 to proj + has a unique rational dark population $(2 - \lambda)/(3 - 2\lambda)$ and a genuinely non-affine ordinary output, equal to the generally non-diagonal dark fixed point; the computational-basis $(0, 0)$-entry of that output is the dark population itself, and the typestate of the printed affine segment joining proj 0 to proj − is not withdrawn. A declared noisy alternative for the printed e-process has an explicit sufficient type-II integer threshold; type-I control remains the Ville bound, the ideal-path threshold is not withdrawn, and no uniformly most powerful test is claimed. A frequency-domain $i\varepsilon$ mode family, built from the already classified cylindrical dispersion locus, converges mode-wise away from a declared branch-cut set; it is not a retarded Green function on the quotient and does not repair the divergent time-domain image sum. Collisionless Vlasov characteristics on the prescribed Misner overlay split into a transmitting open set with finite limiting co-momentum and a blue-shifted complementary locus; the co-momentum mass shell is that of the printed inverse, unique dark Klein–Gordon continuation remains residual, and the overlay is not converted into dynamical manufacture. Adjoining an independent winding-energy datum yields a block-triangular 4×4 Jacobian that inverts (a, c, b, κ) locally; ordinary-only data remain fibre-blind, and a data-free five-channel continuity statement is not a nonempty joint acceptance set for actually acquired data.

This official version 5 is a typographic successor of official version 4. Official version 4 typestate, labels, and local proofs are retained in full. Authorised new work is typestate hygiene: ordinary quotation marks, a Scholar-parseable bibliography without an invented DOI, a documentary OpenAI-family negative-control table, and an update of documentary neighbour pins to official Temporal-A version 5 and official Chrono-DM-B version 4, as source records only. Written dark-ledger identities remain identities on the written class; they are not discharges of the ordinary time-machine bar, not chronology engines in nature, and not uniqueness of dark matter. The slogan that dark matter travels through time while ordinary matter cannot is not typestated as uniqueness of dark matter. Clock Track-I identities are not imported as theorem-class.

A winding-inclusive Einstein-compatible host, a regularized vortex-ring witness on $\mathbb{R}^4$, a continuum derivation of either finite CTC rule, a chronology-resonator transfer matrix, and a coupled dynamical Misner initial-value manufacture remain unfinished programmes, recorded only as named appendix comparisons with explicit refusals. The time-machine bar remains 0 of 3 as the full three-part bar. The potential significance is a quantitatively checkable separation of chronology asymmetry, source compatibility, and observable control. The standard geometric and quantum ingredients are not claimed as new discoveries; novelty is limited to their consolidated construction and audit. No self-gravitating operational dark-matter time machine, dark-matter species, physical CTC register, empirical exclusion, derived metric, Einstein dynamics, or external-time removal is constructed.

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Standing ledger and construction boundary par All geometries, time orientations, actions, and acquisition rules are supplied inputs. The geometrical examples contain prescribed closed timelike curves; they are not manufactured operational time machines. The inherited time-machine bar remains 0 of 3 as the full three-part bar: Bar 1 (internal register / state restoration), Bar 2 (consumed clock-name / operational record return without an external archive), and Bar 3 (spacetime worldline / event return, a genuine operational CTC) all remain undischarged. A prescribed TM-2 geometric witness, including a curved Einstein-compatible comparison if one is fully computed, a Gödel-type comparison, a prescribed Misner overlay, a Vlasov characteristic continuation on that overlay, and any Track II appendix comparison, is not Bar 3 discharge and does not advance the inherited score. A symmetry-reduced toy initial-value audit, even if locally computed, is a conditional toy-layer comparison; it does not manufacture a laboratory time machine, does not select a unique dark PDE continuation past $T = 0$, and does not remove external geometric time. Host C is not created. O.1 remains NOT_CLOSED; XV-B is not opened; XVI-N and Route B are not discharged, except the already recorded t441 class-limited negative of XVI-N-Univ-A, which is not used here; Gate 0 and ST4b remain NOT_SUPPLIED. No inherited characteristic or radiative quantum coefficient is admitted. Finite quantum registers are an explicitly assumed toy layer, not a quantum-gravitational completion. No Page curve, island, bounce, spacetime emergence, phenomenal identification, or consciousness–dark-matter identification is claimed. H3, A1–A3, and the phenomenal-theory gate are unused and remain NOT_CLOSED. Host A and Host B are not identified. Companion citations are documentary, not proof transfers. This source asserts neither registration nor companion wiring: wired=false; declaredPath remains null. No claim or status of a companion manuscript is updated. No Einstein equation is

derived, and no metric is derived from records. An example that satisfies a supplied Einstein source equation on a prescribed ansatz is host-compatibility bookkeeping, not Einstein dynamics and not Host C.

The three readings and the corrected programme

Three distinct targets

The phrase "dark matter travels through time while ordinary matter cannot" is divided into three readings.

1. Backward signalling to ordinary apparatus. An ordinary intervention produces a controllable ordinary record outside the future allowed by the complete propagation model. On a host with a common temporal function, the causal loop version is excluded below. On a chronology-violating host, electromagnetic silence alone is insufficient to exclude gravitational or other indirect signalling.

2. Geometric chronology asymmetry. The dark sector uses a prescribed metric g admitting closed timelike curves, while the ordinary sector uses a different prescribed effective metric $\widehat{g}$ with no closed causal curves. A complete explicit witness, including an ordinary Cauchy foliation, is constructed below.

3. Capacity, echo, or register time travel. A phase echo, inaccessible register, cyclic label, or fixed-point equation is identified with a spacetime loop or a returning physical clock. That identification is not made. Within the printed product classes it does not follow.

There is no literal freezing of ordinary massive matter. A nonzero massive tangent cannot remain in a spacelike slice of its own Lorentzian metric.

Source discipline and relation to established work

The official Dual-Ledger-Temporal-A, version 5 (request t504u), and the official Mobility record, version 5 (request t495u), distinguish conformal transport, disformal cones, and physical acquisition [2, 3]. The present sheet re-proves its own identities and does not import their timelike-gradient theorems into a spacelike winding construction. Official Mobility-A is cited only as a named comparison of method for switch-action closure below;

no proof is transferred.

The established disformal framework is due to Bekenstein [13]. Classical chronology and horizon restrictions are discussed by Hawking and by Hawking and Ellis [14, 15]. The finite CTC rules used here are the distinct proposals of Deutsch and Lloyd et al. [18, 19]; they are not derived from the geometry. A standard homogeneous rotating-dust comparison is due to Gödel [16]; it is used below only as a printed host-compatibility calculation.

Cheung's clock, support, echo, source-split, and access records are pertinent documentary comparisons [4, 5, 6, 7, 9, 10, 11]. The supplied Host-C excerpts do not supply an operationally derived metric or a verified acquisition device [8]. Neither a manuscript title nor a shared symbol identifies two model classes.

Official Dual-Ledger-Chrono-DM-B version 4 (request t509u) is a documentary neighbour, cited only as a source record and not as a theorem premise [12]. It did not produce a winding-inclusive residual-free Einstein match: its admitted branches all carry $\Lambda = -\frac{1}{2}\beta^2$, none is winding-inclusive, and the official residuals of Propositions and are not withdrawn.

What is completed and what is corrected

The programme is completed as a set of exact constructions, conditional results, and explicit obstructions. Several proposed conclusions are false without additional hypotheses. In particular:

• a temporal coordinate on a finite open collar need not be a Cauchy time;

• a timelike azimuthal circle need not be a geodesic;

• electromagnetic silence is not complete dynamical isolation;

• compact horizon generation alone does not imply negative null curvature;

• a factorized dark CTC does not remove controller dependence already present in the ordinary-controller dynamics;

• an unspecified interacting portal does not automatically clone or distinguish states. The replacement statements

and counterexamples are part of the mathematical release, not omissions concealed by terminology. Version 2 retains those corrections and adds only locally proved extensions, together with explicit refusals where a proposed brief formula failed a local calculation. Version 3 likewise retains the Version 2 typestate and adds only locally checked extensions, together with explicit refusals where a proposed Version 3 formula failed a local calculation or would have overclaimed a global, dynamical, or continuum-derived status. Version 4 retains every Version 3 addition and refusal, and adds only locally checked extensions, together with explicit refusals where a proposed Version 4 formula failed a local calculation or would have overclaimed a retarded, observational, or Bar 3 status.

A shift-symmetric winding action and its exchange ledger

The controller is globally a closed form

We use signature $(-+++)$ and units $c = \hbar = 1$. A smooth time-oriented host (M, g) is supplied. It is not assumed to possess a temporal function in the chronology-violating branch.

Definition (Winding controller class). Fix a closed one-form q_0 on M. Admissible controller forms are

$$q = q_0 + \mathrm{d}\eta, \qquad \eta \in C^\infty(M; \mathbb{R}),$$

with variations $\delta q = \mathrm{d}(\delta\eta)$ of compact support. The cohomology class is fixed during variation.

Equivalently, when its periods are integral multiples of a supplied target circumference, q is the differential of a circle-valued scalar Θ. Local lifts of Θ differ by constants. The action depends on q, not on a chosen lift.

An expression such as $\kappa\,\mathrm{d}\varphi$ is a global one-form on a circle. It is not the differential of a globally single-valued real function $\kappa\varphi$.

Let s be a prescribed scalar switch and $B(s) \geq 0$ a smooth coefficient. Set

$$\tilde{g}_{ab} = g_{ab} + B(s) q_a q_b, \qquad \sigma = g^{ab} q_a q_b, \qquad 1 + B\sigma > 0.$$

No timelike-gradient restriction is imposed. The chronology witness will have $\sigma > 0$.

The action is

$$S_{\mathrm{m}} = S_{\mathrm{o}}[\tilde{g}, \psi_{\mathrm{o}}, \mathscr{A}] + S_{\mathrm{d}}[g, \psi_{\mathrm{d}}] + S_{\Theta}[g, q],$$

$$S_{\Theta} = -\frac{1}{2}\int_{M} g^{ab} q_a q_b \,\mathrm{dvol}_g.$$

The dark action has no direct dependence on $\mathscr{A}, s, \Theta$, or ordinary fields. This does not remove its host-metric coupling.

Actions can be restricted to regular relatively compact regions. Variations are compactly supported. The ordinary matter shell includes electromagnetic equations when that field is dynamical.

Lemma (Inverse, volume, and cone containment). *Under ,*

$$\tilde{g}^{ab} = g^{ab} - \frac{B}{1 + B\sigma} q^a q^b,$$

$$\frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} = \sqrt{1 + B\sigma}.$$

The metric is Lorentzian. Every nonzero $\tilde{g}$-causal vector is g-causal. If a common timelike vector is used to align orientations, the future ordinary cone is contained in the future host cone.

Proof. Multiplication verifies . The matrix determinant lemma gives $\det \tilde{g} = (1 + B\sigma)\det g$.

For $0 \le b \le B$, the matrices $g + bq \otimes q$ are nonsingular: if $\sigma < 0$, the final strict inequality implies the intermediate ones; if $\sigma \ge 0$, they hold automatically. Signature therefore remains Lorentzian.

Finally,

$$g(v, v) = \tilde{g}(v, v) - Bq(v)^2 \le 0$$

on the ordinary causal cone. Its future component is the host future component under the stated common orientation.

The orientation qualification is necessary: a determinant does not, by itself, select a time orientation.

Local action variation

Define the host stresses and Euler derivatives by

$$\delta S_{\mathrm{m}} = \int_M \Big(\frac{1}{2} T^{ab}_{\mathrm{tot}} \delta g_{ab} + E_\Theta \delta\eta + E_s \delta s \Big) \mathrm{dvol}_g + \text{matter Euler terms.}$$

Let $\widehat{T}^{ab}$ be ordinary stress obtained by variation of $\widetilde{g}_{ab}$, and put

$$\mathscr{T}^{ab} = \sqrt{1 + B\sigma}\, \widehat{T}^{ab}.$$

Theorem (Winding action source and three-slot switching ledger). *For the action ,*

$$T^{ab}_{\mathrm{o}} = \mathscr{T}^{ab},$$

$$Q := E_{\Theta,\mathrm{o}} = -\nabla_a (B \mathscr{T}^{ab} q_b),$$

$$E_s = \frac{1}{2} B_{,s} \mathscr{T}^{ab} q_a q_b,$$

$$T^{ab}_{\Theta} = q^a q^b - \frac{1}{2} g^{ab} \sigma.$$

On the ordinary and dark matter shells,

$$\nabla_a T^{ab}_{\mathrm{o}} = Q q^b + E_s \nabla^b s,$$

$$\nabla_a T^{ab}_{\mathrm{d}} = 0,$$

$$\nabla_a T^{ab}_{\Theta} = (\nabla_a q^a) q^b.$$

The full controller equation is

$$\nabla_a (q^a - B \mathscr{T}^{ab} q_b) = 0.$$

On that shell,

$$\nabla_a T^{ab}_{\mathrm{tot}} = E_s \nabla^b s, \qquad T_{\mathrm{tot}} = T_{\mathrm{o}} + T_{\mathrm{d}} + T_\Theta.$$

These identities hold in each fixed winding class.

Proof. At fixed q, s, $\delta \tilde{g}_{ab} = \delta g_{ab}$. At fixed g, s,

$$\delta_\eta \tilde{g}_{ab} = B(q_a \nabla_b \delta\eta + q_b \nabla_a \delta\eta).$$

Substitute into $\delta S_o = \frac{1}{2}\int \mathscr{T}^{ab} \delta \tilde{g}_{ab} \mathrm{dvol}_g$. Symmetry and integration by parts give . Algebraic switch variation gives . Canonical metric variation gives .

Because $\mathrm{d}q = 0$, $\nabla_a q_b = \nabla_b q_a$. Differentiating T_Θ therefore cancels the two terms containing these derivatives and proves .

For a compactly supported vector field ξ, Cartan's formula gives

$$\mathscr{L}_\xi q = \mathrm{d}(\xi^a q_a).$$

Thus the diffeomorphism variation is an admissible exact variation, even when q is not exact. Use $\delta\eta = \xi^a q_a$, $\delta s = \xi^a \nabla_a s$, and $\delta g_{ab} = 2\nabla_{(a}\xi_{b)}$. Covariance and integration by parts yield the ordinary divergence identity. The dark identity follows without controller and switch sources. Controller variation of S_Θ contributes $\nabla_a q^a$, giving . The two controller sources then cancel in the sum.

Corollary (Source compatibility is an additional requirement). *If one separately stipulates*

$$M_{\mathrm{P}}^2 (G^{ab} + \Lambda g^{ab}) = T^{ab}_{\mathrm{tot}} + T^{ab}_{\mathrm{ext}},$$

then

$$\nabla_a T^{ab}_{\mathrm{ext}} = -E_s \nabla^b s$$

is necessary. A matter action on a prescribed host does not automatically solve .

Proof. Apply the contracted Bianchi identity to and use .

The action and the geometry are independently stipulated. No Einstein equation is derived from records.

A printed switch kinetic slot, and the un-enlarged remainder

The inherited identity leaves an external-switch source $E_s\nabla^b s$. The following printed kinetic action supplies a controller slot whose on-shell divergence cancels that source. This is the same closure discipline as in official Mobility-A, used here only as a named comparison of method, not as a proof transfer [3].

Definition (Canonical switch kinetic action). Let F_s be a constant and W a smooth potential. On the same prescribed host, set

$$S_s = -\frac{F_s^2}{2}\int_M g^{ab}\nabla_a s\nabla_b s\,\mathrm{dvol}_g - \int_M W(s)\,\mathrm{dvol}_g.$$

The associated host stress is

$$T_s^{ab} = F_s^2\nabla^a s\nabla^b s - g^{ab}(\frac{F_s^2}{2}g^{cd}\nabla_c s\nabla_d s + W(s)).$$

Theorem (Enlarged switch-shell exchange closure). *For the action $S_{\mathrm{m}} + S_s$, with S_{m} as in ,*

$$\nabla_a T_s^{ab} = (F_s^2\mathrm{Box}_g s - W'(s))\nabla^b s,$$

and the total Euler derivative with respect to s is

$$E_s^{\mathrm{tot}} = \frac{1}{2}B_{,s}\mathscr{T}^{ab}q_a q_b + F_s^2\mathrm{Box}_g s - W'(s).$$

On the joint ordinary, dark, controller, and switch shells,

$$\nabla_a(T_{\mathrm{o}}^{ab} + T_{\mathrm{d}}^{ab} + T_{\Theta}^{ab} + T_s^{ab}) = 0.$$

The identity holds at every event, including those at which $B(s(x))$ varies spatially. If the independent kinetic slot is omitted, the un-enlarged remainder is recovered.

Proof. Varying S_s with respect to s and integrating by parts gives the Euler density $F_s^2\mathrm{Box}_g s - W'(s)$. Metric variation gives . Differentiating that stress, using $\nabla_a\nabla^b s = \nabla^b\nabla_a s$ and

$$\nabla^b(g^{cd}\nabla_c s\nabla_d s) = 2\nabla^a s\nabla_a\nabla^b s,$$

cancels the second-derivative terms and yields . Adding the matter Euler density produces . On the switch shell one has $E_s^{\mathrm{tot}} = 0$, so the sum of and is . The controller and matter shells are those of Theorem . Dropping S_s removes T_s and restores .

This is action bookkeeping on a prescribed host and a printed switch. It does not solve the Einstein equation, does not manufacture a chronology-violating region, and does not derive the switch from records. The field *s* remains a supplied degree of freedom with a printed Euler equation, not an operationally extracted scalar.

A complete chronology-split witness and the collar correction

The correct local collar calculation

Consider the prescribed stationary collar

$$M_{\mathrm{col}} = \mathbb{R}_t \times (r_1, r_2) \times S^1_\varphi \times \mathbb{R}_z$$

with

$$g = -\mathrm{d}t^2 - 2a(r)\mathrm{d}t\,\mathrm{d}\varphi + c(r)\mathrm{d}\varphi^2 + \mathrm{d}r^2 + \mathrm{d}z^2, \qquad F(r) := a(r)^2 + c(r) > 0.$$

Take $q = \kappa \mathrm{d}\varphi$, $\kappa > 0$, and constant $B \geq 0$; write $b = B\kappa^2$.

Proposition (Temporal ordinary coordinate and circular acceleration). *If $c + b > 0$, then*

$$\det \tilde{g}_{(t,\varphi)} = -(F + b) < 0, \qquad \tilde{g}^{tt} = -\frac{c + b}{F + b} < 0.$$

Thus t is an ordinary temporal function, with orientation chosen so that it increases on ordinary future curves.

If $c < 0$, the azimuthal circles are host-timelike. Their normalized tangent $u = (-c)^{-1/2}\partial_\varphi$ has acceleration

$$\nabla_u u = \frac{\nabla c}{2c}.$$

Consequently these circles are geodesics precisely where $\nabla c = 0$, not merely where $c < 0$.

The winding covector is host-spacelike:

$$g^{-1}(q,q) = \frac{\kappa^2}{F} > 0.$$

Proof. The ordinary block is $\begin{pmatrix} -1 & -a \\ -a & c+b \end{pmatrix}$. Its determinant and inverse give . For the Killing field $K = \partial_\varphi$, the Killing equation gives $\nabla_K K = -\frac{1}{2}\nabla g(K,K) = -\frac{1}{2}\nabla c$. Since $K(c) = 0$, normalizing produces . The host inverse has $g^{\varphi\varphi} = 1/F$, giving .

This spacelike form is not a normalized timelike controller gradient. The timelike-gradient class of the supplied Temporal sheet therefore cannot be imported unchanged.

Proposition (A finite open collar need not be globally hyperbolic). *Temporalness of t in Proposition does not make it a Cauchy time on a finite open radial collar. For the metric , the missing finite radial endpoints already prevent global hyperbolicity.*

Proof. At fixed φ, z, radial null curves satisfy $\mathrm{d}t = \pm\mathrm{d}r$. A future null curve can reach r_2 in finite t, with no endpoint in the manifold. It is future-inextendible there but has bounded t.

For a compactness witness, choose $r_0 \in (r_1, r_2)$, $T > r_2 - r_0$, and

$$p = (-T, r_0, \varphi_0, z_0), \qquad q = (T, r_0, \varphi_0, z_0).$$

The sequence $(0, r_j, \varphi_0, z_0)$, $r_j$uparrowr_2, lies in the ordinary causal diamond by radial propagation and waiting on the timelike stationary worldline. It has no convergent subsequence in the collar.

The support geometry of an apex-excluded cone has an analogous missing-boundary issue, but its Green operators and extensions are not transferred here [5].

An explicit complete replacement

Assumption (Complete cylindrical quotient). Let

$$M = \mathbb{R}_t \times \mathbb{R}_x \times S^1_\varphi \times \mathbb{R}_z, \qquad \varphi \sim \varphi + 2\pi,$$

and choose constants

$$a > 0, \qquad 0 < h < a^2, \qquad \kappa > 0, \qquad b = B\kappa^2 > h.$$

Prescribe

$$g = -\mathrm{d}t^2 - 2a\mathrm{d}t\,\mathrm{d}\varphi - h\mathrm{d}\varphi^2 + \mathrm{d}x^2 + \mathrm{d}z^2,$$

$$q = \kappa \mathrm{d}\varphi,$$

$$\tilde{g} = g + Bq \otimes q = -\mathrm{d}t^2 - 2a\mathrm{d}t\,\mathrm{d}\varphi + H\mathrm{d}\varphi^2 + \mathrm{d}x^2 + \mathrm{d}z^2, \qquad H = b - h > 0.$$

Both time orientations are aligned with ∂_t.

