Dual-Ledger-Chrono-DM-D Loop Holonomy Memory, a Spectral Filter, and a Prescribed Bimetric Chronology Resonator

Typestate-Honest Completions of Official Monodromy and Resonator Programmes, with Explicit Refusals

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Abstract

Official Dual-Ledger-Chrono-DM-A version 4 already recorded internal monodromy along a closed dark geodesic as a unitary loop phase in U(*N*), typestated as not a single-valued elapsed-time scalar and as not Bar 1, Bar 2, or backward signalling, and recorded resonant gravitational-wave lensing, a chronology resonator, and a Fabry–Pérot scattering theorem as an unfinished programme [2]. That official record is cited only as a documentary source; every identity used below is proved from definitions printed in this sheet. The present manuscript is official version 3 of Dual-Ledger-Chrono-DM-D, a typographic successor of official version 2 [1], itself an expansion of official version 1. Official version 2 typestate, labels, and local proofs are retained. It is not a rewrite of official Chrono-DM-A.

Three loop questions are separated at the typestate level. A smooth single-valued elapsed-time field cannot satisfy $u(\tau) = 1$ around a closed timelike orbit, and a unitary seam monodromy cannot strictly increase von Neumann entropy or rank in one circuit. A Hermitian internal connection does, however, store gauge-invariant data in the conjugacy class of its monodromy: in the general Hermitian case the monodromy takes values in U(*N*), and the traceless subclass takes values in SU(*N*). Two localised SU(2) kicks have an order-insensitive trace, so they do not by themselves typestate gauge-invariant path-order memory; three non-commuting kicks do, once the traces are

shown to differ. Levi–Civita spinor holonomy is computed on each printed host that actually supplies a closed dark geodesic, and is not a scalar-age clock. On the cratered unit-lapse ring the co-rotating connection includes the Darboux shift of the 2π-periodic triad, the Floquet generator is elliptic with invariant frequency 1/6, and the vectorial holonomy angle is $\pi\sqrt{5}/3 \notin 2\pi\mathbb{Z}$. An adiabatic Floquet–Berry split is typestated under explicit gap hypotheses, with a remainder of class $O(L^{-1})$ controlled by the gap and the C^2 norms, and a damped resolvent is typestated as a spectral projection onto the exact monodromy eigenspace of eigenvalue 1, not onto the adiabatic phase-quantisation set. Under a written nondegeneracy hypothesis, an exact unit-eigenvalue orbit lies within $O(L^{-1})$ of each adiabatic root.

A prescribed bimetric scattering slab with completed-square coefficient $F(x) > 0$ is printed in two models, not one: a globally smooth collar, and a piecewise-constant even step-collar. The operative wave operator is that of a massless scalar, except for a printed longitudinal-electric test Maxwell reduction, at $k_z = 0$, in which a potential $\mathscr{A} = \Psi_{\mathrm{EM}}(t, x, \varphi)\,\mathrm{d}z$ satisfies the source-free Maxwell equation on g or on $\tilde{g}$ if and only if Ψ_{EM} satisfies the same massless scalar wave operator. That reduction is a test-field identity on a prescribed host. It is not Einstein dynamics, not Host C, and not a typestate of electromagnetic or gravitational-wave manufacture in general. Axial test-tensor perturbations are typestated only as flat-piece test fields on the constant-coefficient regions, not as a global Regge–Wheeler reduction and not as dynamical gravitational waves. The Liouville potential is derived from the printed inverse metric and volume element. Wronskian flux is conserved for a real potential, and $|R|^2 + |T|^2 = 1$ is typestated only in the two-sided propagating window with matched exteriors. The piecewise-constant model admits an $\mathrm{SL}(2,\mathbb{R})$ transfer matrix in a printed current basis, and the even slab has exact unit transmission at the real roots of an explicit matching condition. On a CTC or horizon core ($c_{\mathrm{core}} \le 0$), the chiral sign $\omega n < 0$ is strictly necessary for a nonempty core-propagating window; same-sign windows can open only for chronology-preserving rotating cores $0 < a_0 < \sqrt{F_0}$. In the thick-barrier regime the matching denominator admits a unique simple pole in a shrinking neighbourhood of each simple real unit-transmission root, with leading width expressed in the printed unimodular data. A Wigner delay $2/\Gamma$ is typestated at the real part of that pole, and

not as an identity at the real unit-transmission root unless the printed residue factor so implies. The ordinary scalar operator, under a prescribed shield $H \geq H_{\min} > 0$, has no negative-c core band. A printed Wilson-line coupling splits host resonances by a shift in effective angular number; the shift is not typestated as the universal increment θ_a/L.

Ordinary Cauchy protection on $(M, \hat{g})$ follows from global hyperbolicity of the ordinary sector and the test-field hypothesis, not from Wigner-delay positivity. Active transient ordinary stress coupled to dynamical Einstein gravity is not typestated as TM-1 isolated; TM-1 remains undischarged. Chirality of the effective potential when $a_0 \neq 0$ is proved. Observation of a trapped co-propagating core resonance at $\omega > 0$ falsifies $c_{\text{core}} \leq 0$ in the even step-collar class. Strict one-chirality is compatible with a CTC or horizon core, but does not by itself prove $c_{\text{core}} \leq 0$. Unique inversion of $(a_0, F_0, b_{\text{cav}})$ from two adjacent resonance frequencies together with a $(\pm n)$ splitting is not typestated: even under a declared barrier calibration, no non-vanishing Jacobian of a complete local inverse is computed in this sheet. A declared rejection contract R − FP − lens is recorded instead, and does not complete, discharge, or replace official Chrono-DM-A's guided-ring Jacobian. Inverse-scattering reconstruction of the full collar profiles, a non-Abelian multi-channel Wilson resonator, and three-dimensional whispering-gallery quasimodes on $\mathbb{R}^4$ are recorded as explicit refusals.

The operational time-machine bar remains 0 of 3 as the full three-part bar. Host C is not created. O.1 stays NOT_CLOSED. H3 stays NOT_CLOSED. Gate 0 and ST4b stay NOT_SUPPLIED. wired = false; declaredPath = null. No compilation or machine verification is asserted.

Bibliographic record

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Scope, three loop questions, and the Standing Ledger

Why a fourth chronology sheet is needed

Official Dual-Ledger-Chrono-DM-A version 4 [2] *Documentary source record; not a theorem premise.* already separated backward signalling to ordinary apparatus (TM-1), bimetric chronology asymmetry on a prescribed host (TM-2), and register or echo non-identifications (TM-3), and it proved an exact TM-2 witness on a complete flat cylindrical quotient. Immediately after its internal-monodromy statement (official Proposition 10.9, labelled prop : monodromy), that sheet recorded that resonant gravitational-wave lensing, a chronology resonator, and a Fabry–Pérot scattering theorem were not proved there, and were an unfinished programme. Official version 1 of the present sheet carried out that attempt by local proofs. Official version 2 expands official version 1 by two typestate repairs, by a promotion of starred environments to the shared per-section theorem counter, and by those analytical enhancements for which a complete local proof is actually written. Official version 3 retains that official version 2 typestate, and does not rewrite official Chrono-DM-A, open XV-B, or create a second official resonator companion. Holonomy and resonator identities remain typestated only on the written non-self-scrambling, portal-silent class, and are not typestated as operational hardware or as a chronology engine in nature.

Official Dual-Ledger-Chrono-DM-B version 4 [3] *Documentary source record; not a theorem premise.* and official Dual-Ledger-Chrono-DM-C version 3 [4] *Documentary source record; not a theorem premise.* are documentary neighbours, used only as source records for the printed hosts that actually supply closed dark geodesics, and for the typestate of those hosts. Official Dual-Ledger-Temporal-A version 5 [5] *Documentary*

source record; not a theorem premise. is likewise a documentary neighbour for clock-rate discipline. No identity is imported as an unproved premise.

Three loop questions are kept distinct throughout.

1. *Monotonic scalar age.* Does a closed future-directed host-timelike geodesic carry a smooth single-valued elapsed-time field with $u(\tau) = 1$? The answer, re-proved locally as Proposition , is negative.

2. *Gauge or Berry holonomy memory.* Can a dark internal connection, or an adiabatic eigenbundle, store loop data in a gauge-invariant conjugacy class without thereby supplying such a scalar age? The answer is affirmative only for the data actually shown to be conjugation-invariant, and only on printed hosts that supply closed dark geodesics (Theorem , Theorem , Theorem).

3. *Passive stationary wave resonance.* Can a prescribed bimetric collar support a unitary, horizon-regular one-dimensional scattering problem with chiral core resonances, and what, exactly, does that typestate about ordinary insulation and parameter inversion? The answer is admitted only in the class actually constructed, and only with the refusals of Sections –.

Signature $(-,+,+,+)$ and units $c = \hbar = 1$ are fixed throughout. The coordinate t is a supplied external geometric time; it is never derived.

Documentary typestate audit of official Chrono-DM-A labels

A commissioning note for the present sheet attached unofficial numbers to official Chrono-DM-A statements. Those unofficial numbers are not repeated as if they were official. The official numbering is the same in official Chrono-DM-A version 3 and official version 4, which share a common theorem counter per section, and official version 4 is the documentary record cited here [2] *Documentary source record; not a theorem premise.*.

Official prop : clockloop is Proposition 4.4 of official Chrono-DM-A: a closed geodesic does not carry a single-valued monotone elapsed-time field. Official Proposition 10.8 is thm : periodic, a closed-flow continuity constraint. Official prop : monodromy is Proposition 10.9: internal monodromy along a closed dark geodesic,

taking values in U(N) for a printed Hermitian Hamiltonian, typestated as not a single-valued elapsed-time scalar and as not Bar 1, Bar 2, or backward signalling. Official Definition 5.3 is off-shell electromagnetic silence, and official Proposition 5.4 (same label prop : silence) states that silence excludes only the declared direct channel, while metric coupling remains present in the dark action. Official Proposition 5.5 is fixed-host direct isolation. Official Proposition 5.7 is prop : leakage, a formal first-order holonomy increment on the cover under a declared Assumption 5.6, not a completed dynamical Einstein theorem. Official Proposition 5.8 is prop : images: the naive periodized retarded sum is not locally finite. Official thm : jointID is Theorem 10.10, a local Jacobian identification of (a, c, b) by a host-null supplement. These statements are named only as documentary comparisons. Every operative lemma below is proved from printed definitions.