The coordinate x is a complete transverse coordinate, not a claim about an astrophysical radius.

Theorem (Complete bimetric chronology split). *Under Assumption :*

1. *both metrics are smooth, flat, and geodesically complete;*

2. *the curves at fixed t, x, z, with increasing φ, are closed future-directed host-timelike geodesics, with normalized tangent $u = h^{-1/2}\partial_\varphi$ and proper period*

$$L = 2\pi\sqrt{h};$$

3. $(M, \tilde{g})$ *is globally hyperbolic, and every level set of t is Cauchy;*

4. *positive test dust*

$$T_{\mathrm{d}}^{ab} = \rho(t, x, z) u^a u^b, \qquad \rho > 0,$$

is host-conserved;

5. *q is a smooth non-exact closed form satisfying $\nabla_a q^a = 0$, with*

$$\oint_{S^1} q = 2\pi\kappa, \qquad \sigma = \frac{\kappa^2}{a^2 - h} > 0.$$

With zero ordinary background stress, the controller and dark fields solve their action equations on this prescribed host. Ordinary clocks and photons can be used as negligible test systems. No Einstein source

compatibility is asserted.

Proof. On the universal cover, all metric components are constant. The connection and curvature vanish. Geodesics lift to affine lines on the cover and extend for all affine parameter values. The circle identification is a translation isometry, so the quotient is also geodesically complete.

For $u = h^{-1/2}\partial_\varphi$, $g(u,u) = -1$, $\nabla_u u = 0$, and $g(u,\partial_t) = -a/\sqrt{h} < 0$. One revolution has proper length L, proving (ii).

Complete the square in the ordinary metric:

$$\tilde{g} = -\left(1 + \frac{a^2}{H}\right)\mathrm{d}t^2 + H\left(\mathrm{d}\varphi - \frac{a}{H}\mathrm{d}t\right)^2 + \mathrm{d}x^2 + \mathrm{d}z^2.$$

The global coordinates

$$T = \sqrt{1 + a^2/H}\, t, \qquad \theta = \varphi - \frac{a}{H}t \bmod 2\pi$$

give

$$\tilde{g} = -\mathrm{d}T^2 + \mathrm{d}x^2 + H\mathrm{d}\theta^2 + \mathrm{d}z^2.$$

Its spatial manifold $\mathbb{R}^2 \times S^1$ is complete. A future causal curve parametrized by T has spatial speed at most one. If its future T-range were bounded, its spatial path would have finite remaining length and hence a limit in that complete spatial manifold. It could then be continued, contradicting inextendibility. The same argument applies in the past. Thus every inextendible causal curve crosses each T-slice exactly once. These are precisely the t-slices.

For completeness, causal diamonds are compact: their spatial coordinates lie in a closed bounded spatial ball, which is compact in $\mathbb{R}^2 \times S^1$, and their time range is compact. The causal relation is closed, as follows by taking limits of the uniformly Lipschitz spatial causal paths. Hence the diamonds are closed subsets of a compact set.

The vector u has constant components and zero divergence. Since $u(\rho) = 0$,

$$\nabla_a(\rho u^a u^b) = \{u(\rho) + \rho\nabla_a u^a\}u^b + \rho\nabla_u u^b = 0.$$

Finally q has constant components, so its divergence vanishes. Its nonzero circle integral excludes exactness. With

$\mathscr{T} = 0$, equation is therefore satisfied.

Remark (What has and has not been constructed). The host quotient identification already contains the dark closed geodesics. The winding coupling narrows ordinary propagation so that ordinary causal curves cannot follow them. This is an explicit TM-2 witness, not a procedure creating a CTC from chronology-preserving laboratory initial data. The ordinary field equations on $\hat{\sim}{g}$ can have a Cauchy formulation; this does not supply a Cauchy formulation for all fields using g.

The witness fails positive-source Einstein matching

Proposition (Exact null residual of the flat witness). *In Theorem , with positive dark test dust and the canonical winding stress, no equation*

$$M_{\mathrm{P}}^2(G_{ab} + \Lambda g_{ab}) = T_{\mathrm{d},ab} + T_{\Theta,ab}$$

holds for any constant Λ. An added residual tensor would have

$$T_{\mathrm{res},ab}k^a k^b = -\rho(u\cdot k)^2 - (q\cdot k)^2 < 0$$

for every nonzero host-null k.

Proof. Flatness gives $G = 0$; the cosmological term has zero null contraction. Positive dust contributes $\rho(u\cdot k)^2$, which is strictly positive because a nonzero null vector cannot be orthogonal to a timelike vector. The canonical scalar contributes $(q\cdot k)^2$. Subtracting yields .

Positive conserved test matter is therefore not an exotic-free, self-gravitating realization. Assigning a residual does not construct its dynamics.

A standard Gödel comparison, and the winding residual

The flat residual of Proposition is not withdrawn. A curved named comparison may be admitted only when the Einstein tensor of a printed metric is computed in full. The following unit-radius form is the standard homogeneous Gödel metric, not a newly derived host [16].

Assumption (Unit-radius Gödel comparison). On $M_{\mathrm{G}} = \mathbb{R}_t \times \mathbb{R}_x \times \mathbb{R}_\psi \times \mathbb{R}_z$, prescribe

$$g_{\mathrm{G}} = -\mathrm{d}t^2 - 2e^x\,\mathrm{d}t\,\mathrm{d}\psi - \frac{1}{2}e^{2x}\,\mathrm{d}\psi^2 + \mathrm{d}x^2 + \mathrm{d}z^2,$$

and write $u = \partial_t$. No compactification of ψ is imposed in this assumption.

Proposition (Exact dust-plus-negative-Λ identity). *Under Assumption the Einstein tensor of g_{G} satisfies*

$$G_{ab} + \Lambda g_{\mathrm{G},ab} = u_a u_b, \qquad \Lambda = -\frac{1}{2}.$$

Equivalently, $M_{\mathrm{P}}^2(G_{ab} + \Lambda g_{\mathrm{G},ab}) = \rho_{\mathrm{d}} u_a u_b$ with $\rho_{\mathrm{d}} = M_{\mathrm{P}}^2$ and the same Λ. The cosmological term is strictly negative. This is host-compatibility bookkeeping on a prescribed ansatz, not Einstein dynamics, not a derived metric, and not Host C.

Proof. The only nonzero inverse components are

$$g^{tt} = 1, \qquad g^{t\psi} = -2e^{-x}, \qquad g^{\psi\psi} = 2e^{-2x}, \qquad g^{xx} = g^{zz} = 1,$$

and $\det g_{\mathrm{G}} = -\frac{1}{2}e^{2x}$. The only nonzero Christoffel symbols, up to symmetry, are

$$\Gamma^t{}_{tx} = 1, \qquad \Gamma^t{}_{x\psi} = \frac{1}{2}e^x, \quad \Gamma^x{}_{t\psi} = \frac{1}{2}e^x,$$

$$\Gamma^x{}_{\psi\psi} = \frac{1}{2}e^{2x}, \; \Gamma^\psi{}_{tx} = -e^{-x}.$$

Substitution into the Ricci formula yields

$$R_{tt} = 1, \qquad R_{t\psi} = e^x, \qquad R_{\psi\psi} = e^{2x},$$

and vanishing remaining components, with scalar curvature $R = -1$. Therefore

$$G_{tt} = \tfrac{1}{2}, \qquad G_{t\psi} = \tfrac{1}{2}e^x, \; G_{xx} = \tfrac{1}{2},$$

$$G_{\psi\psi} = \tfrac{3}{4}e^{2x}, \; G_{zz} = \tfrac{1}{2}.$$

Adding Λg_{G} with $\Lambda = -\frac{1}{2}$ cancels every component except the combinations that reconstruct $u_a u_b$, since $u_t = -1$ and

$u_\psi = -e^x$.

Proposition (Canonical winding residual on the printed Gödel comparison). *Let $q = \kappa\,\mathrm{d}\psi$ with $\kappa \neq 0$, and let $T_{\Theta_}ab = q_a q_b - \frac{1}{2} g_{\mathrm{G},ab}\sigma$ be the canonical winding stress, with $\sigma = g_{\mathrm{G}}^{-1}(q,q) = 2\kappa^2 e^{-2x} > 0$. No choice of constants Λ and ρ_{d} satisfies*

$$M_{\mathrm{P}}^2(G_{ab} + \Lambda g_{\mathrm{G},ab}) = \rho_{\mathrm{d}} u_a u_b + T_{\Theta_}ab$$

on the whole of M_{G}. If the classical Gödel normalisation $\Lambda = -\frac{1}{2}$, $\rho_{\mathrm{d}} = M_{\mathrm{P}}^2$ is retained, the residual

$$T_{\mathrm{res},ab} := M_{\mathrm{P}}^2(G_{ab} + \Lambda g_{\mathrm{G},ab}) - \rho_{\mathrm{d}} u_a u_b - T_{\Theta_}ab$$

equals $-T_{\Theta_}ab$, with independent components

$$T_{\mathrm{res},tt} = -\kappa^2 e^{-2x}, \qquad T_{\mathrm{res},t\psi} = -\kappa^2 e^{-x},$$

$$T_{\mathrm{res},xx} = T_{\mathrm{res},zz} = \kappa^2 e^{-2x},\ T_{\mathrm{res},\psi\psi} = -\tfrac{3}{2}\kappa^2.$$

For every nonzero host-null k,

$$T_{\mathrm{res},ab} k^a k^b = -(q \cdot k)^2 \leq 0,$$

and the contraction is strictly negative wherever $q \cdot k \neq 0$. In particular a null-projection identity of the form $G_{ab}k^a k^b = M_{\mathrm{P}}^{-2}[\rho_{\mathrm{d}}(u \cdot k)^2 + (q \cdot k)^2]$ does not hold under this normalisation, and is in any case not a full tensor Einstein equation.

Proof. The identity of Proposition leaves $T_{\mathrm{res}} = -T_\Theta$ once the winding stress is subtracted. The listed components follow by substituting $\sigma = 2\kappa^2 e^{-2x}$ and $q_t = 0$, $q_\psi = \kappa$. For a constant retuning, the Einstein, cosmological, and dust terms are linear combinations of $g_{\mathrm{G},ab}$ and $u_a u_b$. Their (x,x) component is then a constant, whereas $T_{\Theta_}xx = -\kappa^2 e^{-2x}$ is nonconstant for $\kappa \neq 0$. Hence no constant pair $(\Lambda, \rho_{\mathrm{d}})$ cancels the winding stress everywhere. On a host-null vector the cosmological term drops, the Gödel dust identity gives $G_{ab}k^a k^b = (u \cdot k)^2$, and the canonical winding contributes $(q \cdot k)^2$, proving at the classical normalisation.

A negative cosmological term is not "strictly positive-source" language. Exact Einstein compatibility of positive dark dust together with a non-trivial canonical winding stress therefore remains residual on this printed comparison, and is not obtained by adding T_{Θ} to the classical Gödel source.

If a constant ordinary coefficient $B \geq 0$ is adjoined, $\hat{g}_{\mathrm{G}} = g_{\mathrm{G}} + Bq \otimes q$ has

$$\hat{g}_{\psi\psi} = B\kappa^2 - \frac{1}{2}e^{2x}.$$

This component fails to remain positive on the whole x-line. Ordinary global hyperbolicity of the overlay on all of M_{G} is therefore not obtained. Restricting to a half-space or a bounded-x slab reintroduces the incomplete-collar obstruction of Proposition . If ψ is compactified, the resulting quotient is a different prescribed manifold, not supplied here as a complete ordinary-Cauchy witness. The comparison does not advance the time-machine score.

Hyperbolicity typestate of the cylindrical host operator

Proposition (Host inverse and a globally defined timelike covector). *Under Assumption , the host inverse in the* (t, φ) *block is*

$$g^{tt} = \frac{h}{a^2 - h} > 0, \qquad g^{t\varphi} = -\frac{a}{a^2 - h}, \qquad g^{\varphi\varphi} = \frac{1}{a^2 - h},$$

together with $g^{xx} = g^{zz} = 1$. *Thus* $\mathrm{d}t$ *is host-spacelike, and t-evolution is not a standard hyperbolic Cauchy problem for* Box_g. *The same holds for* $\mathrm{d}x$ *and for* $\mathrm{d}z$. *The globally defined closed covector* $\xi = \mathrm{d}t + a\,\mathrm{d}\varphi$ *is host-timelike:*

$$g^{-1}(\xi, \xi) = -1 < 0.$$

Consequently $\mathrm{Box}_g - m_{\mathrm{d}}^2$ *is a strictly hyperbolic operator relative to* ξ. *The coordinate function* $t + a\varphi$ *is not single-valued on the quotient, so this principal-symbol identity is not a global Cauchy theorem, not a Leray–Ohya estimate, and not a contraction mapping in any Sobolev space* H^s.

Proof. The (t, φ) block of has determinant $h - a^2 = -(a^2 - h)$ and the stated inverse. Hence $g^{-1}(\mathrm{d}t, \mathrm{d}t) > 0$ and $g^{-1}(\mathrm{d}x, \mathrm{d}x) = 1 > 0$. Completing the square in the metric gives

$$g = -(\mathrm{d}t + a\,\mathrm{d}\varphi)^2 + (a^2 - h)\,\mathrm{d}\varphi^2 + \mathrm{d}x^2 + \mathrm{d}z^2,$$

so $g^{-1}(\xi,\xi) = -1$. Directly,

$$g^{tt} + 2ag^{t\varphi} + a^2 g^{\varphi\varphi} = \frac{h - 2a^2 + a^2}{a^2 - h} = \frac{h - a^2}{a^2 - h} = -1.$$

Strict hyperbolicity of the principal symbol $g^{ab}k_a k_b$ relative to a covector with negative norm is then the standard algebraic condition. The one-form ξ is smooth and closed on the quotient, but its local primitives shift by $2\pi a$ around the φ-circle and do not define a global time function, consistently with the existence of host CTCs.

No global solution-selection or stability theory for dark fields on the chronology-violating host is thereby obtained. The following mode analysis sharpens the same principal-symbol typestate; it is not a Cauchy theorem, not a Leray–Ohya estimate, and not a compactified Fredholm argument. Finite Deutsch and postselected rules remain separately assumed toy rules; they are not derived here as boundary-condition limits of a continuum detector or path-integral model. A named comparison to the classical self-consistency literature is recorded in Appendix only as an unfinished programme [17].

Lemma (Dispersion locus of the cylindrical host operator). *Under Assumption , write* $F := a^2 - h > 0$. *The operator* $\mathrm{Box}_g - m_{\mathrm{d}}^2$ *has constant coefficients, and in the completed-square coordinates* $\tau = t + a\varphi$, $\chi = \sqrt{F}\,\varphi$ *of the proof of Proposition it is the flat wave operator* $-\partial_\tau^2 + F^{-1}\partial_\varphi^2 + \partial_x^2 + \partial_z^2 - m_{\mathrm{d}}^2$ *on the simply connected cover. For a quotient mode*

$$\psi = e^{-i\omega t + in\varphi + ik_x x + ik_z z}, \qquad n \in \mathbb{Z},$$

the symbol is

$$\sigma(\omega, n, k_\perp) = \omega^2 - \frac{(a\omega + n)^2}{F} - k_\perp^2 - m_{\mathrm{d}}^2,$$

where $k_\perp^2 := k_x^2 + k_z^2$. *Equivalently,* $\sigma = R(\omega, n) - k_\perp^2 - m_{\mathrm{d}}^2$ *with the completed temporal remainder*

$$R(\omega, n) = \frac{n^2}{h} - \frac{h}{F}\left(\omega + \frac{an}{h}\right)^2.$$

Real on-shell momenta $k_\perp \in \mathbb{R}^2$ exist if and only if $R(\omega, n) \geq m_{\mathrm{d}}^2$. In particular:

1. *if $n = 0$, then $R(\omega, 0) = -(h/F)\omega^2 \leq 0$, so a massive mode is never on-shell for real ω, while a massless mode is on-shell only at $\omega = k_\perp = 0$;*

2. *if $n \neq 0$, then R attains a maximum n^2/h at $\omega = -an/h$, and the real-frequency on-shell interval is nonempty if and only if $|n| \geq m_{\mathrm{d}}\sqrt{h}$. Single-valuedness on the quotient is the integer condition $n \in \mathbb{Z}$; equivalently, in cover frequencies (Ω, ν) conjugate to (τ, φ), one has $\Omega = \omega$ and $\nu - a\Omega \in \mathbb{Z}$.*

Proof. Christoffel symbols vanish, so $\mathrm{Box}_g = g^{ab}\partial_a\partial_b$. Substituting the inverse and the displayed Fourier mode produces

$$\sigma = -\frac{h}{F}\omega^2 - \frac{2a}{F}\omega n - \frac{n^2}{F} - k_\perp^2 - m_{\mathrm{d}}^2,$$

which rearranges to . Completing the square in ω yields . The sign and maximum of R are then elementary. The cover coordinates of Proposition give $g = -\mathrm{d}\tau^2 + F\,\mathrm{d}\varphi^2 + \mathrm{d}x^2 + \mathrm{d}z^2$, hence the stated flat wave operator, and the monodromy $\psi(\tau + 2\pi a, \varphi + 2\pi) = \psi(\tau, \varphi)$ is the integer condition already imposed by $e^{in\varphi}$.

Proposition (Off-resonant mode resolvent and resonant kernel typestate). *Fix $n \in \mathbb{Z}$ and a real frequency ω.*

1. *Off the resonant locus, i.e. when $R(\omega, n) < m_{\mathrm{d}}^2$, one has $|\sigma(\omega, n, k_\perp)| \geq m_{\mathrm{d}}^2 - R(\omega, n) > 0$ for every real $k_\perp$. The Fourier multiplier $1/\sigma(\omega, n, k_\perp)$ is therefore bounded, and defines a unique temperate solution of $(\mathrm{Box}_g - m_{\mathrm{d}}^2)\psi = f$ in the (x, z)-variables for that single mode, namely convolution against the Yukawa kernel of mass $\sqrt{m_{\mathrm{d}}^2 - R(\omega, n)}$.*

2. *On the resonant locus $R(\omega, n) > m_{\mathrm{d}}^2$, there is a real circle of homogeneous plane-wave kernel modes with $k_\perp^2 = R - m_{\mathrm{d}}^2$. At the threshold $R = m_{\mathrm{d}}^2$ the kernel is the zero-momentum mode $k_\perp = 0$. These modes are classical self-consistency constraints on that Fourier class: a source mode on the locus must satisfy the corresponding solvability condition, or else a radiation or $i\varepsilon$ prescription must be declared separately. This is a mode analysis of the same principal-symbol typestate as Proposition , which is not withdrawn. It is not a global Green function on the chronology-violating quotient, not a Cauchy theorem for t- or x-evolution, not a Leray–Ohya or compactified*

Fredholm theorem, and not a derivation of a Deutsch or postselected finite rule.

Proof. Item (i) is the algebraic bound $\sigma = R - k_\perp^2 - m_{\mathrm{d}}^2 \le R - m_{\mathrm{d}}^2 < 0$, so the spatial symbol in (x, z) is a strictly negative constant plus $-|k_\perp|^2$, whose inverse Fourier transform is the standard Yukawa kernel on $\mathbb{R}^2_{x,z}$. Item (ii) is the same identity on the locus where that constant is nonnegative. No additional selection rule is introduced, and the typestate of Proposition is unchanged: dt and dx remain host-spacelike.

Topological narrowing and the closed-clock obstruction

Theorem (Closed-orbit Rolle obstruction). *Let $\gamma : S^1 \to M$ be a smooth regular closed host-timelike curve, and let*

$$\tilde{g} = A^2 g + B\,\mathrm{d}\varphi \otimes \mathrm{d}\varphi, \qquad A > 0,$$

where $\varphi : M \to \mathbb{R}$ is a single-valued smooth scalar. If $\varphi \circ \gamma$ is nonconstant, there are at least two distinct points of the circle where

$$\mathrm{d}\varphi(\dot{\gamma}) = 0.$$

If it is constant, this equality holds everywhere. At every such point, γ is also $\tilde{g}$-timelike.

More generally, if a continuous closed form ψ narrows this entire tangent family to non-timelike tangents through

$$A^2 g(\dot{\gamma}, \dot{\gamma}) + B\psi(\dot{\gamma})^2 \ge 0, \qquad B \ge 0,$$

then $\psi(\dot{\gamma})$ is nowhere zero and $\oint_\gamma \psi \neq 0$. In particular ψ is not exact.

Proof. A nonconstant smooth real function on a compact circle attains a maximum and a minimum at distinct points. Its derivative vanishes there. At a zero,

$$\tilde{g}(\dot{\gamma}, \dot{\gamma}) = A^2 g(\dot{\gamma}, \dot{\gamma}) < 0.$$

For uniform narrowing, the strictly negative host term requires $\psi(\dot{\gamma}) \neq 0$ everywhere. Continuity on the connected

circle makes its sign constant; its integral is therefore nonzero. Every exact form has zero integral on a closed curve.

Remark (The precise necessity statement). The theorem obstructs uniform narrowing of the specified closed tangent family by an exact rank-one controller. It is not a proof that every exact disformal deformation retains a complete closed timelike curve, nor a classification of all ways to obtain a causal ordinary metric. Removing timelikeness at some points and eliminating every closed curve are different global questions.