Standing Ledger

Standing Ledger (unchanged typestate locks). par Host A and Host B stay separate prescribed backgrounds. Host C is not created. O.1 stays NOT_CLOSED. H3 stays NOT_CLOSED. XV-B is not opened. XVI-N and Route B are not discharged, except the class-limited negative of t441 XVI-N-Univ-A, which is not used here. Gate 0 and ST4b stay NOT_SUPPLIED. No quantum coefficient, derived metric, vacuum-energy claim, Page curve, island, bounce, or consciousness–dark-matter identification is admitted. External geometric time remains supplied. The source is not companion-wired: wired = false, and declaredPath remains null. The operational time-machine bar remains 0 of 3 as the full three-part bar. Non-Abelian loop holonomy, an adiabatic Berry decomposition, a coherent spectral filter, a prescribed bimetric scattering slab, a unitary transfer matrix, a Wigner delay, or a declared inverse contract is not Bar 1, not Bar 2, and not Bar 3. Prescribed hosts do not discharge Bar 3. Memory stored in conjugacy classes is not an unbounded elapsed-time counter. A massless scalar on a prescribed host is not electromagnetism and is not linearised gravity, unless a reduction is written. A longitudinal-electric test Maxwell reduction is written in Theorem ; it remains a test-field identity on a prescribed host, not electromagnetic manufacture in general, and not Host C. Axial test-tensor perturbations are typestated only in the class actually decoupled there. A shield inequality $H \geq H_{\min} > 0$ is a prescribed condition, not Einstein manufacture and not Host

C. A prescribed chronology split, a holonomy memory, an adiabatic remainder, a transfer-matrix resonance, a Wigner delay, a Jacobian on a declared calibration, a test-field Maxwell or tensor reduction, or any Track II appendix comparison is not Bar 1, not Bar 2, and not Bar 3.

Printed closed dark geodesics on three declared hosts

A transverse family of closed future-directed *g*-timelike geodesics is typestated only where a printed host actually supplies them. The following proposition re-proves closure on the three classes used below, and records two non-inhabiting comparisons.

Proposition (Closed dark geodesics on three printed hosts).

1. *Constant cylindrical core. On*

$$M = \mathbb{R}_t \times \mathbb{R}_x \times S^1_\varphi \times \mathbb{R}_z, \qquad \varphi \sim \varphi + 2\pi,$$

prescribe the constant-coefficient metric

$$g = -(\mathrm{d}t + a\,\mathrm{d}\varphi)^2 + F\,\mathrm{d}\varphi^2 + \mathrm{d}x^2 + \mathrm{d}z^2,$$

with constants $a > 0$ *and* $F = a^2 - h > 0$, *so that the chronology function is* $c := F - a^2 = -h < 0$. *For every* (t_0, x_0, z_0), *the azimuthal circle* $\gamma(\varphi) = (t_0, x_0, \varphi, z_0)$ *is a smooth, closed, future-directed g-timelike geodesic, with* $g(\partial_\varphi, \partial_\varphi) = -h < 0$, *future orientation* $(\mathrm{d}t + a\,\mathrm{d}\varphi)(\partial_\varphi) = a > 0$, *unit tangent* $u = h^{-1/2}\partial_\varphi$, *and proper period*

$$L = 2\pi\sqrt{h}.$$

2. *Holonomy-shifted ring, restricted class. For real constants* $0 < \alpha < 1$, $\beta \neq 0$, *and* $A_0 > 0$, *prescribe on the same manifold the holonomy-shifted metric*

$$g_{\alpha,\beta,A_0} := -(\mathrm{d}t + (A_0 + \alpha e^{\beta x})\,\mathrm{d}\varphi)^2 + \mathrm{d}x^2 + e^{2\beta x}\,\mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

At the unique radius $x_* = \beta^{-1}\ln(\alpha A_0/(1-\alpha^2))$ *one has* $g_{\alpha,\beta,A_0}(\partial_\varphi, \partial_\varphi) = -A_0^2/(1-\alpha^2) < 0$ *and* $\partial_x g_{\alpha,\beta,A_0}(\partial_\varphi, \partial_\varphi) = 0$. *Hence every azimuthal circle at that radius is a closed future-directed timelike geodesic, of proper period*

$$L = \frac{2\pi A_0}{\sqrt{1-\alpha^2}}.$$

3. *Printed cratered ring on a unit-lapse host. On* $M = \mathbb{R}^4$ *with Cartesian coordinates* (t, x, y, z) *and cylindrical radius* $r = \sqrt{x^2 + y^2}$*, fix* $r_* = 2$ *and* $\Omega_0 = 1$*, and define the local model*

$$C_{\mathrm{loc}}(r, z) := 5 + (r-2)^2 + z^2.$$

Let $\eta \in C_c^\infty(\mathbb{R}^3; [0,1])$ *be even in z, supported in the compact annular box* $\{1 \le r \le 3,\ |z| \le 1\}$*, and identically* 1 *on the flat neighbourhood* $\{(r-2)^2 + z^2 \le \frac{1}{4}\}$*. Set*

$$w(r, z) := \eta(r, z)\,\frac{\sqrt{C_{\mathrm{loc}}(r, z) + r^2}}{r^2}$$

for $r > 0$*, and* $w = 0$ *along the axis. Prescribe the stationary unit-lapse metric*

$$g := -(\mathrm{d}t + \Omega_0 w(r^2, z^2)\,(x\,\mathrm{d}y - y\,\mathrm{d}x))^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2.$$

On the flat neighbourhood one has $C_-(r, z) := \Omega_0^2 r^4 w^2 - r^2 = C_{\mathrm{loc}}(r, z)$*, and* $(r, z) = (2, 0)$ *is a strict local minimum of* C_- *with* $C_-(2, 0) = 5 > 0$ *and* $\nabla C_-(2, 0) = 0$*. The azimuthal circle at* $(r_*, z_*) = (2, 0)$ *is therefore a closed future-directed g-timelike geodesic, of proper period*

$$L = 2\pi\sqrt{5}.$$

Moreover $\Omega_0 r_* w(r_*, 0) = 3/2 > 1$*, so the circle lies in the chronology-violating tube* $C_- > 0$*. In all three cases the geodesic is a prescribed geometric witness. It is not Bar 3, and it is not an Einstein-manufactured torus.*

Proof. For (i), the coefficient $g(\partial_\varphi, \partial_\varphi) = F - a^2 = -h < 0$ is constant, so the azimuthal circle is uniformly timelike, and $(\mathrm{d}t + a\,\mathrm{d}\varphi)(\partial_\varphi) = a > 0$ orients it towards the future. The field $K := \partial_\varphi$ is Killing, and $\nabla_K K = -\frac{1}{2}\mathrm{grad}\,(K \cdot K)$. Because $K \cdot K = -h$ is constant, the gradient vanishes, and the circle is a geodesic. The unit tangent is $u = K/\sqrt{h}$, so $\mathrm{d}\varphi/\mathrm{d}\tau = h^{-1/2}$ and the proper period for $\Delta\varphi = 2\pi$ is .

For (ii), write $A(x) := A_0 + \alpha e^{\beta x}$ and $F(x) := e^{2\beta x}$, so that $K \cdot K = F - A^2 = e^{2\beta x} - (A_0 + \alpha e^{\beta x})^2$. Differentiating in x

gives

$$\partial_x(K \cdot K) = 2\beta e^{2\beta x} - 2(A_0 + \alpha e^{\beta x})\,\alpha\beta e^{\beta x}.$$

The right-hand side vanishes precisely when $e^{\beta x}(1-\alpha^2) = \alpha A_0$. The hypotheses $0 < \alpha < 1$ and $A_0 > 0$ make the logarithm well-defined and supply a unique root x_*. Substituting $e^{\beta x_*} = \alpha A_0/(1-\alpha^2)$ yields $K \cdot K|_{x_*} = -A_0^2/(1-\alpha^2) < 0$. Independence of z and of t kills the remaining first derivatives, so $\nabla_K K = 0$ along every azimuthal circle at x_*. The future orientation is $(\mathrm{d}t + A\,\mathrm{d}\varphi)(\partial_\varphi) = A(x_*) = A_0/(1-\alpha^2) > 0$. The proper period is $2\pi\sqrt{-K \cdot K}$, which is .

For (iii), on $\{\eta = 1\}$ one has $\Omega_0 r^2 w = \sqrt{C_{\mathrm{loc}} + r^2}$, hence $C_- = C_{\mathrm{loc}}$. The jet of C_{loc} at $(2, 0)$ is $C_* = 5$, $\nabla C_{\mathrm{loc}} = 0$, so $g_{\varphi\varphi} = -C_- = -5 < 0$ and $\nabla g_{\varphi\varphi} = 0$ there. Killing's identity again yields $\nabla_K K = 0$ along the azimuthal circle. The future orientation is $(\mathrm{d}t + \Omega_0 w(x\,\mathrm{d}y - y\,\mathrm{d}x))(\partial_\varphi) = a_* = 3 > 0$. The proper period is $2\pi\sqrt{5}$. The identity $\Omega_0 r_* w(r_*, 0) = a_*/r_* = 3/2 > 1$ is immediate from $a_* = \sqrt{C_* + r_*^2} = 3$.

Remark (Two classes that do not inhabit the family). The unshifted family

$$g_{\alpha,\beta} := -(\mathrm{d}t + \alpha e^{\beta x}\,\mathrm{d}\varphi)^2 + \mathrm{d}x^2 + e^{2\beta x}\,\mathrm{d}\varphi^2 + \mathrm{d}z^2$$

with $\alpha > 1$ and $\beta \neq 0$ does admit closed future-directed timelike curves: for $K = \partial_\varphi$ one has $K \cdot K = (1-\alpha^2)e^{2\beta x} < 0$. These circles are not geodesics. Indeed $\partial_x(K \cdot K) = 2\beta(1-\alpha^2)e^{2\beta x} \neq 0$, so $\nabla_K K \neq 0$. Official Chrono-DM-B version 4 further records a closed causal-geodesic obstruction on that unshifted family [3] *Documentary source record; not a theorem premise.*; that obstruction is a documentary neighbour, not a premise, and is not re-opened here. Consequently unshifted $g_{\alpha,\beta}$ is not typestated as supplying the family γ_x of Section .

Official Chrono-DM-C version 3 withdrew the official version 2 single-invariant on-shell pullback of a toroidal Einstein match [4] *Documentary source record; not a theorem premise.*. The printed ring of Proposition (iii) is used only as a prescribed unit-lapse geometric host, not as a coupled Einstein–vortex solution, and not as Host C.

par *Non-claim.* Proposition typestates only geodesic closure and proper periods on printed classes. It does not

discharge Bar 3, it does not manufacture a laboratory time machine, and it does not supply Gate 0 or ST4b. par

Organisation

Section treats internal and spinor holonomy memory. Section treats an adiabatic Floquet–Berry split and a damped spectral filter. Section prints a bimetric scattering slab and analyses two distinct resonator models. Section separates ordinary Cauchy protection from transient back-reaction. Section records a declared chiral contract and refuses an unproved unique inversion. Section consolidates the typestate, Appendix is a debugging table, and Appendix records Track II programmes that remain explicit refusals.

Non-Abelian holonomy memory on closed dark geodesics

Internal connection and monodromy

Let (M, g) be a prescribed host carrying a smooth transverse family of closed future-directed g-timelike geodesics $\gamma_x : [0, L(x)] \to M$, parameterised by proper time, with $\gamma_x(L(x)) = \gamma_x(0)$ and unit tangent $u = \dot{\gamma}_x$. The printed classes of Proposition inhabit the family: the constant cylindrical core, with period ; the holonomy-shifted ring at x_*, with period ; and the cratered unit-lapse ring, with period .

Let $E_{\mathrm{d}} \to M$ be a smooth Hermitian vector bundle of rank N, and let a Hermitian connection be given along each orbit by

$$D_\tau = \partial_\tau + \mathrm{i}\mathscr{H}_{\mathrm{d}}(\tau; x),$$

where $\mathscr{H}_{\mathrm{d}}(\tau; x)$ is a smooth family of Hermitian endomorphisms of the fibre. The parallel-transport monodromy around γ_x is

$$W(x) := \mathscr{P}\exp\left(-\mathrm{i}\int_0^{L(x)} \mathscr{H}_{\mathrm{d}}(\tau; x)\,\mathrm{d}\tau\right) \in \mathrm{U}(N).$$

Existence and unitarity of the path-ordered exponential, for a smooth Hermitian family on a compact interval, are standard: if $U(\tau)$ solves $\mathrm{i}\partial_\tau U = \mathscr{H}_{\mathrm{d}} U$ with $U(0) = \mathrm{I}$, then $U(\tau)^\dagger U(\tau) = \mathrm{I}$ by differentiation, and $W(x) := U(L(x))$. If, in addition, $\mathrm{Tr}\,\mathscr{H}_{\mathrm{d}}(\tau; x) = 0$ for all τ, then $\det U(\tau) = 1$ and $W(x) \in \mathrm{SU}(N)$. In the absence of that traceless condition,

the monodromy is typestated in U(*N*), as official Chrono-DM-A typestated its printed Hermitian Hamiltonian [2] *Documentary source record; not a theorem premise.*. The traceless subclass is recorded separately, and is not silent in the statements that use it.

Scalar age and unitary invariants

Proposition (Scalar-age obstruction and unitary rank–entropy invariance). *Let u generate a closed timelike orbit of proper period* $L > 0$.

1. *There is no smooth single-valued real function* $\tau : M \to \mathbb{R}$ *satisfying* $u(\tau) = 1$ *around that orbit. The accumulated proper time on its lifted parameter line nevertheless increases by L per circuit.*

2. *Let σ be a density matrix on a finite-dimensional fibre, and let the seam map be the unitary conjugation* $\sigma \mapsto W\sigma W^{\dagger}$. *If* $\sigma(L) = \sigma(0)$ *is a seam-periodic state, then the von Neumann entropy* $S(\sigma) = -\mathrm{Tr}\,(\sigma \log \sigma)$ *and the rank of σ are invariant under the circuit. In particular, neither quantity can strictly increase in one circuit. Clause (ii) is a unitary-invariance identity. It is not a thermodynamic second law, not a Bar discharge, and not an archive theorem for a non-unitary seam map. No non-unitary return rule is printed in this sheet.*

Proof. For (i), integrate $u(\tau) = 1$ along one proper-time period:

$$\tau(\gamma(L)) - \tau(\gamma(0)) = L.$$

The geometric endpoints coincide, making the left-hand side vanish, which contradicts $L > 0$. On the lifted parameter line there is no endpoint identification, so accumulated proper time is strictly increasing.

For (ii), unitary conjugation preserves the spectrum of *σ*. Von Neumann entropy is a symmetric function of that spectrum, and rank is the number of nonzero eigenvalues, so both are invariant. A seam-periodic state cannot then exhibit a strict increase of either quantity around the circuit.

Official Chrono-DM-A's documentary Proposition 4.4 is not withdrawn by clause (i); the present proof is local and does not use that statement as a premise.

Conjugacy-class memory

Theorem (Conjugacy memory of the loop monodromy). *Let* $\mathscr{H}_{\mathrm{d}}(\tau;x)$ *be a smooth Hermitian family on* $[0,L(x)]$ *, and let* $W(x)$ *be as in .*

1. *Under a smooth fibrewise unitary gauge transformation* $G(\tau;x)\in \mathrm{U}(N)$ *which is periodic,* $G(L(x);x)=G(0;x)$*, the connection and monodromy transform as*

$$\mathscr{H}_{\mathrm{d}}^{'}=G\mathscr{H}_{\mathrm{d}}G^{-1}+\mathrm{i}(\partial_{\tau}G)G^{-1},$$

$$W^{'}=G(0;x)\,W(x)\,G(0;x)^{-1}.$$

In particular, the conjugacy class of $W(x)$ *is gauge-invariant.*

2. *The gauge-invariant data are the conjugacy classes in* $\mathrm{U}(N)$*: equivalently, the unordered N-tuple of eigenphases* $(\theta_1(x),\ldots,\theta_N(x))\in \mathbb{T}^N/S_N$*, or the characters* $\chi_k(x)=\mathrm{Tr}\,(\wedge^k W(x))$ *for* $k=1,\ldots,N$*.*

3. *If* $\mathrm{Tr}\,\mathscr{H}_{\mathrm{d}}=0$ *and* $G(\tau;x)\in \mathrm{SU}(N)$*, then* $W(x)\in \mathrm{SU}(N)$ *and the eigenphases may be reduced to an* $(N-1)$*-tuple on* $\mathbb{T}^{N-1}/S_N$*, because* $\det W=1$ *forces* $\sum_{a=1}^{N}\theta_a\in 2\pi\mathbb{Z}$*. These data are not an elapsed-time scalar. Proposition (i) is not withdrawn. The identity is not Bar 1, not Bar 2, and not backward signalling into the ordinary sector.*

Proof. Write $D_{\tau}\psi=\partial_{\tau}\psi+\mathrm{i}\mathscr{H}_{\mathrm{d}}\psi$, so that internal parallel transport is $D_{\tau}\psi=0$, or equivalently $\mathrm{i}\partial_{\tau}\psi=\mathscr{H}_{\mathrm{d}}\psi$. If $\psi^{'}=G\psi$, then

$$\mathrm{i}\partial_{\tau}\psi^{'}=\mathrm{i}(\partial_{\tau}G)\psi+G(\mathrm{i}\partial_{\tau}\psi)=(G\mathscr{H}_{\mathrm{d}}G^{-1}+\mathrm{i}(\partial_{\tau}G)G^{-1})\psi^{'},$$

which is . Let $U(\tau)$ and $U^{'}(\tau)$ be the evolution operators for $\mathscr{H}_{\mathrm{d}}$ and $\mathscr{H}_{\mathrm{d}}^{'}$, with $U(0)=U^{'}(0)=\mathrm{I}$. The transformed solution starting at $\psi^{'}(0)$ is $\psi^{'}(\tau)=G(\tau)U(\tau)G(0)^{-1}\psi^{'}(0)$, hence $U^{'}(\tau)=G(\tau)U(\tau)G(0)^{-1}$. Periodicity $G(L)=G(0)$ then yields . Conjugacy classes, spectra, and the characters $\chi_k=\mathrm{Tr}\,(\wedge^k W)$ are invariant under . In the traceless subclass, $\mathrm{Tr}\,\mathscr{H}_{\mathrm{d}}^{'}=\mathrm{Tr}\,\mathscr{H}_{\mathrm{d}}+\mathrm{i}\partial_{\tau}\log\det G=0$ whenever $\det G=1$, so $W^{'}\in \mathrm{SU}(N)$ as well, and $\det W=\exp\left(-\mathrm{i}\int_0^L \mathrm{Tr}\,\mathscr{H}_{\mathrm{d}}\,\mathrm{d}\tau\right)=1$ reduces the eigenphases by one integer constraint, modulo permutations. A single-valued real function with $u(\tau)=1$ remains impossible by Proposition (i), independently of any internal unitary. No coupling to the ordinary

sector is introduced.

Path-order sensitivity

Two localised SU(2) kicks do not typestate gauge-invariant path-order memory. Three non-commuting kicks do, once the traces are shown to differ. The following theorem records both identities.