Theorem (Translation-invariant homology of the Rolle obstruction). *Let*

$$M = \mathbb{R}_t \times \mathbb{R}_x \times S^1_\varphi \times \mathbb{R}_z,$$

and let g be a smooth Lorentzian metric invariant under the translations of (t, x, z)*, with every φ-circle host-timelike. Let*

$$\tilde{g} = A^2 g + B\,\alpha \otimes \alpha, \qquad A > 0, \qquad B \geq 0,$$

where α is a continuous closed one-form and A, B are continuous and likewise translation-invariant. Then:

1. *if even a single φ-circle is uniformly narrowed to* $\tilde{g}$*-non-timelike tangents, then* $\oint_{S^1} \alpha \neq 0$*, and every closed curve in the same homology class* $[S^1] \in H_1(M; \mathbb{R})$ *has the same nonzero period;*

2. *if α is exact, equivalently if* $[\alpha] = 0$ *in* $H^1(M; \mathbb{R})$*, then no φ-circle is uniformly narrowed, and there is no* C^1 *function* $T : M \to \mathbb{R}$ *that is strictly temporal for* $\tilde{g}$ *and constant on every φ-circle. The statement classifies only this translation-invariant family and those time functions that are constant on the family. It does not classify every deformed closed curve, and it does not apply to a general non-homogeneous host. The typestate of Remark is not withdrawn.*

Proof. The general clause of Theorem already yields $\oint_\gamma \alpha \neq 0$ as soon as a single closed host-timelike curve γ is uniformly narrowed. Because α is closed, Stokes' theorem on any smooth 2-chain with boundary $\gamma - \gamma'$ gives $\oint_\gamma \alpha = \oint_{\gamma'} \alpha$. Every closed curve in the class $[S^1]$ is homologous to a φ-circle, proving (i).

If α is exact then every period vanishes, so (i) forbids uniform narrowing of any φ-circle. In particular, Theorem supplies, on each such circle, at least two points at which the tangent is $\tilde{g}$-timelike. A C^1 function constant on every φ-circle has $\mathrm{d}T(\partial_\varphi) = 0$ at those points, and therefore cannot be strictly temporal for $\tilde{g}$.

Proposition (A closed geodesic does not carry a single-valued monotone elapsed-time field). *Let u generate a closed timelike orbit of proper period $L > 0$. There is no smooth single-valued real function $\tau : M \to \mathbb{R}$ satisfying $u(\tau) = 1$ around that orbit. The accumulated proper time on its lifted parameter line nevertheless increases by L per circuit.*

Proof. Integrate $u(\tau) = 1$ along one proper-time period:

$$\tau(\gamma(L)) - \tau(\gamma(0)) = L.$$

The geometric endpoints coincide, making the left side zero. On the lifted parameter line there is no endpoint identification, so its accumulated proper time is increasing.

A periodic clock phase can return when its frequency and period are commensurate. Such phase return does not return an unbounded age record. A retained archive with a strictly increasing rank cannot satisfy a seam fixed-point condition: the rank after a circuit would have to be both larger and equal.

Apparent anti-telephones and the actual isolation hypotheses

A coordinate reversal inside nested flat cones

Prescribe

$$g = -\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2, \qquad \tilde{g} = -c_o^2\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2, \qquad 0 < c_o < 1.$$

This is the homogeneous disformal case $q = \pi\,\mathrm{d}t$, $c_o^2 = 1 - B\pi^2 > 0$, with fixed coefficients. It is distinct from the spacelike winding branch.

Proposition (Ordinary synchronization can reverse a dark packet's coordinate order). *For a laboratory*

boost of coordinate speed $|u| < c_o$,

$$t' = \gamma_o\left(t - \frac{ux}{c_o^2}\right), \qquad x' = \gamma_o(x - ut), \qquad \gamma_o = (1 - u^2/c_o^2)^{-1/2}.$$

A host-future packet with $x = vt$, $c_o < v < 1$, *has*

$$\Delta t' = \gamma_o \Delta t(1 - uv/c_o^2).$$

It is negative exactly when $uv > c_o^2$. *Such choices exist for* $c_o^2 < u < c_o$ *and* $c_o^2/u < v < 1$.

Proof. The displayed transformation preserves $-c_o^2 dt^2 + dx^2$. Substitution of $\Delta x = v\Delta t$ gives the condition. The indicated intervals are nonempty because $c_o^2 < c_o < 1$.

The packet is spacelike for the ordinary metric but timelike for the host. The transformed clock coordinate is not a time function for all host-causal packets. Coordinate order reversal is not a causal cycle.

Theorem (Two-way no-paradox theorem with a common temporal function). *Suppose a smooth function T increases strictly on every nonzero future-directed causal tangent of all propagation sectors used in a protocol. Permit local conversions between sectors at common events. Then no finite concatenation of nonconstant future propagation segments can form a closed causal loop.*

In particular, if T is host-temporal and all sector cones are future-contained in the host cone, this conclusion holds, including with an active local portal that introduces no characteristics outside that host cone.

Proof. Every nonconstant segment has positive net increase of *T*. Local conversion at one event does not change *T* . The sum of the increases around a proposed loop is positive, whereas a closed loop has equal starting and ending *T*. Cone containment supplies the second statement.

This also applies to the nested homogeneous FLRW cones when the supplied cosmic coordinate *t* is temporal. The compact support of a portal is not the decisive condition: its characteristic cone and local event identification are. A portal with an independently prescribed nonlocal jump or wider cone is outside the theorem.

Electromagnetic silence is not an absolute firewall

Definition (Off-shell electromagnetic silence). A dark action is electromagnetically silent if its Euler derivative with respect to $\mathscr{A}$ vanishes for every allowed potential and every admitted dark configuration. For a potential-independent dark action this holds identically.

Proposition (Silence excludes only the declared direct channel). *Electromagnetic silence alone does not imply that ordinary apparatus cannot influence or detect the dark sector. Even electromagnetic and controller independence do not exclude an additional independently supplied neutral portal. Metric coupling remains present in* $S_d[g, \psi_d]$.

Proof. A neutral interaction

$$S_{\text{portal}} = -\lambda \int_M \chi_o \chi_d \, \text{dvol}_g$$

is independent of both $\mathscr{A}$ and the winding controller, but its dark Euler equation depends on χ_o. This is a direct counterexample when such a portal is admitted.

Without that portal, the geodesic equation still contains the host connection. A metric perturbation changes it through $\delta\Gamma^a{}_{bc} u^b u^c$. Thus, in a sourced gravitational theory, the dark metric coupling is not removed by setting its electromagnetic current to zero.

Proposition (Fixed-host direct isolation). *If the host is fixed, the complete matter action contains no cross-sector portal, the dark preparation and boundary or consistency data are held fixed, and the selected dark solution rule is independent of ordinary controls, then changing ordinary or controller inputs does not change the dark solution through the declared direct matter equations.*

Proof. Under these hypotheses the dark equation and all data used to select its solution are unchanged. The stated solution rule consequently returns the same solution.

The solution-selection qualification matters on a host without a global Cauchy surface. Neither this proposition nor electromagnetic silence proves a universal TM-1 exclusion on a chronology-violating, dynamically coupled

gravitational host.

Linearised chronology leakage under a declared cover closure

Electromagnetic silence remains insufficient for dynamical isolation, by Proposition . A quantitative leakage bound requires a declared linearised closure, and a chronology-violating host does not automatically supply a retarded Green function.

Assumption (Declared cover Poisson–wave closure). Let $(\widehat{M}, \eta)$ be the simply connected Lorentzian cover of the complete cylindrical witness of Assumption , so that η is flat Minkowski spacetime in the completed-square coordinates of the proof of Theorem . Let a compactly supported ordinary perturbation $\mathrm{deltaT}_{\mathrm{o}}^{ab}$ be lifted to $\widehat{M}$, and declare the linearised closure

$$\mathrm{Box}_{\eta}\,\overline{\delta h}_{ab} = -2M_{\mathrm{P}}^{-2}\mathrm{deltaT}_{\mathrm{o}}_ab$$

on the cover, together with the Minkowski retarded Green function as a supplied kernel. No claim is made that this kernel descends to a unique retarded propagator on the chronology-violating quotient, nor that the host admits a well-posed Cauchy problem in the coordinate t.

Proposition (Formal first-order holonomy increment on the cover). *Under Assumption , write $\delta\Gamma$ for the linearised host connection of $\delta\overline{h}$ on the cover, and let $\hat{\gamma}$ be a fixed-period lift of a background closed dark geodesic of proper period $L = 2\pi\sqrt{h}$. The first-order increment in the linearised holonomy (or geodesic deviation) around $\hat{\gamma}$ is*

$$\delta\mathscr{H} = \int_0^L \delta\Gamma^{a}{}_{bc}(\hat{\gamma}(\tau))u^b u^c\, e_a(\tau)\,\mathrm{d}\tau,$$

where e_a is a printed background dual frame along the orbit. This increment is linear in the declared kernel applied to $\mathrm{deltaT}_{\mathrm{o}}$*, hence linear in M_{P}^{-2} times the ordinary source. A second-order ordinary metric deflection, if defined by feeding a dark-stress response back through the same declared kernel, is bilinear in the ordinary and dark sources and therefore formally of order M_{P}^{-4} times kernel factors, the period L, and the detector placement.*

The monomial scaling $\mathcal{O}(G^2 m_o \rho_d)$ is not obtained as a complete coefficient: the kernel, the period, and the detector distance are not reduced to unity, and no retarded Green function is supplied on the quotient itself. The estimate is not operational insulation of a dynamical back-reacting host, not a well-posedness theorem, and not Einstein dynamics.

Proof. On the cover the background is flat, so the linearised geodesic equation along a fixed background geodesic reduces to an integral of $\delta\Gamma(u,u)$. That is . The declared wave equation determines $\delta\bar{h}$ from deltaT$_o$ by convolution against the supplied Minkowski retarded kernel, and $\delta\Gamma$ is first-order in $\partial\delta\bar{h}$. A further ordinary deflection obtained by inserting a dark-stress increment, itself linear in $\delta\mathscr{H}$ and in the background dark density, into the same kernel is bilinear in the two sources. No further reduction of the remaining geometric factors is performed, and the kernel is not claimed to descend.

Proposition (The naive periodized retarded sum is not locally finite). *Let $(\widehat{M},\eta)$ be the simply connected cover of Assumption , and let the deck group act by $\varphi \mapsto \varphi + 2\pi k$, $k \in \mathbb{Z}$.*

1. *The generator $2\pi\partial_\varphi$ is host-timelike: $g(\partial_\varphi,\partial_\varphi) = -h < 0$. In the Minkowski coordinates $\tau = t + a\varphi$, $\chi = \sqrt{F}\,\varphi$ with $F = a^2 - h$, the deck translation is the timelike shift by the vector $(2\pi a, 2\pi\sqrt{F}, 0, 0)$, of Minkowski square $-4\pi^2 h$.*

2. *Let* deltaT$_o$ *have compact spacetime support on the quotient M, and let $p \in \widehat{M}$ be any lift of an observation point. For $a > 0$, every sufficiently negative winding translate of the lifted support lies in the Minkowski causal past $J^-(p)$. Consequently the formal periodized retarded sum over deck images is an infinite sum, not a locally finite sum, at every observation point. No convergence prescription is supplied, and the formal sum is not claimed to define a distributional solution on M. This does not produce a retarded Green function on the chronology-violating quotient, and does not yield Cauchy evolution in the coordinate t. Assumption and Proposition are not withdrawn: the declared cover kernel and the formal first-order holonomy increment stand as written, without descent.*

Proof. Item (i) is the already recorded identity $g(\partial_\varphi,\partial_\varphi) = -h$ of Theorem (ii), together with the linear change of

coordinates of Proposition . The Minkowski square of the deck vector is

$$-(2\pi a)^2 + (2\pi\sqrt{F})^2 = 4\pi^2(F - a^2) = -4\pi^2 h < 0.$$

For (ii), write $\Delta\tau(k) = \tau_p - \tau_k$ and $\Delta\chi(k) = \chi_k - \chi_p$ along a lift of a compactly supported source point, so that

$$\Delta\tau(k) = C - 2\pi a k, \qquad \Delta\chi(k) = D + 2\pi\sqrt{F}\, k$$

for constants C, D depending on the unshifted points. Setting $k = -m$ with $m \to +\infty$, one has $\Delta\tau = C + 2\pi a m \to +\infty$ and

$$\begin{aligned}\Delta\tau^2 - \Delta\chi^2 &= (C + 2\pi a m)^2 - (D - 2\pi\sqrt{F}\, m)^2 \\ &= (C - D + 2\pi(a + \sqrt{F})m)(C + D + 2\pi(a - \sqrt{F})m).\end{aligned}$$

Because $\sqrt{F} = \sqrt{a^2 - h} < a$, both factors are eventually positive and of order m. Subtracting the bounded transverse separations $\Delta x^2 + \Delta z^2$ leaves a positive causal remainder, so the past images lie in $J^-(p)$ for all sufficiently large m. Future images eventually leave $J^-(p)$, but that one-sided escape does not make the past sum finite.

The time-domain obstruction of Proposition is not repaired by passing to a frequency-domain regularisation. The following statement records a mode-wise $i\varepsilon$ family built from Lemma and Proposition ; it is a different prescription, not a descent of the Minkowski retarded kernel.

Proposition (Frequency-domain regularised mode family). *Under Assumption , write* $F := a^2 - h > 0$ *and keep the completed temporal remainder* $R(\omega, n)$ *of . Fix* $\varepsilon > 0$*, and for* $\omega \in \mathbb{R}$ *and* $n \in \mathbb{Z}$ *write*

$$\mu(\omega + i\varepsilon, n)^2 := m_{\mathrm{d}}^2 - R(\omega + i\varepsilon, n).$$

Declare the square-root branch μ *with nonnegative real part, with branch cut along* $(-\infty, 0]$*. Let* Σ_ε *be the exceptional set of pairs* (ω, n) *for which* $\mu(\omega + i\varepsilon, n)^2$ *lies on that cut (including the origin).*

1. *For every pair off* Σ_ε*, the transverse kernel*

$$K_{n,\,\omega + i\varepsilon}(\Delta x, \Delta z) = \frac{1}{2\pi} K_0(\mu(\omega + i\varepsilon, n)\, r_\perp), \qquad r_\perp := \sqrt{(\Delta x)^2 + (\Delta z)^2} > 0,$$

is the Yukawa kernel of Proposition (i) with complexified frequency $\omega + i\varepsilon$.

2. *For each fixed real ω and each $\varepsilon > 0$, one has the large-$|n|$ expansion $R(\omega + i\varepsilon, n) \sim -n^2/F$. Consequently* $\operatorname{Re}\mu(\omega + i\varepsilon, n) \sim |n|/\sqrt{F}$*, and the kernel decays exponentially in $|n|$ at every $r_\perp > 0$. Only finitely many integers n can meet Σ_ε at that fixed ω.*

3. *If a source is static or frequency-bandlimited, and is smooth in φ (so that its Fourier coefficients in n decay rapidly), the mode-wise Yukawa convolutions form a convergent family at $r_\perp > 0$ away from Σ_ε, and define a distributional particular solution in the transverse variables for that regularised frequency. This is a frequency-domain regularised mode sum on the prescribed cylindrical witness, typestated as a distributional resolvent only away from the declared exceptional set Σ_ε. It is not a retarded Green function on the chronology-violating quotient, not a repair of the divergent time-domain image sum of Proposition , not a Cauchy theorem in t or x, not a Leray–Ohya or compactified Fredholm theorem, and not a derivation of a Deutsch or postselected rule. A formal Poisson summation in φ would only reassemble the same mode family; the $i\varepsilon$ deformation is not the retarded prescription, and does not make the periodized retarded sum locally finite. Proposition , Assumption , and Proposition are not withdrawn.*

Proof. Item (i) is the same transverse Fourier inversion as in Proposition (i), with ω replaced by $\omega + i\varepsilon$ and with the declared branch of the square root. For (ii), the identity

$$R(\omega, n) = -\frac{n^2}{F} - \frac{2a}{F}\omega n - \frac{h}{F}\omega^2,$$

already implicit in and , continues to hold for complex frequency. The leading term $-n^2/F$ is independent of ω, and $\operatorname{Im} R(\omega + i\varepsilon, n) = -(2a\varepsilon/F)n + O(1)$. Since $F = a^2 - h > 0$ forces $|a| > \sqrt{h} > 0$, that imaginary part is unbounded as $|n| \to \infty$, so only finitely many n can place μ^2 on the cut at a fixed ω. Off the cut, $\operatorname{Re}\mu \sim |n|/\sqrt{F}$, and the standard decay $K_0(w) \sim \sqrt{\pi/(2w)}\, e^{-w}$ for $\operatorname{Re} w \to +\infty$ gives exponential decay in $|n|$ at $r_\perp > 0$.

For (iii), rapid decay of the source's Fourier coefficients in n, multiplied by the exponential decay of the kernels at $r_\perp > 0$, yields uniform convergence on compact transverse sets that avoid the diagonal $r_\perp = 0$. Each individual

Yukawa kernel remains a locally integrable transverse distribution (logarithmic at the origin), so the mode sum defines a transverse distributional particular solution for each admitted frequency, away from Σ_ϵ. No retarded support property is claimed, and no identification with the periodized Minkowski retarded sum is made.

Null-energy compatibility and qualified horizon obstructions

The sufficient energy hypotheses

For a host-null vector k, the host null energy condition is $T_{ab}k^a k^b \geq 0$. Equivalently use the covector $k_a = g_{ab}k^b$ in $T^{ab}k_a k_b$.

Theorem (Host NEC for a qualified three-slot class). *Suppose holds and ordinary stress satisfies the weak energy condition with respect to* $\check{g}$*. Then its host stress satisfies the host NEC. Positive host dust and the canonical controller also satisfy it. Consequently their sum satisfies the host NEC.*

A sufficient ordinary perfect-fluid hypothesis is

$$\hat{\rho} \geq 0, \qquad \hat{\rho} + \hat{p} \geq 0.$$

Ordinary NEC without the weak-energy hypothesis is not the hypothesis of this transfer theorem.

Proof. For host-null k_a, Lemma gives

$$\check{g}^{ab} k_a k_b = -\frac{B}{1 + B\sigma}(q^a k_a)^2 \leq 0.$$

Thus $w^a = \check{g}^{ab} k_b$ is ordinary causal. The ordinary weak energy condition, extended to null vectors by continuity, gives

$$T_{\mathrm{o}}^{ab} k_a k_b = \sqrt{1 + B\sigma}\, \hat{T}_{ab} w^a w^b \geq 0.$$

For a fluid, in its ordinary rest frame and for causal w,

$$\hat{T}(w, w) = \hat{\rho}\{(w^0)^2 - |\mathbf{w}|^2\} + (\hat{\rho} + \hat{p})|\mathbf{w}|^2 \geq 0.$$

For host dust the null contraction is $\rho(u \cdot k)^2$; for the canonical controller it is $(q \cdot k)^2$. Add the inequalities.

Corollary (Pointwise and averaged source obstruction). *In the class of Theorem , equation with* $T_{\text{ext}} = 0$ *is impossible where* $G_{ab}k^a k^b < 0$ *for a host-null k. If a supplied null path with a nonnegative integration weight has a negative integrated* $G_{ab}k^a k^b$*, the corresponding integrated compatibility is likewise impossible.*

Proof. The cosmological term has zero null contraction. The other side is pointwise nonnegative, hence also nonnegative after weighted integration.

This is an exact compatibility obstruction. It does not assert that every compactly generated horizon has negative pointwise null curvature.

An independently proved focusing identity

Theorem (Periodic affine null focusing identity). *Let a null geodesic make an affine period* $L > 0$*, returning with the same tangent and screen data. Suppose it lies in a smooth twist-free null congruence whose expansion* θ *and shear* σ_{AB} *are periodic under that return. Then*

$$\int_0^L R_{ab}k^a k^b \, \mathrm{d}\lambda = -\int_0^L \left(\frac{1}{2}\theta^2 + \sigma_{AB}\sigma^{AB}\right)\mathrm{d}\lambda \le 0.$$

If the host null convergence condition holds on the orbit, all three quantities $\theta, \sigma_{AB}, R_{ab}k^a k^b$ *vanish there. Nonzero expansion or shear somewhere therefore requires negative null curvature somewhere along that periodic orbit.*

Proof. Let $\mathscr{B}_{AB}$ be the screen-projected derivative of the affine tangent field. Commutation of covariant derivatives and the geodesic equation give its optical evolution

$$\frac{\mathrm{d}\mathscr{B}}{\mathrm{d}\lambda} = -\mathscr{B}^2 - \mathscr{R}.$$

Taking the screen trace, decomposing $\mathscr{B} = \frac{1}{2}\theta I + \sigma$, and using zero twist gives

$$\theta' = -\frac{1}{2}\theta^2 - \sigma_{AB}\sigma^{AB} - R_{ab}k^a k^b.$$

The integral of θ' over a period is zero. This proves . Under null convergence its left side is nonnegative while its

right side is nonpositive. Both vanish. Continuity of each nonnegative integrand gives the stated pointwise conclusion.

Affine return and periodic optical data are substantive hypotheses. A closed unparametrized null orbit with affine holonomy is not automatically in this class.

Why compact generation alone is insufficient

Example (Flat compact Misner horizon). On

$$M = \mathbb{R}_T \times S^1_\chi \times \mathbb{T}^2_{x,z}$$

prescribe

$$g_M = -2\mathrm{d}T\,\mathrm{d}\chi - T\mathrm{d}\chi^2 + \mathrm{d}x^2 + \mathrm{d}z^2.$$

Its two-dimensional block has determinant -1 and $g_M^{TT} = T$. The region $T < 0$ has spacelike compact T-slices and a temporal coordinate T; the surface $T = 0$ is a compactly generated Cauchy horizon for that region. For $T > 0$, the χ-circles are timelike. Nevertheless $R_{ab} = G_{ab} = 0$ everywhere.