Throughout, Pauli matrices satisfy $\sigma_j\sigma_k = \delta_{jk}\mathrm{I} + \mathrm{i}\varepsilon_{jkl}\sigma_l$, and a unit vector $n \in \mathbb{R}^3$ determines the rotation

$$R(\alpha, n) := \exp(-\mathrm{i}\alpha\, n \cdot \sigma) = \cos\alpha\,\mathrm{I} - \mathrm{i}\sin\alpha\,(n \cdot \sigma) \in \mathrm{SU}(2).$$

A localised kick of integrated strength α and axis n is the limit of a family of smooth compactly supported Hermitian bumps of integral $\alpha(n \cdot \sigma)$, or, equivalently once that regularisation is declared, an exact stepwise factor in the path-ordered product. The two descriptions agree on the monodromy because the path-ordered exponential is continuous in the L^1 topology of the generator, and the bumps may be taken disjoint.

Theorem (Two-kick traces are order-insensitive; three-kick traces need not be).

1. *Two kicks. Let*

$$W_{21} = R(\alpha_2, n_2)R(\alpha_1, n_1), \qquad W_{12} = R(\alpha_1, n_1)R(\alpha_2, n_2),$$

with $\alpha_j \in \mathbb{R}$ *and unit axes* $n_j \in \mathbb{R}^3$*. Then*

$$\frac{1}{2}\mathrm{Tr}\, W_{21} = \frac{1}{2}\mathrm{Tr}\, W_{12} = \cos\alpha_1\cos\alpha_2 - \sin\alpha_1\sin\alpha_2\,(n_1 \cdot n_2).$$

The conjugacy class in SU(2)*, being determined by the trace, is therefore independent of order. The based monodromy at a marked frame has an order-dependent axis, because the cross-product term in the product formula changes sign upon interchange; that axis is not a gauge-invariant scalar on an unmarked loop.*

2. *Three kicks. For three rotations* $R(\alpha_j, n_j)$*,*

$$\begin{aligned}\tfrac{1}{2}\mathrm{Tr}\,(R(\alpha_3,n_3)R(\alpha_2,n_2)R(\alpha_1,n_1)) &= \cos\alpha_1\cos\alpha_2\cos\alpha_3\\ &\quad-\cos\alpha_1\sin\alpha_2\sin\alpha_3\,(n_2\cdot n_3)\\ &\quad-\cos\alpha_2\sin\alpha_3\sin\alpha_1\,(n_3\cdot n_1)\\ &\quad-\cos\alpha_3\sin\alpha_1\sin\alpha_2\,(n_1\cdot n_2)\\ &\quad+\sin\alpha_1\sin\alpha_2\sin\alpha_3\,n_1\cdot(n_2\times n_3).\end{aligned}$$

The corresponding formula for the reversed product $R(\alpha_1,n_1)R(\alpha_2,n_2)R(\alpha_3,n_3)$ is obtained by replacing the scalar triple product by $n_3\cdot(n_2\times n_1)=-n_1\cdot(n_2\times n_3)$, while the pairwise terms are unchanged. Hence the traces differ by

$$\frac{1}{2}\mathrm{Tr}\,(R_3R_2R_1)-\frac{1}{2}\mathrm{Tr}\,(R_1R_2R_3)=2\sin\alpha_1\sin\alpha_2\sin\alpha_3\,n_1\cdot(n_2\times n_3).$$

Whenever all three sines are nonzero and the axes are not coplanar, the traces differ, and the conjugacy classes therefore distinguish the two orders. Stationary Floquet eigenstates of W do not, by themselves, store a monotonic clock scalar. Proposition is not withdrawn.

Proof. Expand :

$$R(\alpha_2,n_2)R(\alpha_1,n_1)=(\cos\alpha_2\,\mathrm{I}-\mathrm{i}\sin\alpha_2\,n_2\cdot\sigma)(\cos\alpha_1\,\mathrm{I}-\mathrm{i}\sin\alpha_1\,n_1\cdot\sigma).$$

The identity $(n_2\cdot\sigma)(n_1\cdot\sigma)=(n_2\cdot n_1)\mathrm{I}+\mathrm{i}(n_2\times n_1)\cdot\sigma$ isolates the scalar term $\cos\alpha_1\cos\alpha_2-\sin\alpha_1\sin\alpha_2\,(n_1\cdot n_2)$, which is symmetric in the two kicks, and the remainder is a real linear combination of Pauli matrices, hence traceless. This is . Writing $W_{21}=S\,\mathrm{I}-\mathrm{i}(v\cdot\sigma)$, the Pauli vector contains the cross term $\sin\alpha_1\sin\alpha_2\,(n_2\times n_1)$, which changes sign upon interchange, so a based monodromy at a marked frame is order-dependent, while its conjugacy class is not.

For three kicks, expand $R(\alpha_3,n_3)(R(\alpha_2,n_2)R(\alpha_1,n_1))$ and again retain only the coefficient of the identity. The two-kick product contributes the scalar and, in the form $S\,\mathrm{I}-\mathrm{i}(v\cdot\sigma)$, the Pauli vector

$$v=\cos\alpha_2\sin\alpha_1\,n_1+\cos\alpha_1\sin\alpha_2\,n_2+\sin\alpha_1\sin\alpha_2\,(n_2\times n_1).$$

Multiplying on the left by $\cos\alpha_3\,\mathrm{I}-\mathrm{i}\sin\alpha_3\,n_3\cdot\sigma$ produces the additional scalar $-\sin\alpha_3\,(n_3\cdot v)$. The non-cross part of v contracts to the three pairwise terms in , and the cross part contributes $-\sin\alpha_1\sin\alpha_2\sin\alpha_3\,n_3\cdot(n_2\times n_1)=\sin\alpha_1\sin\alpha_2\sin\alpha_3\,n_1\cdot(n_2\times n_3)$. This is . Reversing the product reverses the triple

product and leaves the pairwise terms unchanged, which is . The right-hand side of is nonzero on a nonempty open set of axes and angles, so the conjugacy classes distinguish those orders.

Lorentz-spinor holonomy

Parallel transport of Lorentz spinors is taken with respect to the Levi–Civita connection of the printed host, pulled back along the closed geodesic. Along a geodesic, Fermi–Walker transport of the spatial triad coincides with Levi–Civita parallel transport; the typestate is nonetheless that of Levi–Civita, not of an independently postulated Fermi–Walker law on a non-geodesic curve. Coefficient conventions are derived before any path-ordered formula is displayed.

Let $\{\hat{\Theta}^{\hat{a}}\}$ be a smooth orthonormal coframe, with Levi–Civita connection forms $\omega^{\hat{a}}{}_{\hat{b}}$ determined by $\mathrm{d}\hat{\Theta}^{\hat{a}} + \omega^{\hat{a}}{}_{\hat{b}} \wedge \hat{\Theta}^{\hat{b}} = 0$ and $\omega_{\hat{a}\hat{b}} = -\omega_{\hat{b}\hat{a}}$. On Dirac spinors the connection acts by

$$\nabla\psi = \mathrm{d}\psi + \frac{1}{4}\omega_{\hat{a}\hat{b}}\gamma^{\hat{a}}\gamma^{\hat{b}}\psi,$$

which is the unique lift of the vectorial rule $\nabla e_{\hat{b}} = \omega^{\hat{a}}{}_{\hat{b}} e_{\hat{a}}$ compatible with $\gamma^{(\hat{a}}\gamma^{\hat{b})} = \eta^{\hat{a}\hat{b}}$. Along a curve with unit tangent u, internal parallel transport is therefore $\frac{\mathrm{D}\psi}{\mathrm{d}\tau} + \frac{1}{4}\omega_{\hat{a}\hat{b}}(u)\gamma^{\hat{a}}\gamma^{\hat{b}}\psi = 0$, and the spin holonomy is

$$\mathscr{H}(\gamma) := \mathscr{P}\exp\left(-\frac{1}{4}\int_0^L \omega_{\hat{a}\hat{b}}(u)\,\gamma^{\hat{a}}\gamma^{\hat{b}}\,\mathrm{d}\tau\right) \in \mathrm{Spin}(1,3).$$

The vectorial holonomy in $\mathrm{SO}^+(1,3)$ is the image of $\mathscr{H}(\gamma)$ under the covering map. This is not a scalar-age clock, and it is not Bar 3.

Proposition (Spinor holonomy on the three printed closed geodesics).

1. *Constant cylindrical core. On the coframe* $\Theta^0 = \mathrm{d}t + a\,\mathrm{d}\varphi$, $\Theta^1 = \mathrm{d}x$, $\Theta^2 = \sqrt{F}\,\mathrm{d}\varphi$, $\Theta^3 = \mathrm{d}z$ *of , one has* $\mathrm{d}\hat{\Theta}^{\hat{a}} = 0$, *hence vanishing connection forms. The spin holonomy around every azimuthal geodesic of Proposition (i) is the identity in* $\mathrm{Spin}(1,3)$. *There is no nontrivial sign relative to* $\mathrm{SO}^+(1,3)$.

2. *Holonomy-shifted ring. On the coframe*

$$\Theta^0 = \mathrm{d}t + A(x)\,\mathrm{d}\varphi, \quad \Theta^1 = \mathrm{d}x, \quad \Theta^2 = e^{\beta x}\,\mathrm{d}\varphi, \quad \Theta^3 = \mathrm{d}z,$$

with $A(x) = A_0 + \alpha e^{\beta x}$*, of the metric , the connection forms are*

$$\begin{aligned}
\omega^{\hat{0}}{}_{\hat{1}} &= \omega^{\hat{1}}{}_{\hat{0}} = \tfrac{\alpha\beta}{2}\Theta^2, \\
\omega^{\hat{0}}{}_{\hat{2}} &= \omega^{\hat{2}}{}_{\hat{0}} = -\tfrac{\alpha\beta}{2}\Theta^1, \\
\omega^{\hat{1}}{}_{\hat{2}} &= -\omega^{\hat{2}}{}_{\hat{1}} = -\beta\Theta^2 + \tfrac{\alpha\beta}{2}\Theta^0,
\end{aligned}$$

with all components involving the index $\hat{3}$ *vanishing. Along the closed geodesic at* x_**, the pullbacks are constant:*

$$\begin{aligned}
\omega^{\hat{0}}{}_{\hat{1}}(u) &= \frac{\alpha^2\beta}{2\sqrt{1-\alpha^2}}, \\
\omega^{\hat{0}}{}_{\hat{2}}(u) &= 0, \\
\omega^{\hat{1}}{}_{\hat{2}}(u) &= -\frac{\alpha\beta}{2\sqrt{1-\alpha^2}}.
\end{aligned}$$

The integrated connection over one proper period is therefore the constant $\mathfrak{so}(1,3)$ *element with nonzero components*

$$\int_0^L \omega^{\hat{0}}{}_{\hat{1}}(u)\,\mathrm{d}\tau = \frac{\alpha^2\beta\pi A_0}{1-\alpha^2}, \qquad \int_0^L \omega^{\hat{1}}{}_{\hat{2}}(u)\,\mathrm{d}\tau = -\frac{\alpha\beta\pi A_0}{1-\alpha^2}.$$

For $\alpha\beta A_0 \neq 0$ *this Lie-algebra element is nonzero. The corresponding vectorial holonomy is a combination of a boost in the* $(0,1)$*-plane and a rotation in the* $(1,2)$*-plane, and is analysed as a group element in clause (iv); a nonzero algebra element need not exponentiate to a non-identity group element. The coframe is globally single-valued on the quotient, so the holonomy is not an artefact of a non-closing frame.*

3. *Cratered unit-lapse ring. On the coframe* $\Theta^0 = \mathrm{d}t + \omega_i\,\mathrm{d}x^i$*,* $\Theta^i = \mathrm{d}x^i$ *of , write* $F_{ij} = \partial_i\omega_j - \partial_j\omega_i$ *and* $F_{ij} = \varepsilon_{ijk}B^k_{\mathrm{gm}}$*. The Levi–Civita connection is*

$$\omega^{\hat{0}}{}_{\widehat{\mathrm{imath}}} = \frac{1}{2}F_{ij}\Theta^j, \qquad \omega^{\widehat{\mathrm{imath}}}{}_{\widehat{\mathrm{jmath}}} = \frac{1}{2}F_{ij}\Theta^0.$$

On the flat neighbourhood of the cratered profile, $\omega = a(r,z)\,\mathrm{d}\varphi$ *with* $a = \sqrt{C_{\mathrm{loc}} + r^2}$*. Along the ring* $(r,z) = (2,0)$ *one has* $a_* = 3$*,* $\partial_r a = 2/3$*,* $\partial_z a = 0$*, and*

$$\mathbf{B}_{\mathrm{gm}} = \frac{1}{3}\partial_z \qquad \text{at every point of the ring.}$$

The pullbacks of along the unit tangent $u = \partial_\varphi/\sqrt{5}$ *are the radial boost* $\hat{\omega}^{\hat{0}}{}_{\hat{r}}(u) = 1/(3\sqrt{5})$ *and the vertical rotation* $\hat{\omega}^{\hat{x}}{}_{\hat{y}}(u) = 1/(2\sqrt{5})$. *These Cartesian pullbacks are intermediate evaluations: their integrals over one proper period are the rapidity* $2\pi/3$ *and the half-turn* π, *respectively.*

Passing from (∂_x, ∂_y) *to the* 2π*-periodic co-rotating triad* $E_r = \cos\varphi\,\partial_x + \sin\varphi\,\partial_y$, $E_\varphi = -\sin\varphi\,\partial_x + \cos\varphi\,\partial_y$, $E_z = \partial_z$ *adds the Darboux rotation* $-\mathrm{d}\varphi(u) = -1/\sqrt{5}$. *The co-rotating connection is therefore*

$$\tilde{\omega}^{\hat{r}}{}_{\hat{\varphi}}(u) = \frac{1}{2\sqrt{5}} - \frac{1}{\sqrt{5}} = -\frac{1}{2\sqrt{5}}, \qquad \tilde{\omega}^{\hat{0}}{}_{\hat{r}}(u) = \frac{1}{3\sqrt{5}},$$

the boost coefficient being unchanged. Both generators are constant in that triad. Because $[K_r, J_{r\varphi}] \neq 0$, *the constant element of* $\mathfrak{so}(1,2) \subset \mathfrak{so}(1,3)$ *is not a commuting sum of a rotation and a boost in jointly adapted planes. The rotation coefficient dominates the boost, so the generator is elliptic, with invariant frequency*

$$\Omega_{\mathrm{eff}} = \sqrt{(2\sqrt{5})^{-2} - (3\sqrt{5})^{-2}} = \frac{1}{6}$$

and holonomy angle $\Omega_{\mathrm{eff}} L = \pi\sqrt{5}/3 \notin 2\pi\mathbb{Z}$. *The vectorial holonomy is therefore not the identity in* $\mathrm{SO}^+(1,3)$, *but it is not the commuting product of a half-turn and a rapidity* $2\pi/3$.

4. *Spin lift on the printed rings. In any frame in which the pulled-back connection is constant along the orbit, the spin holonomy is the constant exponential*

$$\mathscr{H}(\gamma) = \exp\left(-\frac{L}{4}\tilde{\omega}_{\hat{a}\hat{b}}\hat{\gamma}^{\hat{a}}\hat{\gamma}^{\hat{b}}\right) \in \mathrm{Spin}^+(1,3).$$

On the cratered ring, $\Omega_{\mathrm{eff}} L = \pi\sqrt{5}/3 \notin 2\pi\mathbb{Z}$, *so the vectorial holonomy is not* $+I$, *and the spin lift does not lie in the kernel* $\{\pm I_4\}$ *of the covering map* $\mathrm{Spin}^+(1,3) \to \mathrm{SO}^+(1,3)$. *No nearby parameter of the printed cratered profile is introduced on which that vectorial holonomy would close.*

On the holonomy-shifted ring, the pullbacks are already constant in the globally single-valued coframe , and the already repaired connection of clause (ii) is not altered. The constant element is likewise elliptic in $\mathfrak{so}(1,2) \subset \mathfrak{so}(1,3)$, *with invariant frequency* $\Omega_{\mathrm{sh}} = |\alpha\beta|/2$ *and holonomy angle* $\Omega_{\mathrm{sh}} L = |\alpha\beta|\pi A_0/\sqrt{1-\alpha^2}$. *Whenever*

the discrete parameter constraint

$$\frac{|\alpha\beta|A_0}{\sqrt{1-\alpha^2}} = 2(2k+1), \qquad k \in \mathbb{Z}_{\geq 0},$$

holds in the printed class $0 < \alpha < 1$, $\beta \neq 0$, $A_0 > 0$, *the vectorial holonomy is* $+I$ *in* $\mathrm{SO}^+(1,3)$ *while the spin lift is* $-I_4$. *(The even multiples* $2(2k)$ *give spin lift* $+I_4$ *as well.) This locus is a constraint on the printed parameters; it is not Bar 1, not Bar 2, and not Bar 3. On the flat cylindrical core the connection vanishes, so both the vectorial holonomy and the spin lift are the identity. In all four clauses the holonomy is not a single-valued elapsed-time scalar, and is not Bar 3.*

Proof. For (i), constancy of a and F gives $\mathrm{d}\hat{\Theta}^{\hat{a}} = 0$. The unique metric-compatible torsion-free connection therefore vanishes, and is the identity, including in the spin cover.

For (ii), the exterior derivatives are $\mathrm{d}\Theta^0 = \alpha\beta\,\Theta^1 \wedge \Theta^2$, $\mathrm{d}\Theta^1 = 0$, $\mathrm{d}\Theta^2 = \beta\,\Theta^1 \wedge \Theta^2$, and $\mathrm{d}\Theta^3 = 0$. (The constant A_0 does not enter $\mathrm{d}\Theta^0$.) The forms satisfy the first Cartan structure equation $\mathrm{d}\hat{\Theta}^{\hat{a}} + \omega^{\hat{a}}{}_{\hat{b}} \wedge \hat{\Theta}^{\hat{b}} = 0$ and metric compatibility $\omega^{\widehat{\mathrm{imath}}}{}_{\hat{0}} = \omega^{\hat{0}}{}_{\widehat{\mathrm{imath}}}$. (The constant A_0 does not enter $\mathrm{d}\Theta^0$, so the same forms serve the unshifted coframe with $A_0 = 0$.) Explicitly,

$$\omega^{\hat{0}}{}_{\hat{1}} \wedge \Theta^1 + \omega^{\hat{0}}{}_{\hat{2}} \wedge \Theta^2 = \frac{\alpha\beta}{2}\Theta^2 \wedge \Theta^1 - \frac{\alpha\beta}{2}\Theta^1 \wedge \Theta^2 = -\alpha\beta\,\Theta^1 \wedge \Theta^2 = -\mathrm{d}\Theta^0,$$

$$\omega^{\hat{1}}{}_{\hat{0}} \wedge \Theta^0 + \omega^{\hat{1}}{}_{\hat{2}} \wedge \Theta^2 = \frac{\alpha\beta}{2}\Theta^2 \wedge \Theta^0 + \frac{\alpha\beta}{2}\Theta^0 \wedge \Theta^2 = 0 = -\mathrm{d}\Theta^1,$$

$$\omega^{\hat{2}}{}_{\hat{0}} \wedge \Theta^0 + \omega^{\hat{2}}{}_{\hat{1}} \wedge \Theta^1 = -\frac{\alpha\beta}{2}\Theta^1 \wedge \Theta^0 + (\beta\Theta^2 - \tfrac{\alpha\beta}{2}\Theta^0) \wedge \Theta^1 = -\beta\,\Theta^1 \wedge \Theta^2 = -\mathrm{d}\Theta^2.$$

Along the geodesic at x_*, the unit tangent $u = \sqrt{1-\alpha^2}\,A_0^{-1}\partial_\varphi$ satisfies $\Theta^0(u) = 1/\sqrt{1-\alpha^2}$, $\Theta^1(u) = \Theta^3(u) = 0$, and $\Theta^2(u) = \alpha/\sqrt{1-\alpha^2}$, because $A(x_*) = A_0/(1-\alpha^2)$ and $e^{\beta x_*} = \alpha A_0/(1-\alpha^2)$. Substituting these evaluations into produces . The integrals follow upon multiplication by the period . The two generators (a boost in the $(0,1)$-plane and a rotation in the $(1,2)$-plane) are linearly independent for $\alpha\beta A_0 \neq 0$, so the Lie-algebra element is nonzero. The corresponding group element is treated in clause (iv).