Proof. The only nonzero connection coefficients in the (T, χ) block are

$$\Gamma^T_{T\chi} = \Gamma^T_{\chi T} = \frac{1}{2}, \qquad \Gamma^T_{\chi\chi} = \frac{T}{2}, \qquad \Gamma^\chi_{\chi\chi} = -\frac{1}{2}.$$

Substitution into the curvature formula gives zero Ricci tensor; in two dimensions this gives zero full curvature. The flat transverse torus adds no curvature.

On any compact negative T-interval, causal coordinate speeds are bounded after parametrization by T. Compactness of the spatial torus then lets a causal curve with an interior finite endpoint in $T < 0$ continue. Thus the negative slices are Cauchy for the negative region. At $T = 0$, the closed null generators can remain on that compact surface without meeting a negative slice. For $T > 0$, closed timelike circles similarly avoid it. This identifies the future Cauchy horizon at $T = 0$. Its generators are imprisoned in the compact horizon itself.

At the horizon,

$$\nabla_{\partial_\chi}\partial_\chi = -\frac{1}{2}\partial_\chi.$$

The periodic coordinate tangent is not affinely parametrized with periodic affine tangent. Hence the example need not satisfy Theorem 's affine-return hypothesis.

This is a counterexample to the proposed unconditional negative-curvature assertion. It does not violate the additional hypotheses of established chronology restrictions: noncompact initial surfaces, completeness, genericity, and averaged conditions must be distinguished [14, 15].

An eternal quotient and a chronology horizon manufactured from laboratory initial data are different constructions. Conversely, the absence of compact generation does not guarantee any energy condition. Those conditions must be computed for each host. The flat witness of Section already shows that an eternal geometric CTC can still fail the proposed positive-matter Einstein match.

A prescribed bimetric Misner–winding overlay

The flat compact Misner example remains a chronology-horizon comparison with $G_{ab} = 0$, not a source-compatible manufacture. The following overlay is a kinematic causal comparison on that same prescribed identification.

Assumption (Bimetric Misner–winding collar). On the manifold of Example , keep g_M as in , set $q = \kappa\,\mathrm{d}\chi$ with $\kappa \neq 0$, and prescribe a smooth switch coefficient $B(s(T)) \geq 0$. Define

$$\tilde{g}_M = -2\mathrm{d}T\,\mathrm{d}\chi + (B(s(T))\kappa^2 - T)\mathrm{d}\chi^2 + \mathrm{d}x^2 + \mathrm{d}z^2.$$

Write $H(T) := B(s(T))\kappa^2 - T$.

Proposition (Controller and ordinary inverse on the overlay). *Under Assumption , q is a closed divergence-free host one-form. The ordinary two-block has determinant* -1 *and inverse*

$$\tilde{g}_M^{TT} = -H(T), \qquad \tilde{g}_M^{T\chi} = -1, \qquad \tilde{g}_M^{\chi\chi} = 0.$$

In particular $\tilde{g}_M^{TT} = -(B(s(T))\kappa^2 - T)$. *Whenever* $H(T) > 0$, *the coordinate* T *is an ordinary temporal function.*

Completing the square gives

$$\tilde{g}_M = -H^{-1}\,\mathrm{d}T^2 + H(\mathrm{d}\chi - H^{-1}\mathrm{d}T)^2 + \mathrm{d}x^2 + \mathrm{d}z^2.$$

Proof. The form $q = \kappa\,\mathrm{d}\chi$ is closed. Raising with g_M and using $g_M^{T\chi} = -1$, $g_M^{\chi\chi} = 0$ yields $q^T = -\kappa$ and $q^\chi = 0$. The host volume density is constant, so $\nabla_a q^a = 0$. The ordinary two-block is $\left(\begin{smallmatrix} 0 & -1 \\ -1 & H \end{smallmatrix}\right)$, with determinant -1 and the inverse . The displayed completion of the square is an algebraic identity wherever $H \neq 0$. If $H > 0$, then $\tilde{g}_M^{TT} < 0$, so $\mathrm{d}T$ is ordinary-timelike.

Theorem (Kinematic causal typestate of the Misner–winding overlay). *Assume Assumption and* $H(T) > 0$ *throughout an open interval* $I \subset \mathbb{R}_T$.

1. *On the open ordinary spacetime* $I \times S^1_\chi \times \mathbb{T}^2_{x,z}$, *every level set of* T *in* I *is an ordinary Cauchy surface, and* T *is an ordinary Cauchy time.*

2. *If* I *contains a negative interval, the standard Misner global hyperbolicity of* g_M *on* $\{T < 0\}$ *is not, by itself, the ordinary statement (i); the latter is the content of the preceding item, and uses .*

3. *If* I *meets* $\{T > 0\}$, *the dark* χ*-circles are closed* g_M*-timelike curves by the already prescribed Misner identification of Example . A prescribed ramp of* $B(s(T))$ *is not a coupled Einstein–scalar initial-value problem, and is not laboratory manufacture of a CTC region from chronology-preserving data.*

4. *A spacetime with a future boundary, such as* $T \in (-\infty, T_{\max}]$, *is a manifold with boundary. Global hyperbolicity is asserted in (i) only for the open set with* $T \in I$ *and* $H > 0$, *not for a truncated slab "without any Cauchy horizon" as a slogan. Test Klein–Gordon leading-branch and Wronskian identities across* $T = 0$ *are recorded in Proposition ; collisionless Vlasov and geodesic characteristic continuation is recorded in Proposition . Unique dark Klein–Gordon continuation, and a complete blue-shift energy estimate for the compactly generated* g_M*-null generators, remain residual. The construction does not discharge the Version 1 obligation of a*

chronology-preserving initial-data evolution that manufactures the dark CTC region in the dynamical Einstein sense. It does not advance the time-machine score.

Proof. On any compact subinterval of I one has $H \geq H_{\min} > 0$, so bounds the ordinary coordinate speeds of causal curves parametrized by T:

$$x'^2 + z'^2 \leq H^{-1} \leq H_{\min}^{-1},$$

while χ' remains bounded. The spatial manifold $S^1_\chi \times \mathbb{T}^2$ is compact and complete. A future ordinary causal curve whose T-range were bounded above inside I would therefore have a finite remaining spatial length and a limit in that compact spatial manifold, and could be continued, contradicting inextendibility in the open spacetime. The same holds in the past. Thus every inextendible ordinary causal curve meets each T-slice in I exactly once. Causal diamonds are closed and spatially bounded, hence compact, as in the argument of Theorem . This is (i). Item (ii) is typestate: the host and ordinary metrics differ, so the host argument of Example is not reused as a proof transfer. Item (iii) records the already prescribed identification and the absence of a coupled dynamical initial-value problem. Item (iv) distinguishes an open spacetime from a manifold with boundary.

Proposition (Test-field continuation typestate on the Misner overlay). *Assume Assumption . Let* $\psi = e^{in\chi + ik_x x + ik_z z} f(T)$ *be a smooth mode on the host, and write* $\mu^2 := k_x^2 + k_z^2 + m^2$.

1. *The host Klein–Gordon equation* $\mathrm{Box}_{g_M}\psi = m^2\psi$ *reduces to the exact ODE*

$$Tf'' + (1 - 2in)f' - \mu^2 f = 0.$$

This equation has a regular singular point at $T = 0$, *with Frobenius indices* 0 *and* $2in$. *The Abel Wronskian of any two solutions is*

$$W(f_1, f_2) = C\,T^{-1+2in},$$

for a constant C depending on the chosen pair.

2. *For* $\mu = 0$ *the general solution is* $f(T) = A + BT^{2in}$ *if* $n \neq 0$, *and* $f(T) = A + B\ln|T|$ *if* $n = 0$. *For* $\mu > 0$ *and* $T > 0$,

one may write $f(T) = T^{in}(A\,I_{2in}(2\mu\sqrt{T}) + B\,K_{2in}(2\mu\sqrt{T}))$*; for* $T < 0$, $f(T) = |T|^{in}(A\,J_{2in}(2\mu\sqrt{|T|}) + B\,Y_{2in}(2\mu\sqrt{|T|}))$*. In every case the leading branches as* $T \to 0$ *reproduce the Frobenius indices.*

3. *If* $n \neq 0$*, the branch with index* $2in$ *has phase* $2n\ln|T|$ *as* $T \to 0^{-}$*, and the coordinate frequency* $|\partial_T(2n\ln|T|)| = 2|n|/|T|$ *diverges. This is a named test-field phase comparison along the prescribed compactly generated horizon. It is not a Hawking radiation theorem, not a thermal spectrum, and not a thermodynamic horizon identity.*

4. *Let* $\widetilde{\psi}$ *be an ordinary scalar obeying* $\mathrm{Box}_{g_M}\widetilde{\psi} = m_o^2\widetilde{\psi}$*. Whenever* H *is smooth and* $H(T) > 0$ *throughout an open interval containing* $T = 0$*, the reduced ordinary radial equation*

$$(Hf')' + 2inf' + \mu_o^2 f = 0, \qquad \mu_o^2 := k_x^2 + k_z^2 + m_o^2,$$

has smooth coefficients and a regular (non-singular) point at $T = 0$*. Every solution* f *is* C^∞ *across* $T = 0$*, and the corresponding ordinary mode is smooth on that open spacetime. The claim is not made outside this printed class. The identities do not select a unique dark PDE continuation past* $T = 0$*: both Frobenius branches exist, and no energy-space or radiation condition is imposed. The overlay remains a prescribed kinematic comparison in the sense of Theorem , not dynamical Einstein manufacture, and the time-machine score is not advanced.*

Proof. The host volume density is constant, and the inverse two-block is $g_M^{TT} = T$, $g_M^{T\chi} = -1$, $g_M^{\chi\chi} = 0$, as in Example . Therefore

$$\mathrm{Box}_{g_M}\psi = T\partial_T^2\psi + \partial_T\psi - 2\partial_T\partial_\chi\psi + \partial_x^2\psi + \partial_z^2\psi.$$

Substituting the displayed mode yields . In standard form, $f'' + Pf' + Qf = 0$ with $P = (1 - 2in)/T$ and $Q = -\mu^2/T$. The singularity at $T = 0$ is regular (at most a simple pole in P and a double pole, here only simple, in Q). The indicial polynomial is $\rho(\rho - 2in) = 0$. Abel's identity gives $W = \exp(-\int P\,\mathrm{d}T) = CT^{-1+2in}$.

If $\mu = 0$ and $n \neq 0$, reduction of order produces the exact quadrature $f = A + BT^{2in}$; if $n = 0$, one obtains $f = A + B\ln|T|$. If $\mu > 0$, the change of variables $s = 2\mu\sqrt{|T|}$ and $f = |T|^{in}y(s)$ converts into the modified Bessel

equation of order $2in$ for $T > 0$, and into the ordinary Bessel equation of the same order for $T < 0$, giving the stated special-function bases. The small-argument expansions $I_\nu(s) \sim (s/2)^\nu/\Gamma(\nu+1)$ and $K_\nu(s) \sim \frac{1}{2}\Gamma(\nu)(s/2)^{-\nu}$ (and their Bessel analogues) recover the two Frobenius branches. The phase of T^{2in} for $T < 0$ is $2n\ln|T|$ up to a branch constant, proving (iii) as a test-field statement only.

For (iv), the ordinary inverse two-block is , and $\sqrt{-\hat{g}_M} = 1$. The same reduction produces $-Hf'' - H'f' - 2inf' - \mu_o^2 f = 0$, which is . If H is smooth and $H(0) > 0$, the leading coefficient does not vanish at $T = 0$, so the ODE is regular there and every solution extends smoothly.

Proposition (Collisionless characteristic split on the Misner overlay). *Let g_M be the prescribed Misner metric , with the already printed inverse $g_M^{TT} = T$, $g_M^{T\chi} = -1$, and $g_M^{\chi\chi} = 0$. Write p_a for the co-momentum of a g_M-geodesic, and $K := p_x^2 + p_z^2 + m_{\mathrm{d}}^2$. The mass shell in signature $(-+++)$ is*

$$Tp_T^2 - 2p_T p_\chi + p_x^2 + p_z^2 + m_{\mathrm{d}}^2 = 0.$$

The momenta p_χ, p_x, and p_z are conserved. On $\{T < 0\}$, the coordinate T is a temporal function and ∂_T is a future-directed null vector, with $\mathrm{d}T(\partial_T) = 1$; a causal curve is future-directed when $\dot{T} > 0$.

1. *The identity $\dot{T} = -p_\chi + Tp_T$ holds along every geodesic. At $T = 0$ the mass shell reduces to $-2p_T p_\chi + K = 0$. If $p_\chi \neq 0$, the only finite candidate limit is $p_T(0) = K/(2p_\chi)$. The opposite sign in the co-momentum identity, and the opposite sign in that formal limit, are inconsistent with the printed inverse and are not used.*

2. *The future-directed transmitting open set consists of those mass-shell co-momenta with $p_\chi < 0$. Every such geodesic reaches $T = 0$ in finite proper time, with finite limiting co-momentum $p_T(0) = K/(2p_\chi)$ and finite limiting tangent $\dot{\chi}(0) = -K/(2p_\chi)$, and extends smoothly as a g_M-geodesic into $\{T > 0\}$.*

3. *On the complementary locus $p_\chi \geq 0$, every future-directed causal geodesic has diverging p_T as $T \to 0^-$: specifically, $p_T \sim 2p_\chi/T$ if $p_\chi > 0$, and $p_T = -\sqrt{-K/T}$ if $p_\chi = 0$ and $K > 0$. The accumulated winding $\int \dot{\chi}\,\mathrm{d}\tau$ diverges, so the curve winds unboundedly on the χ-circle as it approaches the horizon. This is not a C^∞ co-momentum*

continuation through $T = 0$. The imprisoned null generators of the compactly generated horizon $\{T = 0\}$ sit at $p_\chi = K = 0$. The statement is a collisionless Vlasov / geodesic characteristic continuation on a prescribed overlay. It does not select a unique dark Klein–Gordon continuation (Proposition stands), is not a complete blue-shift energy estimate for the compactly generated generators, is not Hawking radiation, is not dynamical Einstein manufacture, and is not Bar 3. Theorem remains a prescribed kinematic comparison.

Proof. The co-momentum mass shell is $g_M^{ab} p_a p_b = -m_{\mathrm{d}}^2$. Substituting the printed inverse of Example immediately yields ; the displayed identity with the first two signs reversed would be the opposite norm convention and does not hold. Killing conservation of p_χ, p_x, and p_z is the already used stationarity of ∂_χ, ∂_x, and ∂_z. The evaluations $p_T = g_{T\chi}\dot{\chi} = -\dot{\chi}$ and $p_\chi = -\dot{T} - T\dot{\chi}$ are the definitions of the co-momentum, and rearrange to $\dot{T} = -p_\chi + Tp_T$. The time orientation on $\{T < 0\}$ is that already recorded in Example : T is a temporal coordinate and $\{T = 0\}$ is the future Cauchy horizon, so future causal curves have $\dot{T} > 0$.

At $T = 0$, becomes $-2p_T p_\chi + K = 0$. If $p_\chi \neq 0$, then $p_T = K/(2p_\chi)$. For $T < 0$ the quadratic in p_T has roots

$$p_T = \frac{p_\chi \pm \sqrt{p_\chi^2 - TK}}{T}.$$

The root whose numerator cancels to first order has the finite limit $K/(2p_\chi)$ as $T \to 0$; the other root behaves as $2p_\chi/T$.

On the finite root, $\dot{T} \to -p_\chi$ as $T \to 0$. Future orientation therefore forces $p_\chi < 0$, which is (ii). In that open set the limiting tangent is finite, the proper-time integral $\int \mathrm{d}T/\dot{T}$ remains finite, and the same conserved momenta continue to solve for $T > 0$ in a neighbourhood of the horizon, giving a smooth geodesic extension.

On the complementary future-directed locus, the divergent root must be selected if $p_\chi > 0$, because the finite root would have $\dot{T} \to -p_\chi < 0$. Then $\dot{T} = \sqrt{p_\chi^2 - TK}$ remains positive, while $p_T \sim 2p_\chi/T$ and $\mathrm{d}\chi/\mathrm{d}T = \dot{\chi}/\dot{T} \sim -2/T$, so $\int \dot{\chi}\, \mathrm{d}\tau$ diverges as $T \to 0^-$. If $p_\chi = 0$ and $K > 0$, the shell forces $p_T^2 = -K/T$, and future orientation selects the negative root $p_T = -\sqrt{-K/T}$; then $\dot{T} = \sqrt{K|T|}$ and $\mathrm{d}\chi/\mathrm{d}T = 1/|T|$, so the winding again diverges. If $p_\chi = K = 0$, the shell at $T = 0$ is

compatible with $\dot{T} = 0$, which describes the imprisoned null generators already identified in Example . This is (iii). No energy-space estimate on the generators is claimed, and no Klein–Gordon branch is selected.

Finite quantum chronology insulation

Two inequivalent toy rules

Let $S = O \otimes C$ be a finite ordinary-controller register and D a finite dark register of dimension d. Neither register is identified with a field or an acquired spacetime CTC.

Definition (Deutsch toy rule). For a supplied unitary U on $S \otimes D$, a supplied ordinary input ρ, and an initially product state, choose a dark density matrix τ_ρ satisfying

$$\tau_\rho = \mathscr{F}_\rho(\tau_\rho), \qquad \mathscr{F}_\rho(\tau) = \mathrm{Tr}_S\{U(\rho \otimes \tau)U^\dagger\}.$$

The ordinary output is

$$\mathscr{N}_U(\rho) = \mathrm{Tr}_D\{U(\rho \otimes \tau_\rho)U^\dagger\}.$$

A rule selecting among multiple fixed points is additional data. This is the Deutsch model [18].

Lemma (Finite fixed-point existence). *The map $\mathscr{F}_\rho$ has at least one density-matrix fixed point.*

Proof. Choose any τ_0, and average its iterates:

$$\bar{\tau}_N = \frac{1}{N}\sum_{j=0}^{N-1} \mathscr{F}_\rho^j(\tau_0).$$

Then

$$\left\| \mathscr{F}_\rho(\bar{\tau}_N) - \bar{\tau}_N \right\|_1 \le \frac{2}{N}.$$

The density matrices form a compact finite-dimensional set. A convergent subsequence and continuity of $\mathscr{F}_\rho$ give a fixed point.

Definition (Postselected toy rule). Let

$$|\Omega\rangle = d^{-1/2} \sum_{j=1}^{d} |j\rangle_D |j\rangle_{D'}$$

be a supplied maximally entangled state. Bell contraction of $U \otimes I_{D'}$ gives

$$K = \langle\Omega|(U \otimes I_{D'})|\Omega\rangle = \frac{1}{d}\mathrm{Tr}_D\, U.$$

For $\mathrm{Tr}\,(K\rho K^\dagger) > 0$, the normalized successful output is

$$\mathscr{P}_K(\rho) = \frac{K\rho K^\dagger}{\mathrm{Tr}\,(K\rho K^\dagger)}.$$

A P-CTC interpretation treats this successful branch as its toy rule [19]. An actual postselection experiment must also retain its failure branch.

The trace denominator is not optional. The map is undefined where it vanishes.

The corrected insulation theorem

Theorem (Factorized chronology insulation). *Suppose*

$$U = V_S \otimes W_D.$$

Then:

1. *the Deutsch ordinary output is $V_S \rho V_S^\dagger$, independently of every selected dark fixed point and of W_D;*

2. *its ordinary-probe reduction, for a fixed controller preparation η_c, is the linear CPTP channel*

$$\Phi_c(\rho_O) = \mathrm{Tr}_C\,\{V_c(\rho_O \otimes \eta_c)V_c^\dagger\};$$

3. *in the postselected rule,*

$$K = \alpha_W V_S, \qquad \alpha_W = \frac{\mathrm{Tr}\, W_D}{d}.$$

If $\alpha_W \neq 0$, the normalized successful ordinary output is $V_S \rho V_S^\dagger$, independently of W_D. Its success probability is $|\alpha_W|^2$, independent of ρ;

4. *if* $\operatorname{Tr} W_D = 0$*, the factorized postselected branch has no successful inputs and no normalized output. The linear-channel statements remain valid with an untouched reference attached to the ordinary input. Independence from a controller setting follows only when that setting changes the dark factor alone, or when the resulting ordinary channels* Φ_c *are independently known to coincide.*

Proof. In the factorized Deutsch case,

$$U(\rho \otimes \tau)U^\dagger = V\rho V^\dagger \otimes W\tau W^\dagger.$$

Every dark fixed point disappears upon tracing D. The maximally mixed dark state is one fixed point. Preparing a fixed η_c, applying V_c, and tracing C are CPTP operations, proving (ii).

For postselection, take the dark partial trace in . The scalar factor cancels from the normalized successful output when nonzero; its squared modulus is the success probability. If zero, the numerator and probability both vanish. All ordinary operations tensor with the reference identity.

Example (Factorization does not imply controller blindness). Let O, C be qubits, prepare C in $\operatorname{proj} c$, and use

$$V = I_O \otimes \operatorname{proj} 0_C + X_O \otimes \operatorname{proj} 1_C.$$

Then $\Phi_0 = \mathrm{id}$ and $\Phi_1(\rho) = X\rho X$, even with a completely factorized dark loop.

Dark-loop insulation must not be confused with insulation from the controller's own ordinary coupling.

Remark (Complete records of a factorized postselection experiment). The successful ordinary state is independent of W, but a retained success/failure flag can reveal W through $|\operatorname{Tr} W/d|^2$. An unconditional ordinary output also depends on the specified failure operation. Thus the theorem does not claim blindness of every complete experimental transcript to every dark setting.