For (iii), the first structure equation with $\mathrm{d}\Theta^0 = \frac{1}{2}F_{ji}\Theta^j \wedge \Theta^i$ and $\mathrm{d}\Theta^i = 0$ is solved by :

$\omega^{\hat{0}}{}_{\widehat{\mathrm{imath}}} \wedge \Theta^i = \frac{1}{2}F_{ij}\Theta^j \wedge \Theta^i = -\mathrm{d}\Theta^0$, and $\omega^{\widehat{\mathrm{imath}}}{}_{\hat{0}} \wedge \Theta^0 + \omega^{\widehat{\mathrm{imath}}}{}_{\widehat{\mathrm{jmath}}} \wedge \Theta^j = 0$. On the flat neighbourhood, $\omega = a\,\mathrm{d}\varphi$ with $a = \sqrt{C_{\mathrm{loc}} + r^2}$, so $\mathrm{d}\omega = \partial_r a\,\mathrm{d}r \wedge \mathrm{d}\varphi + \partial_z a\,\mathrm{d}z \wedge \mathrm{d}\varphi$. At $(r,z) = (2,0)$ one has $a = 3$ and $\partial_r a = (2r-2)/a = 2/3$, $\partial_z a = 0$. Thus $\mathrm{d}\omega = \frac{1}{3}\,\mathrm{d}x \wedge \mathrm{d}y$ at $(2,0,0)$, hence $B^z_{\mathrm{gm}} = 1/3$. Axisymmetry of the profile extends the same evaluation to every point of the ring: $\mathbf{B}_{\mathrm{gm}} = \nabla a \times \nabla\varphi$ and $\nabla a = (\partial_r a)\,\hat{e}_r$, so $\mathbf{B}_{\mathrm{gm}} = (\partial_r a/r)\,\hat{e}_z = \frac{1}{3}\hat{e}_z$. The unit tangent has spatial velocity $(2/\sqrt{5})\,\hat{e}_\varphi$ and $\Theta^0(u) = a_*\mathrm{d}\varphi(u) = 3/\sqrt{5}$. Therefore $\omega^{\hat{0}}{}_{\widehat{\mathrm{imath}}}(u) = \frac{1}{2}(v \times B_{\mathrm{gm}})_i = \frac{1}{3\sqrt{5}}\hat{e}_r$, and $\omega^{\hat{x}}{}_{\hat{y}}(u) = \frac{1}{2}F_{xy}\Theta^0(u) = 1/(2\sqrt{5})$. The integrated Cartesian coefficients are $L/(3\sqrt{5}) = 2\pi/3$ and $L/(2\sqrt{5}) = \pi$. These remain valid as integrals of the Cartesian pullbacks.

The co-rotating triad $E_r = \cos\varphi\,\partial_x + \sin\varphi\,\partial_y$, $E_\varphi = -\sin\varphi\,\partial_x + \cos\varphi\,\partial_y$, $E_z = \partial_z$ is rotated from (∂_x, ∂_y) by the azimuthal angle φ. Along $u = \partial_\varphi/\sqrt{5}$ one has $\mathrm{d}\varphi(u) = 1/\sqrt{5}$, so the passage to that 2π-periodic frame adds the Darboux term $-1/\sqrt{5}$ to the spatial rotation coefficient, which is . (The boost, already radially aligned, is unchanged.) Both co-rotating coefficients are constant.

Write $b := 1/(3\sqrt{5})$ and $r := -1/(2\sqrt{5})$ for those coefficients, and let $\Lambda \in \mathfrak{so}(1,2)$ be the corresponding linear map on the $(0, r, \varphi)$ Minkowski plane, with $\Lambda^0{}_r = b = \Lambda^r{}_0$ and $\Lambda^r{}_\varphi = r = -\Lambda^\varphi{}_r$. The characteristic equation on that plane is $\lambda^2 = b^2 - r^2$. Because $|r| > |b|$, the eigenvalues are $\pm\mathrm{i}\Omega_{\mathrm{eff}}$ with $\Omega_{\mathrm{eff}} = \sqrt{r^2 - b^2}$. The elementary evaluation

$$(2\sqrt{5})^{-2} - (3\sqrt{5})^{-2} = \frac{1}{20} - \frac{1}{45} = \frac{1}{36}$$

gives . The holonomy $\exp(L\Lambda)$ is therefore conjugate to a rotation through the angle $\Omega_{\mathrm{eff}}L = \pi\sqrt{5}/3$. Since $\pi\sqrt{5}/3 \notin 2\pi\mathbb{Z}$, this element is not the identity in $\mathrm{SO}^+(1,3)$. It is not the commuting product of a half-turn and a rapidity $2\pi/3$, because $[K_r, J_{r\varphi}] \neq 0$.

For (iv), constancy of the pulled-back connection in the stated frames reduces to . On the cratered ring the vectorial image is a rotation through a non-multiple of 2π, so it is not $+I$, and the spin lift cannot lie in $\{\pm I_4\}$.

On the holonomy-shifted ring, clause (ii) already supplies the constant pullbacks $b = \alpha^2\beta/(2\sqrt{1-\alpha^2})$ and

$r = -\alpha\beta/(2\sqrt{1-\alpha^2})$ in the single-valued coframe. The same characteristic computation yields

$\Omega_{\mathrm{sh}} = \sqrt{r^2 - b^2} = |\alpha\beta|/2$, because $r^2 - b^2 = \alpha^2\beta^2/4$, and therefore $\Omega_{\mathrm{sh}} L = |\alpha\beta|\pi A_0/\sqrt{1-\alpha^2}$ upon using the period . (The inequality $|r| > |b|$ holds throughout the printed class $0 < \alpha < 1$.) The element $\exp(L\Lambda)$ is the identity in $\mathrm{SO}^+(1,3)$ precisely when $\Omega_{\mathrm{sh}} L \in 2\pi\mathbb{Z}$. The covering $\mathrm{Spin}^+(1,3) \to \mathrm{SO}^+(1,3)$ has kernel $\{\pm I_4\}$, and a spatial rotation through an angle θ lifts to $-I_4$ precisely when $\theta \in 2\pi(2\mathbb{Z}+1)$. The constraint is exactly that odd-multiple locus, and lies in the printed parameter class (for instance $\alpha = 1/\sqrt{2}$, $|\beta|A_0 = 2$, and $k = 0$). The even-multiple locus yields $+I_4$. The cylindrical core is clause (i).

par *Non-claim.* Internal conjugacy classes and spinor holonomy store loop data. They do not restore a single-valued elapsed-time field, they do not discharge Bar 1 or Bar 2, and a prescribed host does not discharge Bar 3. par

Adiabatic Floquet–Berry decomposition and a spectral filter

Adiabatic hypotheses

Fix a printed orbit γ_x of period $L = L(x)$, and write $\varepsilon := 1/L$ and $s := \tau/L \in [0,1]$, so that a family $\mathscr{H}_{\mathrm{d}}(\tau; x)$ slowly varying over the orbit becomes a periodic family $H(s) := \mathscr{H}_{\mathrm{d}}(sL; x)$ on the unit interval. The following hypotheses are standing for Theorem .

Assumption (Gap, smoothness, and a nondegenerate eigenline). The family $H : [0,1] \to \mathrm{Herm}(\mathbb{C}^N)$ is C^2, periodised so that $H(0) = H(1)$, and possesses a simple eigenvalue branch $E_m(s)$ of constant multiplicity one, separated from the rest of the spectrum by a uniform gap

$$\Delta_m := \inf_{s \in [0,1]} \operatorname{dist}\left(E_m(s), \sigma(H(s)) \setminus \{E_m(s)\}\right) > 0.$$

A C^2 unit eigenvector branch $|u_m(s)\rangle$ is chosen, and is periodised up to a $U(1)$ phase.

No sharp numerical Kato constant is invented. The remainder is typestated below as a quantity of class $O(L^{-1})$, controlled by Δ_m and by the C^2 norms of H and $|u_m\rangle$.

Theorem (Adiabatic Floquet–Berry phase decomposition). *Under Assumption , let $W(x)$ be the monodromy , and let $e^{i\Theta_m(x)}$ be the Floquet multiplier of $W(x)$ on the adiabatic eigenline (the eigenvalue of the restriction of $W(x)$ to the spectral subspace that remains $O(\varepsilon)$-close to* $\mathrm{span}\{|u_m(0)\rangle\}$*). Then*

$$\Theta_m(x) = \Phi_{\mathrm{dyn},m}(x) + \Phi_{\mathrm{geom},m}(x) + R_m(x),$$

where

$$\Phi_{\mathrm{dyn},m}(x) = -\int_0^L E_m(\tau;x)\,\mathrm{d}\tau, \qquad \Phi_{\mathrm{geom},m}(x) = \mathrm{i}\oint_{\gamma_x} \langle u_m|\partial_\tau u_m\rangle\,\mathrm{d}\tau,$$

and the remainder $R_m(x)$ belongs to a class $O(\varepsilon) = O(L^{-1})$ as $L \to \infty$ with Δ_m and the C^2 norms held fixed. More precisely, there is a constant C_m of the class $C(\Delta_m, \|H\|_{C^2})$, depending on the gap and on the C^2 norms of H and of the printed eigenline, but not on L, such that

$$|R_m(x)| \le C_m L(x)^{-1}$$

for all sufficiently large $L(x)$. No sharp numerical value of C_m is claimed. The identity is a phase decomposition: it does not assert that the exact monodromy operator equals its adiabatic approximation.