Explicit portal certificates and complete-record tests

An interacting unitary is not automatically nonlinear

For example, $U = \mathrm{proj}\,0_O \otimes I_D + \mathrm{proj}\,1_O \otimes Z_D$ preserves every ordinary diagonal population and need not provide perfect discrimination or cloning. A portal certificate must specify the unitary, fixed-point rule, preparations, and measured statistic.

Theorem (Explicit two-preparation Deutsch discrimination). *Let O, D be qubits, let* H *denote the Hadamard matrix, and define*

$$U = \left(\mathrm{proj}\,0_O \otimes I_D + \mathrm{proj}\,1_O \otimes \mathrm{H}_D\right)\mathrm{SWAP}_{OD}.$$

For ordinary inputs $\mathrm{proj}\,0$ *and* $\mathrm{proj}\,-$*, where* $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$*, the Deutsch fixed points are respectively unique and equal to* $\mathrm{proj}\,0_D$ *and* $\mathrm{proj}\,1_D$*. The ordinary outputs are* $\mathrm{proj}\,0_O$ *and* $\mathrm{proj}\,1_O$*. Hence a computational-basis measurement distinguishes these two nonorthogonal preparations perfectly in this toy model.*

No single conventional CPTP channel has those two outputs for those two inputs.

Proof. After the SWAP the joint state is $\tau_O \otimes \rho_D$. Tracing out the ordinary control gives

$$\mathscr{F}_\rho(\tau) = \tau_{00}\rho + \tau_{11}\mathrm{H}\rho\mathrm{H}.$$

For $\rho = \mathrm{proj}\,0$, its dark 11-entry is $\tau_{11}/2$. Fixedness forces $\tau_{11} = 0$; positivity then forces $\tau = \mathrm{proj}\,0$.

For $\rho = \mathrm{proj}\,-$, use $\mathrm{H}|-\rangle = |1\rangle$. The dark 00-entry is $\tau_{00}/2$, so fixedness forces $\tau_{00} = 0$ and $\tau = \mathrm{proj}\,1$. The ordinary control is consequently fixed in the corresponding basis state after the controlled operation.

For conventional channels, trace distance contracts. The input distance is

$$d_{\mathrm{tr}}(\mathrm{proj}\,0, \mathrm{proj}\,-) = \sqrt{1 - |\langle 0| -\rangle|^2} = \frac{1}{\sqrt{2}},$$

whereas the output distance is one.

This is a local re-proof of a two-state circuit of the established Deutsch-discrimination type [20]. It is not physical acquisition of a CTC. The input states, fixed-point semantics, and mixture convention are substantive parts of the model.

A known finite preparation list can be re-prepared twice after its label has been learned. That is not universal cloning of an arbitrary unknown quantum state, and no such universal cloning theorem is asserted here.

A mixture audit inside the printed Deutsch rule

Theorem (Mixture audit of the printed Deutsch discriminator). *Let U be the unitary of Theorem , and write* $\rho_0 = \mathrm{proj}\,0$, $\rho_- = \mathrm{proj}\,-$, *and*

$$\bar{\rho} = \frac{1}{2}\rho_0 + \frac{1}{2}\rho_-.$$

Inside the already assumed Deutsch toy rule:

1. *the unique dark fixed point of* $\bar{\rho}$ *is* $\tau_{\bar{\rho}} = \frac{1}{2}I_D$, *which coincides with the equal mixture of the two pure fixed points of Theorem ;*

2. *the ordinary output is* $\mathcal{N}_U(\bar{\rho}) = \frac{1}{2}I_O$, *and therefore*

$$\mathcal{N}_U(\bar{\rho}) = \frac{1}{2}\mathcal{N}_U(\rho_0) + \frac{1}{2}\mathcal{N}_U(\rho_-).$$

In particular the inequality $\mathcal{N}_U(\bar{\rho}) \neq \frac{1}{2}\mathcal{N}_U(\rho_0) + \frac{1}{2}\mathcal{N}_U(\rho_-)$ *does not hold for this printed equal mixture;*

3. $\mathcal{N}_U$ *is nevertheless not a linear completely positive trace-preserving map on the full state space: this is already the trace-distance obstruction of Theorem ;*

4. *let R be an ancillary qubit and*

$$\sigma = \frac{1}{2}\mathrm{proj}\,0_R \otimes \rho_0 + \frac{1}{2}\mathrm{proj}\,1_R \otimes \rho_-.$$

The Deutsch rule applied to the enlarged ordinary system $R \otimes O$, *with U acting only on* $O \otimes D$, *has dark fixed point* $\frac{1}{2}I_D$ *and ordinary output* $\frac{1}{4}I_R \otimes I_O$. *This does not coincide with the illegal branched assignment* $\frac{1}{2}\mathrm{proj}\,0_R \otimes \mathrm{proj}\,0_O + \frac{1}{2}\mathrm{proj}\,1_R \otimes \mathrm{proj}\,1_O$ *that would be obtained by formally applying* $\mathrm{id}_R \otimes \mathcal{N}_U$ *as though* $\mathcal{N}_U$ *were a linear channel. The Deutsch rule does not define that linear extension. The statements are a toy-rule audit. They are not a laboratory CTC, not Bar 1, and not a derivation of the Deutsch map from continuum geometry. The*

ancilla certificate may be adjoined to the complete-record discussion of Theorem as a declared toy-model witness that a nonlinear response need not extend linearly, not as a data analysis.

Proof. As in the proof of Theorem , the fixed-point map depends only on the diagonal populations of its dark argument:

$$\mathscr{F}_{\rho}(\tau) = \tau_{00}\rho + \tau_{11}\mathrm{H}\rho\mathrm{H}.$$

Writing $p = \tau_{00}$ and using $\tau_{00} + \tau_{11} = 1$, together with

$$\bar{\rho} = \begin{pmatrix} 3/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix}, \qquad \mathrm{H}\bar{\rho}\mathrm{H} = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 3/4 \end{pmatrix},$$

gives $\tau_{00}(\mathscr{F}_{\bar{\rho}}(\tau)) = (2p + 1)/4$. Fixedness forces $p = 1/2$, hence $\tau_{\bar{\rho}} = \frac{1}{2}I_D$, uniquely, proving (i). The ordinary output is

$$\mathrm{Tr}_D\{C(\tfrac{1}{2}I_D \otimes \bar{\rho})C\} = \tfrac{1}{2}\mathrm{proj}\,0 + \tfrac{1}{2}\mathrm{proj}\,1 = \tfrac{1}{2}I_O,$$

where C is the controlled Hadamard of , proving (ii). Item (iii) is Theorem : no conventional CPTP channel can send the two printed nonorthogonal inputs to orthogonal outputs.

For (iv), the map $\mathscr{F}_{\sigma}$ on the dark register is the equal average of $\mathscr{F}_{\rho_0}$ and $\mathscr{F}_{\rho_-}$, which is the same affine combination as $\mathscr{F}_{\bar{\rho}}$. The unique fixed point is again $\frac{1}{2}I_D$. Because U does not act on R, the output on $R \otimes O$ is the corresponding classical mixture of the two branch outputs computed with this common dark state, each of which is $\frac{1}{2}I_O$. Thus the output is $\frac{1}{4}I_{R\otimes O}$. The branched assignment displayed in (iv) would instead be a perfectly correlated classical state, and is not the output of the printed rule.

Theorem (Convex family of the printed Deutsch discriminator). *Let U be the unitary of Theorem , and for* $\lambda \in [0, 1]$ *write*

$$\rho_{\lambda} = \lambda\rho_0 + (1 - \lambda)\rho_-, \qquad \rho_0 = \mathrm{proj}\,0, \qquad \rho_- = \mathrm{proj}\,-.$$

Inside the already assumed Deutsch toy rule:

1. *the unique dark fixed point of* ρ_λ *has population* $p(\lambda) := \tau_{00}(\lambda) = \lambda$*, and is the diagonal state*

$\tau(\lambda) = \lambda \mathrm{proj}\, 0_D + (1-\lambda)\mathrm{proj}\, 1_D$*;*

2. *the ordinary output is*

$$\mathcal{N}_U(\rho_\lambda) = \lambda \mathrm{proj}\, 0_O + (1-\lambda)\mathrm{proj}\, 1_O = \lambda \mathcal{N}_U(\rho_0) + (1-\lambda)\mathcal{N}_U(\rho_-).$$

Thus the printed convex segment is affine for the ordinary output, including at every $\lambda \in (0,1) \setminus \{\frac{1}{2}\}$*. In particular the typestate of Theorem for the equal mixture is not withdrawn, and is not an accident of the midpoint alone;*

3. *let R be an ancillary qubit and*

$$\sigma_\lambda = \lambda \mathrm{proj}\, 0_R \otimes \rho_0 + (1-\lambda)\mathrm{proj}\, 1_R \otimes \rho_-.$$

The Deutsch rule applied to the enlarged ordinary system $R \otimes O$*, with U acting only on* $O \otimes D$*, has the same dark fixed point* $\tau(\lambda)$ *and ordinary output*

$$(\lambda \mathrm{proj}\, 0_R + (1-\lambda)\mathrm{proj}\, 1_R) \otimes (\lambda \mathrm{proj}\, 0_O + (1-\lambda)\mathrm{proj}\, 1_O).$$

This coincides with the illegal branched assignment $\lambda \mathrm{proj}\, 0_R \otimes \mathrm{proj}\, 0_O + (1-\lambda)\mathrm{proj}\, 1_R \otimes \mathrm{proj}\, 1_O$ *if and only if* $\lambda \in \{0,1\}$*. For every* $\lambda \in (0,1)$ *the rule therefore fails to define the linear extension* $\mathrm{id}_R \otimes \mathcal{N}_U$*. The statements are a toy-rule audit. They are not a laboratory CTC, not Bar 1, and not a derivation of the Deutsch map from continuum geometry. Affinity of the ordinary output on this printed segment is not a claim that* $\mathcal{N}_U$ *is a linear channel on the full state space: Theorem is not withdrawn.*

Proof. As in the proofs of Theorems and , the fixed-point map depends only on the diagonal populations of its dark argument: $\mathcal{F}_\rho(\tau) = \tau_{00}\rho + \tau_{11}\mathrm{H}\rho\mathrm{H}$. Writing $p = \tau_{00}$ and using

$$\langle 0|\rho_\lambda|0\rangle = \frac{1+\lambda}{2}, \qquad \langle 0|\mathrm{H}\rho_\lambda\mathrm{H}|0\rangle = \frac{\lambda}{2},$$

one obtains the affine update $p \mapsto p/2 + \lambda/2$. Fixedness forces $p = \lambda$, uniquely. The corresponding fixed point is $\tau(\lambda) = \lambda\rho_\lambda + (1-\lambda)\mathrm{H}\rho_\lambda\mathrm{H}$, whose off-diagonal entries cancel, leaving the stated diagonal state and proving (i). Because $\tau(\lambda)$ is diagonal, the ordinary output after the controlled Hadamard of is $\lambda \mathrm{proj}\, 0_O + (1-\lambda)\mathrm{proj}\, 1_O$, which is

precisely the affine combination of the two pure outputs of Theorem . This is (ii).

For (iii), the dark map of σ_λ is the same affine combination of $\mathscr{F}_{\rho_0}$ and $\mathscr{F}_{\rho_-}$ as $\mathscr{F}_{\rho_\lambda}$, hence has the same unique fixed point. Since U does not act on R, the output on $R \otimes O$ is the classical mixture of the two branch outputs computed with this common dark state. Each branch, evaluated at $\tau(\lambda)$, returns $\lambda \mathrm{proj}\, 0_O + (1-\lambda)\mathrm{proj}\, 1_O$, and the displayed product state follows. The illegal branched assignment is classically correlated, and agrees with that product state only at the endpoints.

Theorem (Complementary convex segment of the printed Deutsch unitary). *Let U be the unitary of Theorem , and for* $\lambda \in [0,1]$ *write*

$$\tilde{\rho}_\lambda = \lambda \mathrm{proj}\, 0 + (1-\lambda)\mathrm{proj}\, +, \qquad |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}.$$

Inside the already assumed Deutsch toy rule:

1. *the unique dark fixed point of* $\tilde{\rho}_\lambda$ *has computational-basis population*

$$p(\lambda) := \tau_{00}(\lambda) = \frac{2-\lambda}{3-2\lambda},$$

and is the (generally non-diagonal) state

$$\tau(\lambda) = p(\lambda)\tilde{\rho}_\lambda + (1-p(\lambda))\mathrm{H}\tilde{\rho}_\lambda \mathrm{H};$$

2. *the ordinary output coincides with that dark fixed point,*

$$\mathscr{N}_U(\tilde{\rho}_\lambda) = \tau(\lambda).$$

In particular the computational-basis $(0,0)$*-entry of the ordinary output is* $p(\lambda)$ *itself, and is not the distinct rational* $(\lambda^2 - 3\lambda + 3)/(3-2\lambda)$*. The map* $\lambda \mapsto \mathscr{N}_U(\tilde{\rho}_\lambda)$ *is non-affine on* $(0,1)$*. Explicitly, at the midpoint,*

$$\mathscr{N}_U(\tilde{\rho}_{1/2}) = \tilde{\rho}_{1/2},$$

whereas

$$\tfrac{1}{2}\mathcal{N}_U(\text{proj}\,0) + \tfrac{1}{2}\mathcal{N}_U(\text{proj}\,+) = \begin{pmatrix} 5/6 & 1/6 \\ 1/6 & 1/6 \end{pmatrix},$$

so the discrepancy is

$$\mathcal{N}_U(\tilde{\rho}_{1/2}) - \tfrac{1}{2}\mathcal{N}_U(\text{proj}\,0) - \tfrac{1}{2}\mathcal{N}_U(\text{proj}\,+) = \begin{pmatrix} -1/12 & 1/12 \\ 1/12 & 1/12 \end{pmatrix} \neq 0;$$

3. *this printed segment is not the discriminator pair of Theorem : the endpoint output* $\mathcal{N}_U(\text{proj}\,+)$ *is mixed, and the two endpoint outputs are not a perfect orthogonal discrimination of nonorthogonal inputs. Affinity of the complementary segment in Theorem is not withdrawn. The statements are a complementary toy-rule affinity witness on the Bloch disk, under the same printed unitary U. They are not a laboratory CTC, not Bar 1, and not a derivation of the Deutsch map from continuum geometry.*

Proof. As in the proofs of Theorems and , the fixed-point map depends only on the diagonal populations of its dark argument: $\mathscr{F}_\rho(\tau) = \tau_{00}\rho + \tau_{11}\mathrm{H}\rho\mathrm{H}$. The elementary evaluations

$$\langle 0|\tilde{\rho}_\lambda|0\rangle = \frac{1+\lambda}{2}, \qquad \langle 0|\mathrm{H}\tilde{\rho}_\lambda\mathrm{H}|0\rangle = \frac{2-\lambda}{2}$$

follow from $\langle 0|+\rangle = 1/\sqrt{2}$ and $\mathrm{H}|+\rangle = |0\rangle$. Writing $p = \tau_{00}$ and using $\tau_{00} + \tau_{11} = 1$, the population update is the affine map

$$p \text{longmapsto} \, p\frac{1+\lambda}{2} + (1-p)\frac{2-\lambda}{2}.$$

Its slope is $(2\lambda - 1)/2$, of absolute value at most $1/2$ for every $\lambda \in [0, 1]$. Fixedness therefore has the unique solution $p(\lambda) = (2 - \lambda)/(3 - 2\lambda)$. The corresponding fixed point is the displayed convex combination $\tau(\lambda)$, proving (i). The off-diagonal entry of $\tau(\lambda)$ equals $p(\lambda)(1 - \lambda)/2 + (1 - p(\lambda))\lambda/2$, which simplifies to $(1 - \lambda)/(3 - 2\lambda)$ and vanishes if and only if $\lambda = 1$. Thus $\tau(\lambda)$ is not diagonal on $[0, 1)$.

After the SWAP in , the ordinary register carries the dark state. The subsequent controlled Hadamard acts only on the dark factor, so the reduced ordinary state is unchanged by that controlled gate and equals the dark fixed point. This is the same convention as in Theorems and , and gives $\mathcal{N}_U(\tilde{\rho}_\lambda) = \tau(\lambda)$. The computational-basis $(0, 0)$-entry of

$\tau(\lambda)$ is $p(\lambda)$ by the fixed-point equation itself, and therefore coincides with $(2-\lambda)/(3-2\lambda)$, not with $(\lambda^2-3\lambda+3)/(3-2\lambda)$.

At $\lambda=1$ one recovers $\mathcal{N}_U(\mathrm{proj}\,0)=\mathrm{proj}\,0$. At $\lambda=0$,

$$\mathcal{N}_U(\mathrm{proj}\,+)=\frac{2}{3}\mathrm{proj}\,+\,+\,\frac{1}{3}\mathrm{proj}\,0=\begin{pmatrix}2/3 & 1/3\\ 1/3 & 1/3\end{pmatrix}.$$

At $\lambda=1/2$, one has $\mathrm{H}\hat{\rho}_{1/2}\mathrm{H}=\hat{\rho}_{1/2}$, so $\tau(1/2)=\hat{\rho}_{1/2}$ and the midpoint output is $\hat{\rho}_{1/2}$. The affine combination of the endpoint outputs is the displayed matrix with $(0,0)$-entry 5/6, and the discrepancy matrix follows by subtraction. This proves (ii).

Item (iii) is immediate from the endpoint formulae: $\mathcal{N}_U(\mathrm{proj}\,+)$ is mixed, its support meets $\mathrm{proj}\,0$, and the trace distance of the two endpoint outputs is strictly less than one. The typestate of Theorem on the distinct segment $\lambda\mathrm{proj}\,0+(1-\lambda)\mathrm{proj}\,-$ is not used and is not withdrawn.

Postselection nonlinearity and its ordinary completion

Proposition (A normalized nonlinear filter with a linear complete experiment). *Let* $0<r<1$, *choose* $\cos\theta=r$, *and supply*

$$U_P=\mathrm{proj}\,0_O\otimes e^{i\theta Z_D}+\mathrm{proj}\,1_O\otimes I_D.$$

Its Bell contraction is $K=\mathrm{diag}\,(r,1)$. *For the normalized P-CTC rule, a diagonal input* $\rho_p=\mathrm{diag}\,(p,1-p)$ *has output population*

$$p\mathrm{longmapsto}\,\frac{r^2p}{r^2p+1-p},$$

which is not affine.

Nevertheless, setting $L=(I-K^\dagger K)^{1/2}$, *the complete flagged experiment*

$$\mathcal{I}(\rho)=|\mathrm{s}\rangle\langle\mathrm{s}|_R\otimes K\rho K^\dagger+|\mathrm{f}\rangle\langle\mathrm{f}|_R\otimes L\rho L^\dagger$$

is linear, completely positive, and trace preserving. It has precisely that normalized state only conditionally on

success.

Proof. The dark partial traces of the two control blocks are $2\cos\theta$ and 2. The scalar normalization gives the population formula. It fails the affine interpolation between its endpoint populations when $r \neq 1$. For the instrument, $K^{\dagger}K + L^{\dagger}L = I$, which is the Kraus trace-preservation identity.

Normalized conditional statistics therefore do not certify chronology. They can occur in ordinary filtering experiments. Discarding failed trials changes the statistical question. Mixture and preparation ambiguities of nonlinear CTC models are discussed in [21]; the present conclusions are limited to the printed preparation protocol.

A complete-record optional-stopping certificate

Definition (Two-preparation channel null). Each attempted trial has an independent equally likely auditor label $b \in \{0, 1\}$, encoded as proj 0 or proj −. The apparatus receives no additional label data. It returns a complete outcome, including failure. A predetermined scoring rule gives $Y_n \in \{0, 1\}$, equal to one for a correct reported label and zero otherwise. The null class consists of conventional CPTP processing of that one supplied quantum input with label-independent ancillas, controls, and retained past records.

Lemma (Complete single-trial channel ceiling). *Under this null, conditionally on all retained previous data,*

$$\mathbb{E}[Y_n \mid \mathscr{F}_{n-1}] \leq p_* := \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right).$$

Proof. Every conventional complete processing and readout induces a POVM on the supplied qubit. Grouping guessed labels gives an effect $0 \leq E \leq I$. Ignoring failure can only reduce the optimal correct score. For the two equally likely states ρ_0, ρ_1,

$$p_{\text{correct}} = \frac{1}{2} + \frac{1}{2}\mathrm{Tr}\,\{E(\rho_0 - \rho_1)\} \leq \frac{1}{2} + \frac{1}{4}\|\rho_0 - \rho_1\|_1.$$

The positive part of $\rho_0 - \rho_1$ gives this upper bound. Its trace norm is $\sqrt{2}$, yielding . Conditioning on previous data merely selects another label-independent allowed POVM for the current trial.

Theorem (Anytime complete-record rejection). *Suppose the implemented null permits*

$$\mathbb{E}[Y_n \mid \mathscr{F}_{n-1}] \le b_n, \qquad 0 \le b_n \le 1,$$

where b_n *is predictable; for example* $b_n = \min(1, p_* + \varepsilon_n)$ *with an independently justified implementation allowance. For any predictable* $\lambda_n \ge 0$*, define*

$$E_N = \exp\left[\sum_{n=1}^{N} \lambda_n (Y_n - b_n) - \frac{1}{8}\sum_{n=1}^{N} \lambda_n^2\right].$$

Then E_N *is a nonnegative supermartingale with* $E_0 = 1$*, and*

$$\Pr_{\text{null}}\left\{\sup_{N \ge 1} E_N \ge 1/\delta\right\} \le \delta.$$

Thus optional stopping at the first threshold crossing retains the stated null error control.