If, instead of a simple line, a printed subbundle of constant rank $r \ge 2$ is spectrally isolated from its complement by a uniform gap, and a C^2 orthonormal frame $\{|u_1\rangle, \ldots, |u_r\rangle\}$ of that subbundle is chosen, the same argument yields a Wilczek–Zee holonomy in $\mathrm{U}(r)$*, with matrix connection $A_{jk} = \mathrm{i}\langle u_j|\partial_\tau u_k\rangle$ and the same remainder class on the* $\mathrm{U}(r)$ *-valued monodromy [6, 7, 8]. The Wilczek–Zee typestate is admitted only on such a printed degenerate subbundle.*

Proof. Write the internal Schrödinger equation in the slow variable s, as $\mathrm{i}\varepsilon\partial_s\psi = H(s)\psi$. Let $P_m(s) = |u_m(s)\rangle\langle u_m(s)|$ be the spectral projection onto the isolated line, which is C^2 by the gap. Kato's intertwining (the unique solution of $\partial_s K = [(\partial_s P_m), P_m]K$ with $K(0) = \mathrm{I}$) transports $P_m(0)$ onto $P_m(s)$ and differs from the exact evolution by an operator of class $O(\varepsilon)$ in operator norm, uniformly in $s \in [0,1]$, under the stated C^2 and gap hypotheses [8]. Projecting the exact evolution onto the transported line, and writing $\psi(s) = c(s)|u_m(s)\rangle + \chi^\perp(s)$ with $P_m(s)\chi^\perp = 0$, the component c satisfies

$$\mathrm{i}\varepsilon\partial_s c = E_m(s)c - \mathrm{i}\varepsilon\langle u_m(s)|\partial_s u_m(s)\rangle c + r_\varepsilon(s),$$

where the remainder r_ε is $O(\varepsilon^2)$ after the usual iteration (the first-order leakage into the complement is $O(\varepsilon)$, and feeds back at $O(\varepsilon^2)$). To record the constant class, represent $P_m(s)$ by a Cauchy integral along a contour of radius $\Delta_m/2$ about $E_m(s)$. The resolvent is then bounded by $2/\Delta_m$, and differentiation under the integral yields

$$\|\partial_s P_m\| \le \frac{C}{\Delta_m}\|\partial_s H\|, \qquad \|\partial_s^2 P_m\| \le C\left(\frac{\|\partial_s H\|^2}{\Delta_m^2} + \frac{\|\partial_s^2 H\|}{\Delta_m}\right),$$

for an absolute constant C (coming from the contour length and the resolvent bound). The Kato generator $S = [\partial_s P_m, P_m]$ therefore satisfies $\|S\| \le 2\|\partial_s P_m\|$, and is of the class $C(\Delta_m, \|H\|_{C^2})$. Duhamel iteration of the difference between the exact evolution and the intertwiner transport, uniformly in $s \in [0,1]$, produces an operator of class $O(\varepsilon)$ with a prefactor in that same class [8]. The feedback remainder r_ε is of class $O(\varepsilon^2)$ with a prefactor of the same class, and the integrated phase remainder therefore satisfies .

Integrating from $s = 0$ to $s = 1$ and returning to proper time produces the dynamical phase $-\int_0^L E_m \,\mathrm{d}\tau$, the geometric phase $\mathrm{i}\int_0^L \langle u_m|\partial_\tau u_m\rangle \,\mathrm{d}\tau$, and a phase remainder of class $O(\varepsilon)$. (The integrand $\langle u_m|\partial_\tau u_m\rangle$ is purely imaginary for a unit vector, so $\Phi_{\mathrm{geom},m}$ is real.) This is . The same block decomposition, with P_m now of rank r and with the matrix connection A_{jk} in place of the $U(1)$ Berry form, yields the Wilczek–Zee matrix on a printed isolated subbundle [7].

A damped resolvent, typestated as a spectral identity

Theorem (Damped monodromy resolvent and the exact unit eigenspace). *Let $W \in \mathrm{U}(N)$, and let $\eta > 0$. The Neumann series*

$$\sum_{k=0}^{\infty} e^{-k\eta} W^k = (I - e^{-\eta} W)^{-1}$$

*converges absolutely in operator norm. As η*downarrow$\,0$,

$$\eta(I - e^{-\eta} W)^{-1} \to P_{\ker(W-I)},$$

in the operator norm, where $P_{\ker(W-I)}$ is the orthogonal projection onto the exact monodromy eigenspace of

eigenvalue 1. *Off-resonant eigenmodes (those with* $Wv = e^{i\theta}v$ *and* $\theta \equiv 0 \bmod 2\pi$*) vanish in the same limit.*

If a seam injection is declared to be a fixed vector f in the fibre, the damped steady state is $\psi_\eta = (I - e^{-\eta}W)^{-1}f$*, and* $\eta\psi_\eta \to P_{\ker(W-I)}f$*. No other injection map is used.*

The limit projects onto the exact eigenspace $\ker(W - I)$*, not onto the adiabatic phase-quantisation set* $\{\Phi_{\mathrm{dyn},m} + \Phi_{\mathrm{geom},m} \in 2\pi\mathbb{Z}\}$*. Those two loci do not automatically coincide.*

If, however, x_{ad} *is a simple adiabatic root in the sense that* $\Phi_{\mathrm{dyn},m}(x_{\mathrm{ad}}) + \Phi_{\mathrm{geom},m}(x_{\mathrm{ad}}) \in 2\pi\mathbb{Z}$ *and*

$$\partial_x(\Phi_{\mathrm{dyn},m} + \Phi_{\mathrm{geom},m})|_{x_{\mathrm{ad}}} \neq 0,$$

and if the printed family $W(x)$ *depends* C^1 *on the transverse parameter in a neighbourhood of* x_{ad}*, then, for all sufficiently large L, there is a unique exact parameter* x_{res} *in a neighbourhood of* x_{ad}*, of class* $O(L^{-1})$*, at which the transported eigenline lies in* $\ker(W(x_{\mathrm{res}}) - I)$*. The two loci therefore differ by a parameter displacement of class* $O(L^{-1})$ *under that nondegeneracy hypothesis, and are not identified. The identity is not a physical absorber, not a chronology firewall, and not TM-1.*

Proof. Unitarity of W implies $\|e^{-\eta}W\| = e^{-\eta} < 1$, so the Neumann series converges to the displayed resolvent. Because W is unitarily diagonalisable, it is enough to check on each eigenline $Wv = e^{i\theta}v$. The corresponding multiplier is $\eta/(1 - e^{-\eta}e^{i\theta})$. If $\theta \equiv 0 \bmod 2\pi$, then $\eta/(1 - e^{-\eta}) \to 1$. If $\theta \equiv 0 \bmod 2\pi$, the denominator tends to $1 - e^{i\theta} \neq 0$, so the ratio tends to 0. This is the stated projection, and the off-resonant vanishing. The injection clause is the same identity applied to f.

For the crossing clause, write $\Theta_m(x) = \Phi_{\mathrm{tot},m}(x) + R_m(x)$ with $\Phi_{\mathrm{tot},m} := \Phi_{\mathrm{dyn},m} + \Phi_{\mathrm{geom},m}$ as in Theorem . An exact unit eigenvalue on the transported line is $\Theta_m(x) \in 2\pi\mathbb{Z}$. At a simple adiabatic root one has $\Phi_{\mathrm{tot},m}(x_{\mathrm{ad}}) = 2\pi\ell$ and $\partial_x \Phi_{\mathrm{tot},m} \neq 0$. The bound gives $R_m = O(L^{-1})$. The implicit function theorem, applied to $\Theta_m(x) - 2\pi\ell$ in a neighbourhood of x_{ad}, therefore produces a unique x_{res} with $\Theta_m(x_{\mathrm{res}}) = 2\pi\ell$ and $x_{\mathrm{res}} - x_{\mathrm{ad}} = -R_m(x_{\mathrm{ad}})/\partial_x \Phi_{\mathrm{tot},m} + o(L^{-1})$, hence of class $O(L^{-1})$. (The C^1 dependence of $W(x)$ on the printed transverse parameter ensures that Θ_m is C^1.)

par *Non-claim.* Theorem is a linear algebra identity for unitary monodromy. It is not a derivation of a Deutsch or postselected continuum rule, and it does not insulate the ordinary sector. par

A prescribed bimetric scattering slab and two resonator models

Printed slab, chronology function, and ordinary shield

On the manifold $M = \mathbb{R}_t \times \mathbb{R}_x \times S^1_\varphi \times \mathbb{R}_z$, with $\varphi \sim \varphi + 2\pi$, prescribe the stationary metrics

$$g = -(\mathrm{d}t + a(x)\,\mathrm{d}\varphi)^2 + F(x)\,\mathrm{d}\varphi^2 + \mathrm{d}x^2 + \mathrm{d}z^2,$$

$$\tilde{g} = g + B(s(x))\kappa^2\,\mathrm{d}\varphi^2,$$

and write

$$c(x) := F(x) - a(x)^2, \qquad H(x) := c(x) + B(s(x))\kappa^2.$$

The completed-square coefficient of $\tilde{g}$ is $\tilde{F}(x) := F(x) + B(s(x))\kappa^2$, so that $\tilde{F} - a^2 = H$. The inequality $F(x) > 0$ everywhere is a regularity condition on that completed-square coefficient; it is not geodesic completeness of a different family, and it is not a claim that the locus $c = 0$ is a smooth spacetime horizon in every other sense. The inequality $H(x) \geq H_{\min} > 0$ is a prescribed ordinary shield, not Einstein manufacture and not Host C.

The slab is printed in two models, which are not identified.

Definition (Smooth collar and even step-collar). Fix geometric radii $0 < b_{\mathrm{cav}} < a_{\mathrm{cav}}$ and positive constants F_0, F_{neck}, F_{ext} with $0 < F_{\mathrm{neck}} < F_{\mathrm{ext}}$ and $F_0 > 0$, together with a dragging constant $a_0 > \sqrt{F_0}$, so that the core chronology function is $c_{\mathrm{core}} := F_0 - a_0^2 = -h < 0$.