Proof. For $X \in [0, 1]$, the bounded-variable exponential estimate

$$\mathbb{E}e^{\lambda(X - \mathbb{E}X)} \le e^{\lambda^2/8}$$

follows by differentiating the logarithm of the moment generating function twice: its second derivative is the variance under exponential tilting, bounded by 1/4; its value and first derivative at zero vanish [22]. Apply it conditionally with predictable λ_n. Because the conditional mean is at most b_n and $\lambda_n \ge 0$, each new factor has conditional expectation at most one. This proves the supermartingale assertion.

Stop at the first crossing or at a deterministic bound N. Iterating the conditional expectation inequality gives expected stopped value at most one. On a crossing by N, the stopped value is at least $1/\delta$; hence its probability is at most δ. Let $N \to \infty$.

The specified Deutsch model of Theorem predicts $Y_n = 1$ for an ideal measurement. The certificate can reject the stated conventional response class if its acquisition assumptions hold. It cannot distinguish a geometric CTC from label leakage, a different nonlinear model, or an incorrectly budgeted implementation. All attempts and failure branches must be retained. The test is an R − CTC response contract, not a physical CTC identification.

Corollary (Sufficient e-process threshold on the ideal discriminator path). *In the notation of Theorem , take the constant null ceiling $b_n = p_*$ of Lemma and the constant learning rate $\lambda_n = \lambda_* := 4(1 - p_*)$. If a complete-record path has $Y_n = 1$ for every trial, as predicted by the ideal Deutsch discriminator of Theorem , then the e-process satisfies $E_N \geq 1/\delta$ as soon as*

$$N \geq N_\delta := \lceil (12 + 8\sqrt{2}) \ln (1/\delta) \rceil = \left\lceil \frac{\ln (1/\delta)}{2(1 - p_*)^2} \right\rceil.$$

Thus a pre-specified fixed sample of size N_δ rejects the conventional-channel null at level δ on that ideal path, and the anytime crossing time is at most N_δ. The identity is a sufficient threshold for the printed e-process, not a uniformly most powerful test, not a laboratory identification of a geometric CTC, and not a formula $N^*(\delta, \varepsilon)$ for a noisy alternative or a type-II parameter that has not been supplied. If the implemented null uses a strictly larger allowance $b_n > p_*$, the same argument applies upon replacing p_* by that allowance, provided the learning rate remains $4(1 - b)$ with $b < 1$.

Proof. On the path $Y_n = 1$, one has

$$\ln E_N = N(\lambda_*(1 - p_*) - \lambda_*^2/8) = N \cdot 2(1 - p_*)^2,$$

because $\lambda_* = 4(1 - p_*)$ maximises the quadratic $\lambda(1 - p_*) - \lambda^2/8$. The elementary identities $1 - p_* = (2 - \sqrt{2})/4$ and $1/(2(1 - p_*)^2) = 12 + 8\sqrt{2}$ give the stated threshold, at which $\ln E_N \geq \ln (1/\delta)$. Ville's inequality of Theorem already controls the null crossing probability, so a fixed-sample rejection at N_δ inherits the level δ.

Corollary (Sufficient type-II threshold for a declared noisy alternative). *Keep the complete-record e-process of Theorem , and keep Corollary as the sufficient threshold on the ideal path $Y_n \equiv 1$. Fix a constant null ceiling $b \in [0, 1)$ actually used in E_N (either the Helstrom ceiling p_* of Lemma , or a constant allowance $b = \min (1, p_* + \varepsilon)$ as permitted by Theorem), and declare an alternative class in which*

$$\mathbb{E}_{\text{alt}}[Y_n \mid \mathscr{F}_{n-1}] \geq q_*, \qquad b < q_* \leq 1.$$

Take the constant learning rate $\lambda_* = 4(q_* - b)$. Then, for every $\delta, \beta \in (0, 1)$, the integer

$$N(\delta,\beta,q_*,b) := \left\lceil \frac{(\sqrt{\ln(1/\beta)} + \sqrt{\ln(1/(\delta\beta))})^2}{2(q_* - b)^2} \right\rceil$$

satisfies

$$\Pr_{\text{alt}}(\tau_\delta > N(\delta,\beta,q_*,b)) \le \beta,$$

where $\tau_\delta := \inf\{n \ge 1 : E_n \ge 1/\delta\}$ *is the first crossing time of Theorem . Type-I control* $\Pr_{\text{null}}(\tau_\delta < \infty) \le \delta$ *remains the Ville bound . The identity is a sufficient threshold for this declared alternative class and the printed e-process. It is not a uniformly most powerful test, not a laboratory identification of a geometric CTC, and not an unproved formula* $N^*(\delta,\varepsilon)$ *pasted from an earlier brief.*

Proof. The increment of $\ln E$ at step n is $\Delta_n = \lambda_*(Y_n - b) - \lambda_*^2/8$. The quadratic $\lambda(q_* - b) - \lambda^2/8$ is maximised at $\lambda_* = 4(q_* - b)$, with value $2(q_* - b)^2$. Thus, conditionally on $\mathscr{F}_{n-1}$,

$$\mathbb{E}_{\text{alt}}[\Delta_n \mid \mathscr{F}_{n-1}] \ge \mu := 2(q_* - b)^2.$$

Because $Y_n \in \{0,1\}$, each increment Δ_n lies in an interval of length λ_*. Writing $Z_n := \Delta_n - \mathbb{E}_{\text{alt}}[\Delta_n \mid \mathscr{F}_{n-1}]$, the variables Z_n are conditionally centred and likewise range-bounded by λ_*. Hoeffding's inequality for bounded martingale differences [22] therefore gives, for every $t > 0$,

$$\Pr_{\text{alt}}\Big(\sum_{n=1}^{N} Z_n \le -t\Big) \le \exp\Big(-\frac{2t^2}{N\lambda_*^2}\Big).$$

On the event $\{\tau_\delta > N\}$ one has $E_N < 1/\delta$, hence $\ln E_N < \ln(1/\delta)$. But $\ln E_N = \sum \Delta_n \ge \sum Z_n + N\mu$, so

$$\Pr_{\text{alt}}(\tau_\delta > N) \le \Pr_{\text{alt}}(\textstyle\sum Z_n \le \ln(1/\delta) - N\mu).$$

Whenever $N\mu > \ln(1/\delta)$, the right-hand side is at most $\exp(-2(N\mu - \ln(1/\delta))^2/(N\lambda_*^2))$. Substituting $\lambda_* = 4(q_* - b)$ and $\mu = 2(q_* - b)^2$, the inequality $2(N\mu - \ln(1/\delta))^2/(N\lambda_*^2) \ge \ln(1/\beta)$ is a quadratic constraint on $\sqrt{N}$. Its positive root is exactly the threshold displayed in , at which the exponent is at least $\ln(1/\beta)$ and automatically $N\mu \ge \ln(1/\delta)$. The same supermartingale and Ville bound of Theorem control the null crossing probability, independently of the alternative class.

Echoes, accessible records, and proper-time non-return

Assumption (Uncoupled geometry and finite register layer). A model is a product (m, K), where m contains prescribed metrics, worldlines, and their time orientations, and K contains finite registers, Hamiltonians, pulses, and readouts. A gate on K does not change m. Echo intervals are hosted on a supplied future ordinary timelike curve on a common-temporal experiment region.

Theorem (Four-pulse and Weyl echoes do not create a loop). *Let a qubit Hamiltonian be* $\mathrm{H}_0 = (\omega/2)Z$*, let* $E = e^{-i\mathrm{H}_0\Delta}$*,* $\Delta > 0$*, and use ideal instantaneous pulses* X, Y, X, Y*, each following an equal free interval. The complete propagator is*

$$U_{\text{echo}} = YE\,XE\,YE\,XE = -I.$$

Thus every density matrix refocuses, although the hosted elapsed proper time is $4\Delta > 0$*.*

For a d-level register, define

$$X|j\rangle = |j + 1 \bmod d\rangle, \qquad Z|j\rangle = \omega_d^j |j\rangle, \qquad \omega_d = e^{2\pi i/d}.$$

Then

$$ZXZ^\dagger X^\dagger = \omega_d I.$$

This phase echo also leaves every register density matrix unchanged.

Neither identity changes a spacetime event or produces a closed causal curve in the supplied common-temporal region.

Proof. Both X and Y anticommute with Z, so conjugation flips the free Hamiltonian sign. The four toggled free phases have coefficients $1, -1, 1, -1$ and cancel. The pulse product is $YXYX = -I$. For the Weyl identity, evaluate $ZX = \omega_d XZ$ on each basis vector.

On the hosted ordinary timelike curve, $d\hat{\tau} > 0$; host proper time is also positive by cone containment. Every finite

free interval contributes positive elapsed time. Pulses act only on the register layer. The common temporal function increases along the unchanged hosted trajectory, excluding event closure.

Remark (Positive proper time alone is not chronology protection). A genuine prescribed CTC also has positive local proper-time increments. What excludes a loop in Theorem is the common temporal host and the fact that the echo does not change it, not positivity of proper time alone. On an already chronology-violating host, an echo neither creates nor removes its geometric CTCs.

Proposition (Register labels do not supply clock geometry). *Let a finite clock readout have a fixed complete record–residual channel. Changing an external dictionary $k \mapsto t_k$ to $k \mapsto at_k$, $a > 0$, leaves the channel unchanged. Consequently the channel alone supplies neither a time unit, a worldline, nor a geometric return event.*

Proof. The dictionary is not an argument of the channel. Every matrix element and outcome law is unchanged. The associated geometric meanings therefore require additional supplied data.

This is the operational separation documented in [4, 6, 10]. Unread capacity, archive restoration, and a returning clock are distinct targets [11]. No identification of inaccessible register capacity with a physical dark halo is made.

Sagnac, winding stress, and periodic-density inverse problems

The common rejection contract

Definition (Calibrated comparison package). A package fixes a model class θ, its supplied host and boundary data, nuisance set $\mathcal{N}$, forward statistics $F_j(\theta, \eta)$, confidence regions $\mathcal{C}_j$, and independently bounded approximation and implementation allowances ε_j. Reject the parameter instance by test j when

$$\inf_{\eta \in \mathcal{N}} \operatorname{dist}\left(F_j(\theta, \eta), \mathcal{C}_j\right) > \varepsilon_j.$$

Statistics, units, and complete outcome alphabets are fixed before applying the rule.

Proposition (Coverage and non-identification). *If the true statistic belongs to $\mathcal{C}_j$ with probability at least $1 - \delta_j$,*

and the forward discrepancy is bounded by ε_j at a true nuisance value, the true instance is rejected with probability at most δ_j. A finite collection of tests has union error at most $\sum_j \delta_j$.

For a declared data map $\mathscr{D}$, a real target F is identified exactly when it is constant on every data fibre. A common datum with target values F_1, F_2 has two-point absolute minimax error $|F_1 - F_2|/2$.

Proof. On the coverage event the infimum in is at most the allowance. Apply the union bound. A single-valued data rule must be constant on each fibre; conversely assign the common fibre value. The triangle inequality gives the half-gap risk, attained by the midpoint.

R − Sagnac: exact guided-ring travel times

Use the stationary collar of , with $\tilde{g}_{\varphi\varphi} = c + b =: H_r > 0$. A supplied guide constrains light to fixed r, z. On a variable collar these guided paths need not be free null geodesics; the guide is part of the experiment. A stationary receiver has $\mathrm{d}\overset{\sim}{\tau} = \mathrm{d}t$.

Theorem (Exact Sagnac forward and chronology-blind completion fibre). *The future arrival times for one positive and one negative azimuthal revolution are*

$$t_+ = 2\pi\left(\sqrt{a^2 + H_r} - a\right), \qquad t_- = 2\pi\left(\sqrt{a^2 + H_r} + a\right).$$

Thus

$$\Delta\overset{\sim}{\tau} = t_- - t_+ = 4\pi a, \qquad a = \frac{t_- - t_+}{4\pi}, \qquad H_r = \left(\frac{t_- + t_+}{4\pi}\right)^2 - a^2.$$

The Sagnac difference is independent of b.

For constant $a > 0, H_r > 0$, the same complete ordinary metric and the same ideal ordinary travel-time data have:

1. *a chronology-preserving host completion with $b_1 = H_r/2$, $c_1 = H_r/2 > 0$;*

2. *a chronology-violating host completion with $b_2 = H_r + a^2/2$, $c_2 = -a^2/2 < 0$. Both have $a^2 + c_i > 0$, $B_i = b_i/\kappa^2 > 0$, and $\tilde{g} = g_i + B_i q \otimes q$. Therefore ideal ordinary Sagnac data alone do not identify the host chronology in this*

declared completion class.

Proof. Writing $y = \mathrm{d}t/\mathrm{d}\varphi$, the guided null equation is

$$-y^2 - 2ay + H_r = 0.$$

Its roots are $-a \pm \sqrt{a^2 + H_r}$. Integrate the positive root over 2π and the negative root over -2π, giving . The reconstruction follows by sums and differences.

The two completions have the same $c_i + b_i = H_r$. For $c_1 > 0$, completing the square as in Theorem gives a complete globally hyperbolic host product. For $c_2 < 0$, the constant-component circles are closed timelike geodesics. Their determinants are negative by the stated inequalities. Every ordinary forward depending only on the fixed $\hat{\sim}$ and identical acquisition data agrees.

This exact fibre is one of the main findings. A nonzero Sagnac effect also occurs on chronology-preserving hosts. Rejecting a measured shift coefficient is possible through ; inferring a dark CTC from it requires an additional host-to-ordinary bridge or winding calibration. Gravitational-wave, lensing, or source-sensitive data may refine the fibre, but are not silently included in this statistic.

R − GW − ring: a declared host-null arrival forward

The ordinary Sagnac non-identification of Theorem is not withdrawn. The following declared additional forward uses guided host-null arrivals, computed from the host null equation rather than by silent analogy with .

Theorem (Host-null guided arrivals and fibre reduction). *On the stationary collar , a supplied guide constrains a host-null signal to fixed r, z. Writing $y = \mathrm{d}t/\mathrm{d}\varphi$, the host null equation is*

$$-y^2 - 2ay + c = 0,$$

with roots $y = -a \pm \sqrt{F}$ and $F = a^2 + c > 0$. The product of the roots is $-c$. Align the host orientation with ∂_t, so that a tangent $y\partial_t + \partial_\varphi$ is future-directed precisely when $y > -a$.

1. *In the chronology-preserving class $c > 0$ with $a > 0$, both future guided arrivals exist, with*

$$t_+^{(g)} = 2\pi(\sqrt{F} - a), \qquad t_-^{(g)} = 2\pi(\sqrt{F} + a),$$

both strictly positive. In that domain,

$$c = \left(\frac{t_+^{(g)} + t_-^{(g)}}{4\pi}\right)^2 - a^2,$$

and c is identified once a is known (for instance from the ordinary Sagnac reconstruction). The reconstruction is not used outside this domain.

2. *In the chronology-violating class* $c < 0$*, both roots have the same sign, opposite to the sign of a. For* $a > 0$ *the increasing-φ future root has* $t_+^{(g)} < 0$*: a stationary observer whose clock increases with t does not receive a two-way pair of future guided host-null arrivals. The fibre-breaking signature is then unidirectional azimuthal cone tilt, or a one-sided future arrival, not a two-time reconstruction. The same conclusions hold, with* $c = -h < 0$*, on the complete witness of Assumption . If the host-null times of (i) are supplied as data, together with the ordinary identification of a, the ordinary Sagnac fibre of Theorem is reduced and the two printed completions are separated. The theorem does not manufacture a host-bridge device, does not discharge an observational test as a data analysis, and does not identify a dark CTC from ordinary Sagnac times alone.*

Proof. The host null equation at fixed r, z is $-\dot{t}^2 - 2a\dot{t}\dot{\varphi} + c\dot{\varphi}^2 = 0$, which is . The roots and their product $-c$ are elementary. The inner product $g(y\partial_t + \partial_\varphi, \partial_t) = -y - a$ is negative precisely when $y > -a$, so only the root $-a + \sqrt{F}$ is future-directed for increasing φ. Integrating that root over $\Delta\varphi = +2\pi$ and the past root $-a - \sqrt{F}$ over $\Delta\varphi = -2\pi$ produces . If $a > 0$ and $c > 0$, then $\sqrt{F} > a$, so both times are positive, and the sum identifies $\sqrt{F}$, hence $c = F - a^2$. If $a > 0$ and $c < 0$, then $\sqrt{F} < a$, so $t_+^{(g)} < 0$, while the roots have positive product and are therefore of one sign, both negative. The complete witness has $c = -h < 0$ and $a > 0$, and falls under (ii).

Proposition (Ordinary Fermi–Walker precession does not split *b* from *c*). *On the collar , with smooth* $a(r)$*,* $c(r)$*, and* $b(r) = B\kappa^2$*, write* $H := c + b$*. Assume* $a^2 + H > 0$ *and* $F = a^2 + c > 0$*.*

1. *The ordinary static congruence* $u = \partial_t$ *has vanishing four-acceleration. Its vorticity has the only independent*

component $\omega_{r\varphi} = -\frac{1}{2}\partial_r a$*, and the Fermi–Walker precession rate in the ordinary rest frame is*

$$|\Omega|_{\mathrm{o}} = \frac{|\partial_r a|}{2\sqrt{a^2 + H}}.$$

2. *The host static congruence likewise has vanishing acceleration, with*

$$|\Omega|_g = \frac{|\partial_r a|}{2\sqrt{F}}.$$

3. *If* $c < 0$*, the host azimuthal circles have radial acceleration* $\nabla_u u = \nabla c/(2c)$*, as already recorded in . There are no ordinary timelike azimuthal circles while* $H > 0$*.*

4. *Ordinary Fermi–Walker data therefore determine a combination of* a*,* $\partial_r a$*, and* H*, and do not by themselves isolate* $\partial_r b$ *from* $\partial_r c$*. If a host-sector datum from (ii) or (iii) is independently supplied, the printed radial derivatives may be separated locally:* H *and* F *determine* $b = H - c$*, while (iii) determines* $\partial_r c$*, and hence* $\partial_r b$ *once* $\partial_r H$ *is obtained from the ordinary reconstruction of* $H(r)$*. This is a declared two-sector comparison on the printed collar. It is not a unique global reconstruction of* (a, b, c) *on an unspecified host, and not a discharged observational test.*

Proof. The ordinary two-block and its inverse are those of Proposition . Direct computation of the Levi-Civita connection yields, in particular, $\Gamma^r_{tt} = 0$ and $\Gamma^r_{t\varphi} = \frac{1}{2}\partial_r a$, $\Gamma^r_{\varphi\varphi} = -\frac{1}{2}\partial_r H$. For $u = \partial_t$ the acceleration $\nabla_u u = \Gamma^r_{tt}\partial_r$ vanishes. The only nonzero vorticity components are $\omega_{r\varphi} = -\omega_{\varphi r} = -\frac{1}{2}\partial_r a$. Raising with the ordinary inverse gives $\omega_{ab}\omega^{ab} = (\partial_r a)^2/(2(a^2 + H))$, so the proper precession rate $|\Omega|_{\mathrm{o}} = \sqrt{\omega_{ab}\omega^{ab}/2}$ is . Replacing H by c throughout gives . Item (iii) is Proposition , together with ordinary temporalness $H > 0$, which makes the φ-circles ordinary-spacelike. Item (iv) is then a comparison of the printed scalars: ordinary data see $H = c + b$, while a host datum sees c or F, and the difference is b.

R − wind: action-dependent controller stress

Proposition (Winding energy in a stationary collar). *For the canonical action , the host stationary observer* $w = \partial_t$ *in measures*

$$\rho_{\Theta} = T_{\Theta}(w, w) = \frac{\kappa^2}{2F(r)}.$$

On the spinning-string-type collar

$$g_{\mathrm{str}} = -(\mathrm{d}t + a\mathrm{d}\varphi)^2 + \mathrm{d}r^2 + r^2\mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad a > 0,$$

one has $F = r^2$ *and*

$$\rho_{\Theta} = \frac{\kappa^2}{2r^2}.$$

Here $q = \kappa\mathrm{d}\varphi$ *satisfies the free controller equation, but timelike azimuthal circles in* $r < a$ *have nonzero acceleration*

$$(\nabla_u u)^r = \frac{r}{r^2 - a^2}.$$

They are not free dark circular dust orbits.

Proof. The observer is host-normalized and $q(w) = 0$. Using gives $T_{\Theta}(w, w) = \sigma/2 = \kappa^2/(2F)$. Expansion of gives $c = r^2 - a^2$, $F = r^2$. The raised winding form has only stationary t, φ components, so

$$\nabla_a q^a = \frac{1}{r}\partial_a(rq^a) = 0.$$

Finally substitute $\partial_r c = 2r$ into .

A mandatory $1/r^2$ law is therefore justified in this particular canonical string collar, not on every cylindrical host. The complete constant-coefficient witness instead has constant winding density.

Corollary (A lower stress bound needs a coupling ceiling). *In the string collar at* $r < a$*, ordinary temporalness requires* $B\kappa^2 > a^2 - r^2$*. If an independent calibration supplies* $0 < B \le B_{\max}$*, then*

$$\rho_{\Theta} > \frac{a^2 - r^2}{2B_{\max}r^2}.$$

Without a coupling ceiling or fixed winding normalization, the cone-flip inequality alone gives no universal

positive numerical lower bound on controller energy.

Proof. Divide the cone-flip inequality by $2Br^2$. For the last statement, in a class allowing different target radii and couplings, hold $b = B\kappa^2$ fixed and let $\kappa \to 0$, $B = b/\kappa^2 \to \infty$. The metric deformation stays fixed while the canonical density tends to zero. A fixed circle target would instead quantize its winding and must be declared separately.

A stress residual inferred from rotation or lensing requires an actual source-compatibility and ordinary-light acquisition model. Equation is not itself a measured mass residual. The flat string collar, like the complete flat witness, cannot match its positive canonical winding stress without additional source accounting.

R − hol: periodic density and internal monodromy

Theorem (Closed-flow continuity constraint). *Let a smooth normalized dark congruence u contain a closed proper-time orbit γ of period $L > 0$. Let $\theta = \nabla_a u^a$, and suppose*

$$\nabla_a(\rho u^a) = 0, \qquad \rho > 0.$$

Then

$$\rho(\gamma(L)) = \rho(\gamma(0))\exp\left(-\int_0^L \theta(\gamma(\tau))\,d\tau\right).$$

Single-valued positive density on the closed orbit requires

$$\int_0^L \theta\,d\tau = 0.$$

If a separately supplied internal quantum seam rule is $\sigma \mapsto V\sigma V^\dagger$, its periodic states satisfy $V\sigma V^\dagger = \sigma$. That internal rule is additional to dust continuity.

Proof. Continuity is $d\rho/d\tau = -\theta\rho$. Integrate this scalar ODE. Equal geometric endpoints and positive single-valued density give . The internal assertion is precisely its supplied return condition.

In Theorem , $\theta = 0$, so the constraint holds exactly. The density is constant along each orbit. The orbit itself must

return in phase space as well as position; this was proved there by constant normalized tangent.

The R − hol contract rejects only a specified closed-flow model whose periodicity forward fails its supplied allowance. A phase recurrence or fitted periodic density alone is not a dark-clock acquisition or proof of a spacetime CTC.

Proposition (Internal monodromy along a closed dark geodesic). *Let γ be a closed future-directed host-timelike geodesic of proper period $L > 0$, as in Theorem (ii), and let a printed internal Hermitian Hamiltonian $H_{\mathrm{d}}(\tau)$ act on a finite-dimensional fibre $\mathbb{C}^N$ along γ. The monodromy*

$$W = \mathscr{P}\exp\left(-i\int_0^L H_{\mathrm{d}}(\tau)\,\mathrm{d}\tau\right) \in \mathrm{U}(N)$$

is well-defined and unitary. If v is an eigenvector with $Wv = e^{i\alpha}v$, the corresponding solution of the internal Schrödinger equation is Floquet: it returns to a phase multiple of itself after one circuit. In particular, if H_{d} is constant along γ, then $W = \exp(-iH_{\mathrm{d}}L)$ and the phases are $-L$ times the eigenvalues of H_{d}. These discrete phases and eigenspaces encode global integrals around the loop. They are not a single-valued elapsed-time scalar; Proposition is not withdrawn. The identity is not Bar 1, not Bar 2, and not backward signalling into the ordinary sector.

Proof. Existence and unitarity of the time-ordered exponential for a smooth Hermitian family on a compact interval are standard. The eigenvalue equation $Wv = e^{i\alpha}v$ is the definition of a Floquet mode. If H_{d} is constant, time ordering is unnecessary and the exponential formula follows. A single-valued real function τ with $u(\tau) = 1$ remains impossible by Proposition , independently of any internal unitary. No coupling to the ordinary sector is introduced.

Resonant gravitational-wave lensing, a chronology resonator, and a Fabry–Pérot scattering theorem are not proved. They are recorded in Appendix as an unfinished programme, not as official theorems. A local regularity identity for a complex-scalar vortex core is likewise recorded there as such, without constructing an asymptotically flat localised time machine on $\mathbb{R}^4$.

Joint observational scope

Theorem (Local Jacobian identification of (a, c, b) by a host-null supplement). *On the stationary collar , write $H := c + b$ and $F := a^2 + c$, and assume $H > 0$ and $F > 0$. Let (t_+, t_-) be the ordinary guided Sagnac arrivals of Theorem , and let a host-null guided arrival be taken from Theorem : either the two-way future pair in the chronology-preserving class $c > 0$, or the one-sided future arrival $t_-^{(g)} = 2\pi(a + \sqrt{F})$ in the chronology-violating class $c < 0$ with $a > 0$.*

1. *At any single radius where the named host-null future arrival exists, the map*

$$(a, c, b) \mathrm{longmapsto} (t_+, t_-, \tau_g)$$

is a local diffeomorphism onto its image, where $\tau_g = t_+^{(g)}$ if $c > 0$ and $\tau_g = t_-^{(g)}$ if $c < 0$. The Jacobian determinant is

$$\det \frac{\partial(t_+, t_-, \tau_g)}{\partial(a, c, b)} = \frac{4\pi^3}{\sqrt{F(a^2 + H)}} \neq 0.$$

Consequently (a, c, b) are locally inverted at that radius. If the same data are supplied on an open radial interval, the inversion holds pointwise on that interval.

2. *The winding scale κ is not a coordinate of this map, and is not inverted. Ordinary Fermi–Walker precession determines a combination of a, $\partial_r a$, and H; if Sagnac data are already known on an open radial interval, then $\partial_r a$ is obtained by differentiation, and the gyroscope datum is a consistency check rather than an independent third host coordinate.*

3. *Host azimuthal circular acceleration , when $c < 0$, determines the logarithmic derivative $\partial_r \log |c|$ and therefore an ordinary differential equation for $c(r)$. A boundary value of c at one radius remains necessary. This supplement does not, by itself, invert (a, c, b) without that constant of integration. Ordinary-only data remain fibre-blind in the sense of Theorem , which is not withdrawn. Proposition is not withdrawn. The theorem uses neither the winding-stress channel* $\mathrm{R} - \mathrm{wind}$ *nor the monodromy channel* $\mathrm{R} - \mathrm{hol}$*, and it does not prove a nonempty joint acceptance set across all five named comparison channels. A host-null or other host-sector datum is required. The*

statement is a declared kinematic map on the printed collar, not a data analysis and not a discharged observational test.

Proof. The ordinary arrivals are the diffeomorphism of Theorem from (a, H) onto $\{t_{\pm} > 0\}$. Differentiating the three component functions $t_{\pm} = 2\pi(\sqrt{a^2 + H} \mp a)$ and $\tau_g = 2\pi(\sqrt{F} - a)$ or $\tau_g = 2\pi(\sqrt{F} + a)$, according as the named host-null datum is $t_+^{(g)}$ or $t_-^{(g)}$, with respect to (a, c, b), and using $H = c + b$, yields the Jacobian matrix whose determinant simplifies algebraically to $4\pi^3/\sqrt{F(a^2 + H)}$. The denominator is positive on the open set $F > 0$, $a^2 + H > 0$, which is implied by the standing collar inequalities together with $H > 0$. This proves (i). Item (ii) is the already computed ordinary precession , together with the observation that none of $t_{\pm}$ or τ_g depends on κ separately from the combination $b = B\kappa^2$. Item (iii) is Proposition : $\nabla_u u = \nabla c/(2c)$ rearranges to $\partial_r c = 2c\,\alpha^r$, whose solutions differ by a positive multiplicative constant.

The three geometric contracts and the finite R − CTC response contract therefore continue to have different forward domains. No nonempty joint acceptance set across R − Sagnac, R − GW − ring, R − gyro, R − wind, and R − hol is proved. Electromagnetic silence does not make all dark effects invisible, and does not force every test to be "purely gravitational." It excludes the selected direct electromagnetic channel; other interactions must be audited as in Section .

A refined data package can distinguish some of the completion fibres of Theorem . It must publish the additional data and calibration rather than relabel the old statistic. Theorem is one such declared supplement, not a relabelling of the ordinary Sagnac statistic.

The typestate of Theorem is not withdrawn: ordinary-only data remain fibre-blind, κ is not inverted by (t_+, t_-, τ_g) alone, and no nonempty joint acceptance set across the five named comparison channels is proved in the observational sense of actually acquired data. The next two statements adjoin an independently supplied winding-energy datum, and record a data-free continuity statement, without converting that typestate into a discharged observational test.

Theorem (Four-parameter Jacobian with an independent winding-energy datum). *On the stationary collar , keep the notation and hypotheses of Theorem , and adjoin the canonical winding-energy datum* $\rho_\Theta = \kappa^2/(2F)$ *of Proposition , with* $\kappa > 0$.

1. *The map*

$$(a, c, b, \kappa) \text{longmapsto} (t_+, t_-, \tau_g, \rho_\Theta)$$

is a local diffeomorphism onto its image on the open set $F > 0$, $a^2 + H > 0$, *and* $\kappa > 0$. *Its Jacobian is block triangular, and*

$$\det \frac{\partial(t_+, t_-, \tau_g, \rho_\Theta)}{\partial(a, c, b, \kappa)} = \frac{4\pi^3 \kappa}{F^{3/2}\sqrt{a^2 + H}} \neq 0.$$

Consequently (a, c, b, κ) *are locally inverted at any radius where the named host-null future arrival exists and* ρ_Θ *is independently supplied. The coupling is then recovered by the bookkeeping identity* $B = b/\kappa^2$.

2. *This inversion holds only when* ρ_Θ *is independently supplied. The datum* ρ_Θ *is a host-stationary measurement of canonical winding stress (*R − wind*), not an ordinary-only operational channel. Ordinary-only data remain fibre-blind. Theorems and are not withdrawn. The statement is a declared kinematic map on the printed collar, not a data analysis and not a discharged observational test. In the chronology-violating class the named host-null coordinate remains the one-sided future arrival* $t_-^{(g)}$ *of Theorem , not a two-future-time product.*

Proof. The first three observables $t_\pm$ and τ_g do not depend on κ separately from the combination $b = B\kappa^2$. The 4×4 Jacobian is therefore block triangular, with upper-left 3×3 block equal to the Jacobian of Theorem and with vanishing upper-right column. The remaining diagonal entry is

$$\frac{\partial \rho_\Theta}{\partial \kappa} = \frac{\kappa}{F},$$

because $F = a^2 + c$ is independent of κ. Multiplying the determinant by κ/F yields , which is nonzero on the stated open set. The bookkeeping identity $B = b/\kappa^2$ is the definition of b. Item (ii) records the typestate already proved in

Theorems and , together with the host-stationary character of .

Proposition (Data-free synthetic five-channel continuity). *Let* $(a_0, c_0, b_0, \kappa_0)$ *be a parameter tuple of the constant cylindrical witness of Assumption , with* $c_0 = -h < 0$, $F_0 = a_0^2 - h > 0$, $H_0 = b_0 - h > 0$, *and* $\kappa_0 > 0$. *Let the five named forward maps be the ordinary Sagnac arrivals of Theorem , the one-sided future host-null arrival* $t_-^{(g)}$ *of Theorem , the ordinary Fermi–Walker rate of Proposition , the winding energy of Proposition , and the closed-flow integral* $\int \theta \, d\tau$ *of Theorem . Each of these maps is continuous at the true tuple. Consequently, if* $\mathscr{C}_1, \ldots, \mathscr{C}_5$ *are open neighbourhoods of the corresponding exact forward images (equivalently: synthetic regions with positive tolerances), there exists an open neighbourhood of* $(a_0, c_0, b_0, \kappa_0)$ *that is mapped into every* $\mathscr{C}_j$.

On this constant witness, $\partial_r a = 0$, *so the ordinary precession rate vanishes identically and the gyroscope forward is a consistency check rather than an independent third host coordinate, as already typestated in Theorem (ii). Likewise* $\theta = 0$ *along the closed dark congruence of Theorem , so the density constraint holds identically; an internal monodromy as in Proposition depends on a separately printed Hamiltonian and on the already determined period* $L = 2\pi\sqrt{h}$, *and is not claimed as an additional inverting coordinate.*

This is a data-free continuity statement. It is not a nonempty joint acceptance set for actually acquired data, and it does not withdraw the typestate, already recorded after Theorem , that no nonempty joint five-channel acceptance set was proved in that observational sense.

Proof. The maps (t_+, t_-), $t_-^{(g)}$, and ρ_Θ are rational (or algebraic) functions of (a, c, b, κ) on the open set $F > 0$, $a^2 + H > 0$, and $\kappa > 0$, hence continuous there. On a C^1 collar neighbourhood of the constant witness, $|\Omega|_o = |\partial_r a|/(2\sqrt{a^2 + H})$ is continuous and vanishes at the constant point. The expansion scalar θ of the closed dark congruence of Theorem vanishes identically on that witness, and remains continuous under C^1 perturbations of the congruence, so the integral $\int \theta \, d\tau$ is continuous and vanishes at the true tuple. The intersection of the five preimages of the open sets $\mathscr{C}_j$ is therefore a neighbourhood of the true tuple. No acquired-data acceptance region is constructed, and no vacuous constraint is re-labelled as an independent inverting coordinate.

Consolidated release, novelty, and remaining obligations

Theorem (Mathematical release of Dual-Ledger-Chrono-DM-A). *For the explicitly defined model classes:*

1. *a fixed-winding action has the local exchange and external-switch ledger of Theorem ; a printed switch kinetic slot closes the enlarged ledger on the switch shell, as action bookkeeping (Theorem);*

2. *the complete cylindrical quotient has closed host timelike geodesics, positive conserved test dust, and an ordinary global Cauchy foliation; its host d'Alembertian is strictly hyperbolic relative to a globally defined timelike covector, without a global Cauchy theorem (Proposition); a Fourier mode analysis supplies an explicit dispersion locus and an off-resonant mode resolvent, without a global Cauchy or Fredholm theorem (Lemma , Proposition); a frequency-domain $i\varepsilon$ mode family is a regularised resolvent away from a declared branch-cut set, and is not a retarded Green function on the quotient (Proposition);*

3. *uniform narrowing of a closed host-timelike tangent family by a closed rank-one controller requires a nonzero controller period; an exact real scalar cannot provide it; on the printed homogeneous quotient this obstruction extends to the translation-invariant homology class and to time functions constant on that class (Theorems and);*

4. *common temporal propagation excludes a two-way anti-telephone loop, even when ordinary synchronization reverses the coordinate order of a dark packet;*

5. *electromagnetic silence does not prove complete operational isolation; the qualified fixed-host isolation statement is proved separately; a declared cover closure yields only a formal first-order holonomy increment, not the monomial leakage scaling rejected in Appendix ; a naive periodized retarded sum on the cover is not locally finite, because the deck translations are timelike (Proposition), and that obstruction is not withdrawn by the frequency-domain regularisation of Proposition ;*

6. *positive-source null compatibility and periodic affine focusing have the printed obstructions, but compact horizon generation alone does not supply negative null curvature; the flat witness retains its strictly negative required null residual; a standard Gödel comparison is Einstein-compatible for dust plus a negative cosmological*

term, and acquires an explicit nonzero residual once the canonical winding stress is adjoined;

7. *a prescribed bimetric Misner–winding overlay is a kinematic causal comparison with a written ordinary Cauchy foliation on every open interval where* $H(T) > 0$*, and is not dynamical manufacture; test Klein–Gordon leading branches and ordinary smoothness on the class* $H > 0$ *are recorded, without unique dark continuation (Proposition); collisionless Vlasov characteristics split into a transmitting open set and a blue-shifted locus, without converting the overlay into dynamical manufacture (Proposition);*

8. *factorized finite CTC rules insulate ordinary evolution from the dark loop under the stated fixed-preparation conditions; controller dependence and zero-probability postselection exceptions are explicit;*

9. *the specified Deutsch portal distinguishes two nonorthogonal preparations, while the specified complete postselection experiment is a linear CPTP instrument; the equal mixture of those preparations happens to have an affine ordinary output, and an ancilla audit shows that the toy rule does not define a linear extension (Theorem); the ordinary output remains affine on the entire printed convex segment joining the two discriminator inputs, while the ancilla extension fails for every interior weight (Theorem); the complementary segment joining* $\mathrm{proj}\,0$ *to* $\mathrm{proj}\,+$ *has a rational non-affine ordinary output under the same unitary, and is not the discriminator pair (Theorem);*

10. *complete-record optional stopping controls rejection error for the stated conventional-channel null; a sufficient e-process threshold on the ideal discriminator path is recorded (Corollary); a sufficient type-II integer threshold is recorded for a declared noisy alternative class, without withdrawing the ideal-path typestate (Corollary);*

11. *echo identities do not change the supplied host or acquire a returned geometric clock; an internal monodromy along a closed dark geodesic is a Floquet fact, not a returned elapsed-time scalar;*

12. *Sagnac, winding-stress, and periodic-density forwards have exact calibrated formulas, including an ordinary Sagnac fibre containing both host chronology types; a declared host-null forward reduces that fibre when both future arrivals exist, and is unidirectional in the chronology-violating class; ordinary Fermi–Walker precession*

does not, by itself, split the winding coefficient from the host profile; a printed Jacobian identifies (a, c, b) *locally when ordinary Sagnac data are paired with a named host-null arrival, without inverting* κ *and without a five-channel joint acceptance set (Theorem); adjoining an independent winding-energy datum inverts* (a, c, b, κ) *by a block-triangular* 4×4 *Jacobian (Theorem), while a data-free five-channel continuity statement is not an observational joint acceptance set (Proposition). These results do not construct a self-gravitating operational dark-matter time machine, do not evade Hawking chronology protection, and do not advance the inherited time-machine score, which remains* 0 *of* 3 *as the full three-part bar.*

Proof. Items (i)–(iii) are Theorem , Theorem , Theorem , Proposition , Proposition , Lemma , Proposition , Theorem , Theorem , and Proposition . Items (iv)–(vii) are Proposition , Theorem , Propositions and , Proposition , Proposition , Proposition , Theorem , Corollary , Theorem , Example , Propositions and , Theorem , Proposition , and Proposition . Items (viii)–(x) are Theorem , Theorem , Theorem , Theorem , Theorem , Proposition , Theorem , Corollary , and Corollary . Items (xi)–(xii) are Theorem , Proposition , Proposition , Theorem , Theorem , Theorem , Theorem , Theorem , Proposition , Proposition , Proposition , Corollary , and Theorem . The final exclusion follows from the printed construction domains, the explicit failure of source matching for the principal flat witness, the residual winding obstruction on the Gödel comparison, and the kinematic typestate of the Misner overlay.

Potential impact and honest novelty

The most useful conclusions are not the original slogan. They are the exact separation

$$\text{boxed prescribed dark CTC} \neq \text{?ordinary backward signal} \neq \text{?clock return} \neq \text{?manufactured time machine.}$$

The complete quotient replaces an incorrect open-collar Cauchy claim by a genuine global example. The Sagnac completion fibre locates a specific observational ambiguity rather than asserting merely that "more data are needed." The quantum insulation theorem locates the precise factorization and success-probability conditions, and the complete-record certificate avoids confusing conditional filtering with a nonlinear physical channel.

These are potentially useful mathematical and methodological findings. Constant Lorentzian quotients, winding fields, disformal metrics, focusing equations, fixed-point CTC models, and quantum discrimination are established ingredients. No first-priority claim is made for those ingredients or for the standard discrimination circuit. The contribution is their self-contained assembly with explicit global hypotheses, exact observational fibres, and corrected operational boundaries.

Outstanding construction tasks

The following Version 1 construction obligations remain, in the exact residual senses now recorded.

• a chronology-preserving initial-data evolution that manufactures the dark CTC region, in the dynamical Einstein sense: the prescribed Misner–winding overlay of Theorem remains a kinematic comparison, Proposition is a test-field calculation on that overlay, and Proposition is a collisionless characteristic comparison on the same overlay; none of these discharges this obligation;

• a source-compatible, globally coupled ordinary–dark–winding solution with the desired chronology split: the flat residual of Proposition stands, and the standard Gödel comparison of Propositions and is Einstein-compatible only for dust plus a negative cosmological term, with an explicit nonzero residual once the canonical winding stress is adjoined; no winding-inclusive replacement is admitted in the official spine (Appendix);

• a global solution-selection and stability theory for all fields propagating on the chronology-violating host: Proposition identifies a hyperbolic principal symbol, and Lemma with Proposition classify resonant modes and an off-resonant multiplier; Proposition is a frequency-domain regularised mode family, not a retarded Green function and not a Cauchy, Leray–Ohya, or compactified Fredholm theorem;

• an ordinary device that prepares, modulates, and reads a physical dark loop with complete calibration;

• a field-theoretic derivation of either finite CTC rule: both rules remain separately assumed toy models; Theorems , , and are audits inside the Deutsch rule, not such a derivation;

• an empirical likelihood, exclusion contour, or nonempty joint observational acceptance region across all five

named comparison channels: Theorem is a declared kinematic map for a named host-null supplement, Theorem adjoins an independent winding-energy datum, and Proposition is a data-free continuity statement, not a data analysis and not that observational five-channel region;

• a removal of the external geometric clock or a derivation of Host C. A compactly supported Ori–Sofer or vortex-ring host on $\mathbb{R}^4$, a gravitational Fabry–Pérot or chronology resonator, a continuum scattering derivation of either finite CTC rule, and a coupled dynamical Misner-type manufacture, remain unfinished programmes (Appendix , Appendix), not official theorems. These are construction obligations, not conclusions that can be supplied by changing the name of a register or host.

appendix

Debugging the proposed programme

toprule Proposed inference Corrected statement

midrule endfirsthead toprule Proposed inference Corrected statement

midrule endhead bottomrule endfoot

A positive azimuthal ordinary component makes t Cauchy It makes dt timelike in the stated block. Finite open radial endpoints can still destroy global hyperbolicity. A complete quotient is proved separately.

Every timelike azimuthal circle is freely traversed by dark dust Its acceleration is $\nabla c/(2c)$. Free circular dust requires the additional geodesic condition. The complete constant-component witness satisfies it.

The determinant changes by $-B\kappa^2$ for every stationary host That expression assumes $g_{tt} = -1$. For a block with $g_{tt} = -N^2$, the change is $-N^2 B\kappa^2$. The local theorem fixes the normalization explicitly.

Winding is the same timelike scalar-gradient class as Temporal-A The winding covector has positive host norm in this collar. It is a non-exact closed form or a circle-valued scalar differential, not a global real scalar with timelike gradient.

Rolle's theorem proves a complete ordinary CTC remains It proves local timelikeness of the original tangent at extrema and obstructs uniform narrowing of that tangent family. It does not classify every remaining closed curve.

Electromagnetic silence blocks every signal on a CTC host Metric coupling and independently supplied neutral portals remain. Complete isolation needs fixed host, complete factorization, and control-independent selection data.

A boosted negative $\Delta t'$ closes an anti-telephone The common host temporal coordinate still increases on every admissible segment. Ordinary boosts are not symmetries giving arbitrary host-past packet preparation.

Compact generation forces $G_{ab}k^{a}k^{b} < 0$ Additional global focusing hypotheses are essential. The flat compact Misner horizon has zero null curvature. A periodic affine focusing identity is proved under its own stronger hypotheses.

Every ordinary NEC tensor transfers to host NEC The sufficient transfer used here is the ordinary weak energy condition. Positive perfect-fluid density and $\widehat{\rho} + \widehat{p} \geq 0$ provide it.

An eternal or noncompact CTC host is automatically positive-source compatible Energy conditions and source matching must be computed. The principal flat quotient has a strictly negative required null residual despite positive conserved test dust.

Factorization makes the ordinary channel independent of every controller setting It removes dependence on the dark loop. The ordinary-controller unitary or controller preparation can still change the ordinary channel.

Every factorized P-CTC gives an ordinary output If the dark unitary has zero trace, its Bell-contracted success operator vanishes. No normalized output exists.

Every portal clones or discriminates nonorthogonal states A portal must be specified and analysed. One explicit Deutsch unitary discriminates the printed pair. No universal cloning theorem is supplied.

A normalized postselection map is a complete nonlinear experiment The normalized successful branch omits its probability and failures. The complete flagged instrument is linear and CPTP.

Optional stopping can be applied to selected successful trials The theorem applies to all attempted trials under the complete conditional-mean null. Hidden postselection changes that null.

A four-pulse echo returns a worldline event It refocuses a register state on an unchanged hosted trajectory. Positive elapsed time and external geometry remain.

A nonzero Sagnac delay proves a dark CTC The same ordinary metric has both chronology-preserving and chronology-violating host completions when winding calibration is omitted.

Winding always produces a mandatory $1/r^2$ energy law That law holds for the specified canonical spinning-string collar. Other hosts have other norms; a numerical lower bound also needs fixed normalization or a coupling ceiling.

A periodic density or holonomy is an acquired dark clock It is a consistency condition on a supplied closed-flow model. A single-valued unbounded elapsed-time field cannot satisfy $u(\tau) = 1$ around a closed orbit.

A two-time host-null formula identifies c on every collar, including the CTC witness Both future guided host-null arrivals exist only in the domain where they are proved, typically $c > 0$. For $c < 0$ the roots have the same sign, and the fibre-breaking signature is unidirectional tilt. The ordinary Sagnac fibre is not withdrawn.

Ordinary Fermi–Walker precession reconstructs (a, b, c) uniquely It determines a combination of a, $\partial_r a$, and $H = c + b$. A host-sector datum is required to split $\partial_r b$ from $\partial_r c$. No unique global reconstruction on an unspecified host is claimed.

Chronology leakage scales as $\mathcal{O}(G^2 m_o \rho_d)$ on the CTC host No retarded Green function is supplied on the quotient. A formal cover identity is linear in a declared kernel, and the printed monomial is not obtained as a complete coefficient.

A switch kinetic action derives the switch or manufactures a CTC region It is on-shell bookkeeping for an enlarged printed action on a prescribed host. The un-enlarged remainder is retained when the kinetic slot is omitted.

The Deutsch output of the equal mixture differs from the mixture of outputs For the printed unitary and equal mixture both sides equal $\frac{1}{2}I$. Nonlinearity is the already proved trace-distance obstruction, and the failure of a linear ancilla extension, not that false inequality.

A Gödel–winding ansatz is an exact positive-source Einstein match with $T_{\text{res}} \equiv 0$ The printed unit-radius Gödel metric matches dust plus a negative cosmological term. Adjoining the canonical winding stress produces the explicit residual . A null projection is not a full tensor equation, and a negative Λ is not strictly positive-source language.

A prescribed Misner overlay manufactures a dark CTC from laboratory initial data The identification is already prescribed. A ramp of $B(s(T))$ is a kinematic comparison, not a coupled Einstein–scalar initial-value problem. A truncated slab with boundary is not styled as globally hyperbolic without qualification.

The host d'Alembertian is well posed by t- or x-evolution, and D-CTC / P-CTC are derived from it On the cylindrical witness, $g^{tt} > 0$ and $g^{xx} = 1$, so neither t nor x is a hyperbolic time. A principal-symbol identity relative to $\xi = \mathrm{d}t + a\,\mathrm{d}\varphi$ is not a Cauchy or Fredholm theorem. The finite rules remain assumed toy models.

An Ori–Sofer vortex ring realises a localised astrophysical time machine in a Minkowski exterior No complete, printed, asymptotically flat metric, regularized core, CTC confinement, and ordinary global hyperbolicity on all of $\mathbb{R}^4$ are proved. A local polar regularity identity $\Phi = \rho e^{i\Theta}$ with $\Phi(0) = 0$ may be recorded as such, and is not that construction.

A dark-loop monodromy is a returned clock, a Bar 1 or Bar 2 discharge, or a chronology resonator It is a Floquet or parallel-transport fact along a prescribed closed geodesic. Scalar age remains obstructed. Resonant gravitational-wave lensing is not proved.

A Fourier analysis supplies Fredholm solvability, a global Green resolver, or a unique evolution on the CTC host It supplies a dispersion locus and a mode-wise off-resonant multiplier (Lemma , Proposition). Neither t nor x becomes a Cauchy time. The finite CTC rules are not derived.

A method-of-images sum of the Minkowski retarded kernel is locally finite and exact on the quotient The deck translations are timelike. Infinitely many past images lie in the causal past of any observation point, so the periodized sum is not locally finite (Proposition). No retarded Green function is obtained on the quotient.

The Misner test-field phase $2n\ln|T|$ is Hawking radiation, or selects a unique dark continuation, or manufactures the overlay dynamically It is a named test-field phase comparison on a prescribed host (Proposition). Both Frobenius branches exist. Ordinary smoothness holds only on the printed class $H > 0$. Theorem is not withdrawn.

The Deutsch output of ρ_λ is non-affine for every $\lambda \in (0,1) \setminus \{\frac{1}{2}\}$ On the printed segment the ordinary output is affine for every $\lambda \in [0,1]$ (Theorem). The equal-mixture typestate is not withdrawn. Nonlinearity remains the trace-distance obstruction and the failure of the linear ancilla extension for every interior weight.

A 4×4 Jacobian inverts (a, c, b, κ) without a host-sector datum, or a five-channel joint acceptance set is proved Ordinary Sagnac plus a named host-null arrival invert (a, c, b) locally where the printed Jacobian has full rank (Theorem). The scale κ is inverted only when the winding-energy datum ρ_Θ is independently supplied (Theorem). Circular acceleration still needs a boundary value of c. Ordinary-only data remain fibre-blind. A data-free continuity statement is not a nonempty joint acceptance set for actually acquired data (Proposition).

The ordinary output of $\widetilde{\rho}_\lambda = \lambda\text{proj}\,0 + (1-\lambda)\text{proj}\,+$ has computational-basis $(0,0)$-entry $(\lambda^2 - 3\lambda + 3)/(3 - 2\lambda)$ Under the same convention as Theorem , that entry equals the dark population $(2-\lambda)/(3-2\lambda)$ (Theorem). The displayed rational is not used. The complementary segment is genuinely non-affine; the printed affine segment of Theorem is not withdrawn.

The Misner co-momentum identity is $2p_T p_\chi - Tp_T^2 + p_x^2 + p_z^2 + m_{\mathrm{d}}^2 = 0$ The printed inverse gives $Tp_T^2 - 2p_T p_\chi + p_x^2 + p_z^2 + m_{\mathrm{d}}^2 = 0$ (Proposition). The finite transmitting limit is $p_T(0) = K/(2p_\chi)$ on the future-directed open set $p_\chi < 0$. The opposite signs are not used. Unique dark Klein–Gordon continuation is not selected.

An $i\varepsilon$ mode resolvent repairs the divergent retarded image sum, or yields a retarded Green function on the quotient Proposition stands: the periodized retarded sum is not locally finite. Proposition is a frequency-domain regularised

mode family away from a declared branch-cut set, not a retarded Green function and not a repair of that time-domain sum.

Writing this companion changes the preceding Standing Ledger No companion status is updated. A prescribed CTC witness is not an operational time machine. The inherited score remains 0 of 3 as the full three-part bar.

Named Track II comparisons and explicit refusals

The following items were proposed as Version 4 discharges of outstanding obligations in Section and of unfinished programmes already recorded above. They are not official Dual-Ledger-Chrono-DM-A theorems, and they do not withdraw Propositions and , Theorem , or the axiomatic typestate of the finite CTC rules. No official Dark-Vortex-CTC-A, Dual-Ledger-Chrono-DM-C, Dark-Chrono-Gate-A, Chrono-Resonator-A, or Chrono-Misner-Dyn-A package is created. Official Chrono-DM-B version 4 is cited only as a documentary source record [12], and is not treated as having resolved the winding residual.

Winding-inclusive Einstein-compatible host. Propositions and stand in the official spine. Official Dual-Ledger-Chrono-DM-B version 4 already refused a residual-free winding-inclusive match on its printed rotating family, kept an azimuthal winding residual, and did not withdraw those official A residuals [12]. A Som–Raychaudhuri, charged-dust, Maxwell, Proca, anisotropic homogeneous, or switch-potential supplement would be a different source class, not a silent replacement of the canonical winding slot on the printed Gödel comparison, and not a slogan imported from a neighbour. No complete local Einstein identity of that class is written here. Even if such a bookkeeping identity were later checked on a printed ansatz, a negative cosmological term would still bar "strictly positive-source" language, and a growing disformal profile making $\hat{g}$ globally hyperbolic would remain a prescribed overlay, not Einstein dynamics and not Host C.

Regularized complex-scalar vortex ring on $\mathbb{R}^4$. The Ori–Sofer / vortex-ring programme remains unfinished. A local polar regularity identity for a complex scalar $\Phi = f(r,z)e^{in\varphi}$ with $f \sim r^{|n|}$ near the axis, as already recorded in the table above, is not a complete, scratch-free, asymptotically flat metric with a compactly supported toroidal frame-dragging core, CTC confinement inside a torus, axis regularity, and global ordinary hyperbolicity on all of

$\mathbb{R}^4$. That construction is not supplied. Official Chrono-DM-C, including the official currently delivered version 3, is not a premise and is not cited. Even a future complete prescribed-host comparison of this type would not advance the time-machine bar and would not create an official Dark-Vortex-CTC-A companion.

Continuum scattering to finite P-CTC or D-CTC rules. Finite Deutsch and postselected maps remain axiomatic finite-register toy rules in the official spine (Section). Theorems , , and are audits inside the already assumed Deutsch rule, not a derivation. Proposition is a frequency-domain mode family, not a continuum derivation of either finite rule. No Unruh–DeWitt, Gaussian, or Keldysh path-integral reduction to the trace-normalised postselected contraction, and no general theorem that Deutsch's nonlinear map cannot arise from any linear path-integral boundary condition, is admitted on the A-spine from this brief. No quantum coefficient is admitted as a chronology theorem. The comparison is not Bar 1, not Bar 2, and not Bar 3. No official Dark-Chrono-Gate-A package is created.

Gravitational Fabry–Pérot / chronology resonator. A piecewise-constant cylindrical step-collar, and a unitary 2×2 transfer-matrix theorem with proved whispering-gallery resonances in a host-null polarization, are not supplied. The programme remains unfinished, as already recorded after Proposition . A named comparison of this type, even if later checked, would not replace Theorem or Theorem as an observational identification theorem. No official Chrono-Resonator-A package is created.

Coupled dynamical Misner-type bifurcation. Theorem remains a prescribed overlay in the official spine. Proposition is a test-field calculation on that overlay, and Proposition is a collisionless characteristic comparison on the same overlay; neither is an initial-value problem for the host metric. No symmetry-reduced 1 + 1, dilaton, Brans–Dicke, or polarized Gowdy initial-value problem is solved here, and no complete argument is claimed that the host metric dynamically develops a compactly generated Cauchy horizon at $T = 0$ while $H(T)$ stays strictly positive. Even a future complete toy-layer audit of that kind would be a conditional comparison only: it would not create Host C, would not manufacture a laboratory time machine from ordinary matter, would not select a unique dark PDE continuation past $T = 0$, would not remove external geometric time, and would not advance the inherited

score of 0 of 3. A symmetry-reduced toy layer is not Bar 3. No official Chrono-Misner-Dyn-A package is created.

Bibliographic and verification discipline

This official version 5 is a typographic successor of official version 4 (request t506u). Official version 4 typestate, labels, and local proofs are retained. Authorised Version 5 work is typestate hygiene, ordinary quotation marks, a Scholar-parseable bibliography without an invented DOI, the documentary negative-control table of Section , and an update of documentary neighbour pins to official currently delivered Temporal-A version 5 and Chrono-DM-B version 4. Those updates are source-record citation hygiene, not imported typestate and not a citation of any unwritten t563u successor. Card 11 remains 0 of 3.

Official Dual-Ledger-Temporal-A version 5 (request t504u) and official Dual-Ledger-Mobility-A version 5 (request t495u) are documentary source records, not previously compiled premises and not theorem imports [2, 3]. Official Dual-Ledger-DM-Capacity-A version 5 and official Chrono-Return-NonCollapse-A version 3 are cited in the same documentary sense [11, 10]. Official Dual-Ledger-Chrono-DM-B version 4 (request t509u) is likewise a documentary source record, not a winding-inclusive residual-free match, and not a theorem premise [12]. Official Chrono-DM-C is not cited.

The supplied Host-C excerpts are a local documentary selection, not a scan of every companion manuscript. They do not establish that every named future enlargement has been classified. Neither their source relationships nor their shared vocabulary are formal premises here.

The Frontiers publisher record provides the Enter-calculus title, volume, article number, and DOI [1]. The present manuscript does not claim independent proof verification of that article.

No registration or browse entry is asserted for this Dual-Ledger-Chrono-DM-A source. No invented deposit, citation-index update, submission, publisher endorsement, or Scholar-indexing campaign is claimed. No landing-page URL is invented for a companion manuscript.

Declarations

Authorship and submission. The author of this official sheet is Ngo Cheung. No affiliation is invented. No endorsement by a cited author or publisher is inferred. Book, figshare, arXiv, and Zenodo deposit, and any Scholar campaign, remain out of scope.

Physical assumptions. The host metrics, quotient identifications, winding sector, actions, matter closures, finite quantum rules, and observational packages are declared. Einstein source compatibility, when considered, is an independently stipulated equation on a prescribed ansatz. It is host-compatibility bookkeeping, not Einstein dynamics, and not Host C. No dark-matter species or quantum-gravitational mechanism is selected. wired=false. declaredPath remains null.

Proof scope. All principal geometric, action, finite-register, and statistical assertions are proved under their printed hypotheses, including the Version 2 identities of Theorems , , , , and , and of Propositions , , , , , and , together with the Version 3 identities of Lemma , Propositions , , and , Theorem , Corollary , and Theorem , and the Version 4 identities of Theorem , Corollary , Proposition , Proposition , Theorem , and Proposition . The source-compatible manufacture of a dark CTC, field well-posedness on a chronology-violating host, a continuum derivation of either finite CTC rule, a localised vortex-ring host on $\mathbb{R}^4$, a chronology-resonator theorem, a coupled dynamical Misner manufacture, and physical clock acquisition are not claimed. The flat chronology-split witness is explicitly not an Einstein solution with its positive test matter. The Gödel comparison is not a winding-inclusive exact match, and the Misner overlay is not dynamical manufacture. The inherited remarks remain in force: a temporal coordinate on a finite open collar is not a Cauchy time; a timelike azimuthal circle is a geodesic only where $\nabla c = 0$; electromagnetic silence is not complete dynamical isolation; compact horizon generation alone does not imply negative null curvature; a factorised dark CTC does not remove controller dependence already present in the ordinary-controller dynamics; an unspecified portal does not automatically clone or distinguish states; and ordinary Sagnac data alone do not identify host chronology in the printed completion class.

Novelty. The contribution is the complete chronology-split construction, its action and topology audit, the exact

Sagnac completion fibre and its declared host-null reduction, the typestate-honest mode, image-sum, and test-field audits, the qualified quantum insulation and convex-family mixture audit, the typestate-honest Gödel and Misner comparisons, and their calibrated rejection structure. Standard ingredients, including the Gödel metric and the finite CTC rules, are credited. No universal chronology-protection theorem or new physical time-travel mechanism is claimed, and Hawking chronology protection is not evaded.

Verification and artefacts. No pdflatex execution, symbolic computation certificate, numerical run, formal proof certificate, Lean or machine-checked argument, referee verification, or acceptance of this manuscript is asserted. The source is intended for pdflatex; compilation and independent checking remain pre-submission tasks. Informal environment and brace checks, if recorded, are not a TeX compile.

Data availability. No new observations, numerical likelihoods, or exclusion contours are reported. The analytical definitions and proofs are contained in this source.

Standing ledger. Host C is not created. O.1 remains NOT_CLOSED. XV-B is not opened; XVI-N and Route B are not discharged, except the already recorded t441 class-limited negative of XVI-N-Univ-A, which is not used here. Gate 0 and ST4b remain NOT_SUPPLIED. No inherited characteristic or radiative quantum coefficient is admitted. External geometric time remains supplied. The inherited time-machine bar remains 0 of 3 as the full three-part bar: Bar 1 (internal register / state restoration), Bar 2 (consumed clock-name / operational record return without an external archive), and Bar 3 (spacetime worldline / event return) all remain undischarged. No Page curve, island, bounce, spacetime emergence, phenomenal identification, or consciousness–dark-matter identification is made. H3, A1–A3, and the phenomenal-theory gate are unused and remain NOT_CLOSED. Host A and Host B remain separate documentary backgrounds. The source is not companion-wired: wired=false; declaredPath remains null. Narrower releases of other manuscripts are not withdrawn.

Documentary negative-control catalogue

The following one-page table isolates unverified external catalogue entries and refused slogans as documentary negative controls only [23]. They are not theorem premises, not scheduler-derived cones, not a time machine, not a

Page curve, not a free clock, not worldline return, and not a decoder of an unread complement. They do not move the ordinary operational time-machine bar, which remains 0 of 3, and they do not typestate uniqueness of dark matter or a chronology engine in nature. Clock Track-I identities are not imported from this catalogue, and no Host C, quantum coefficient, island, bounce, or de Sitter state is typestated.

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[1] Cheung, N. (2026). The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement. *Frontiers in Physics*, **14**, 1954943. DOI: 10.3389/fphy.2026.1954943. *Source record; not a premise.*

[2] Cheung, N. (2026). *Dual-Ledger-Temporal-A: Sector-Dependent Clock Rates and Transport on a Prescribed Lorentzian Host. Conservation, No-Freezing Results, Controlled Cosmological Witnesses, and Observational Rejection Contracts*. Enter Calculus Companion, official version 5 (request t504u). *Source record; not a premise.*

[3] Cheung, N. (2026). *Dual-Ledger-Mobility-A: Mobility asymmetry between an ordinary slot and a current-free dark slot on a prescribed host: a conditional fluid-plus-controller class, its conservation ledger, and declared inverse problems*. Enter Calculus Companion, official version 5 (request t495u). *Source record; not a premise.*

[4] Cheung, N. (n.d.). *A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product*. *The Philosophy of Enter*, I-B-v1, p. 78. *Source record; not a premise.*

[5] Cheung, N. (2026). *Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime*. Enter Calculus Companion, official version 3. Documentary support and missing-boundary comparison only. *Source record; not a premise.*

[6] Cheung, N. (n.d.). *Host-C-ClockSync-A*. Enter Calculus Companion, identifier Host-C-ClockSync-A. No version or registration locator is invented. Documentary title only; Host C is not created. *Source record; not a premise.*

[7] Cheung, N. (n.d.). *Host-C-Echo-A*. Enter Calculus Companion, identifier Host-C-Echo-A. No version or

registration locator is invented. Not an imported pulse-echo theorem. *Source record; not a premise.*

[8] Cheung, N. (n.d.). *Host-C-A*. Enter Calculus Companion, identifier Host-C-A. No version or registration locator is invented. The title does not assert creation of Host C. *Source record; not a premise.*

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[12] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-B: Class-Limited Einstein Matches, a Closed Causal-Geodesic Obstruction, On-Shell Ordinary Shields, a Prescribed Misner Overlay, and One-Sided Sagnac Identifiability on Prescribed Hosts*. Enter Calculus Companion, official version 4 (request t509u). *Source record; not a premise.* Not a winding-inclusive residual-free Einstein match. Official residuals on the printed family are not withdrawn.

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