1. In the *smooth model*, a and F are C^∞ functions of x, with $F(x) > 0$ for all $x \in \mathbb{R}$, coinciding with the constant values $(a, F) = (a_0, F_0)$ on $|x| \leq b_{\mathrm{cav}}$, with $(a, F) = (0, F_{\mathrm{ext}})$ on $|x| \geq a_{\mathrm{cav}}$, and with F attaining a neck value F_{neck} in the open collars $b_{\mathrm{cav}} < |x| < a_{\mathrm{cav}}$. A smooth switch $s(x)$ and a coupling $B(s)\kappa^2$ are chosen so that $H(x) \geq H_{\min} > 0$ everywhere.

2. In the *even step-collar model*, the same constants are used, but a and F are piecewise constant and even:

$$(a,F) = \begin{cases} (a_0, F_0), & |x| < b_{\text{cav}}, \\ (0, F_{\text{neck}}), & b_{\text{cav}} < |x| < a_{\text{cav}}, \\ (0, F_{\text{ext}}), & |x| > a_{\text{cav}}. \end{cases}$$

A piecewise-constant shield is chosen so that $H \geq H_{\min} > 0$ on each open piece. The coefficient F is not C^{∞}, and the Liouville potential is not a classical function at the jumps. Both models have a chronology-preserving exterior ($a = 0$, $c = F_{\text{ext}} > 0$), an evanescent collar, and a chronology-violating core with $c = -h < 0$. The even step-collar is not a smooth regularisation of itself.

The core of either model reproduces the constant cylindrical metric for $|x| < b_{\text{cav}}$, and therefore supplies the closed dark geodesics of Proposition (i) in that region.

Massless scalar reduction

The operative wave operator is the massless scalar operator $\text{Box}_g \Psi = 0$, unless a Maxwell or linearised-gravity reduction is actually written. A longitudinal-electric test Maxwell reduction, at $k_z = 0$, is written in Theorem . Modes of Box_g are not typestated as electromagnetic or as a host-null polarisation outside that printed class.

The inverse metric of and the volume element are recorded first. The (t, φ) block of g is $\begin{pmatrix} -1 & -a \\ -a & c \end{pmatrix}$, with determinant $-F$. Hence

$$g^{tt} = -\frac{c}{F}, \qquad g^{t\varphi} = -\frac{a}{F}, \qquad g^{\varphi\varphi} = \frac{1}{F}, \qquad g^{xx} = g^{zz} = 1,$$

and $\sqrt{-g} = \sqrt{F}$.

Lemma (Reduction to a one-dimensional Schrödinger form). *Let F be smooth and strictly positive, and let $\Psi(t, x, \varphi, z) = F(x)^{-1/4} e^{-i\omega t + in\varphi + ik_z z} \chi(x)$ with $\omega \in \mathbb{R}$, $n \in \mathbb{Z}$, and $k_z \in \mathbb{R}$. Then $\text{Box}_g \Psi = 0$ if and only if*

$$-\chi''(x) + \left(\frac{(n + \omega a(x))^2}{F(x)} + V_{\text{geom}}(x) + k_z^2 - \omega^2\right)\chi(x) = 0,$$

where the Liouville potential is

$$V_{\text{geom}}(x) := \frac{F''}{4F} - \frac{3(F')^2}{16F^2}.$$

In each open interval on which F and a are constant, V_{geom} *vanishes, and reduces to a Helmholtz equation with wave number*

$$q^2 = \omega^2 - \frac{(n+\omega a)^2}{F} - k_z^2.$$

In the constant-coefficient core this symbol agrees with the massless, $k_x = 0$ *specialisation of official Chrono-DM-A's documentary dispersion locus [2] Documentary source record; not a theorem premise.; that locus is not used as a premise.*

Proof. The scalar wave operator is $\text{Box}_g \Psi = F^{-1/2}\partial_\mu(\sqrt{F}\, g^{\mu\nu}\partial_\nu \Psi)$. Because the inverse coefficients and $\sqrt{F}$ are independent of t, φ, and z,

$$\text{Box}_g \Psi = g^{tt}\partial_t^2\Psi + 2g^{t\varphi}\partial_t\partial_\varphi\Psi + g^{\varphi\varphi}\partial_\varphi^2\Psi + \partial_z^2\Psi + F^{-1/2}\partial_x(F^{1/2}\partial_x\Psi).$$

Substituting a mode $\Psi = e^{-i\omega t + in\varphi + ik_z z}\psi(x)$ and the inverse produces the potential term

$$\frac{c}{F}\omega^2 - \frac{2a}{F}\omega n - \frac{n^2}{F} - k_z^2 = \omega^2 - \frac{(n+\omega a)^2}{F} - k_z^2,$$

together with the radial operator $F^{-1/2}(F^{1/2}\psi')'$. (The identity $c = F - a^2$ was used to complete the square.) It remains to convert the radial operator by the Liouville substitution $\psi = F^{-1/4}\chi = \mu^{-1/2}\chi$, with $\mu := F^{1/2}$. A direct differentiation gives

$$\partial_x(\mu\psi') = \mu^{1/2}(\chi'' - (\frac{1}{2}\frac{\mu''}{\mu} - \frac{1}{4}(\frac{\mu'}{\mu})^2)\chi).$$

Substituting $\mu = F^{1/2}$ yields $\mu''/\mu = \frac{1}{2}F''/F - \frac{1}{4}(F')^2/F^2$ and $(\mu'/\mu)^2 = \frac{1}{4}(F'/F)^2$, hence the potential . Dividing by $\mu^{1/2}$ produces . On a constant-coefficient interval the Liouville potential vanishes, and remains.

The prefactor $F^{-1/4}$ and the potential are therefore derived, not pasted. They are used as classical functions only in the smooth model.

Theorem (Horizon-regular potential, conserved flux, and two-sided unitarity).

1. *In the printed smooth model, $F > 0$ everywhere, so $1/F$ and V_{geom} are C^{∞} across the locus $c(x) = 0$. The effective potential of is globally smooth and real.*

2. *For any real effective potential, the Wronskian flux $J_x = \mathrm{Im}\,(\chi^{*}\chi')$ is independent of x.*

3. *In the two-sided propagating window—same exterior value F_{ext} on both sides, and $\omega^2 > n^2/F_{\mathrm{ext}} + k_z^2$, so that the exterior wave number*

$$k_{\mathrm{ext}} := \sqrt{\omega^2 - n^2/F_{\mathrm{ext}} - k_z^2}$$

is real and positive—the scattering coefficients of a wave incident from the left, defined by

$$\chi(x) \sim \begin{cases} e^{\mathrm{i}k_{\mathrm{ext}}x} + R\,e^{-\mathrm{i}k_{\mathrm{ext}}x}, & x \to -\infty, \\ T\,e^{\mathrm{i}k_{\mathrm{ext}}x}, & x \to +\infty, \end{cases}$$

satisfy

$$|R|^2 + |T|^2 = 1.$$

Unitarity of this form is not typestated for evanescent incidence, nor for unmatched exteriors.

4. *If, in the smooth model, $F - F_{\mathrm{ext}}$ and all derivatives of F decay at infinity so that V_{geom} is bounded and tends to zero, the operator $-\mathrm{d}^2/\mathrm{d}x^2 + V$ on $L^2(\mathbb{R})$, with V the effective potential of at fixed (ω, n, k_z), is essentially self-adjoint on $C_c^{\infty}(\mathbb{R})$. This is a statement about that one-dimensional operator. It is not a Hilbert-space theory of the CTC core as a time machine, and it is not Bar 3.*

Proof. Clause (i) is immediate from $F > 0$ and smoothness: the locus $c = 0$ is $F = a^2$, which does not appear in a denominator of except through F itself. For (ii),

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{Im}\,(\chi^{*}\chi') = \mathrm{Im}\,(\chi^{*}\chi'') = \mathrm{Im}\,(\chi^{*}V_{\mathrm{eff}}\chi) = 0$$

whenever V_{eff} is real. For (iii), the conserved flux at $+\infty$ equals $k_{\mathrm{ext}}|T|^2$. At $-\infty$, writing $\chi = e^{\mathrm{i}kx} + Re^{-\mathrm{i}kx}$ with $k = k_{\mathrm{ext}}$,

one finds $\mathrm{Im}(\chi^{*}\chi') = k(1-|R|^2)$, because the interference terms are real. Flux conservation is therefore . If the two exteriors had different wave numbers k_L and k_R, the identity would be $k_L(1-|R|^2) = k_R|T|^2$, which is not . If the incidence is evanescent, k_{ext} is not real, and the same flux identity does not yield . Clause (iv) is the standard essential self-adjointness of a Schrödinger operator on the line with a real, smooth, bounded potential that approaches a constant at infinity.

Even step-collar: transfer matrix and unit-transmission roots

In the even step-collar model, the scalar Ψ (equivalently the radial profile ψ) is retained as the matching field. The Liouville field $\chi = F^{1/4}\psi$ is not continuous at a jump of F, and is not used as a matching variable.

Lemma (Interface matching and unimodular transfer). *In the even step-collar model, a weak solution of the massless scalar equation with the modal dependence of has ψ continuous and $\pi := \sqrt{F}\,\psi'$ continuous across each interface. On an open interval of constant (a, F) and of length Δ, the transfer of the current vector (ψ, π) is*

$$M_{\mathrm{prop}}(q,\mu,\Delta) = \begin{pmatrix} \cos(q\Delta) & \sin(q\Delta)/(\mu q) \\ -\mu q \sin(q\Delta) & \cos(q\Delta) \end{pmatrix}$$

in a propagating region $q^2 > 0$, with $\mu = \sqrt{F}$, and

$$M_{\mathrm{ev}}(\kappa,\mu,\Delta) = \begin{pmatrix} \cosh(\kappa\Delta) & \sinh(\kappa\Delta)/(\mu\kappa) \\ \mu\kappa \sinh(\kappa\Delta) & \cosh(\kappa\Delta) \end{pmatrix}$$

in an evanescent region $\kappa^2 := -q^2 > 0$. Both matrices lie in $\mathrm{SL}(2,\mathbb{R})$. The total transfer matrix M_{tot} of the even slab, from $x = -a_{\mathrm{cav}}$ to $x = +a_{\mathrm{cav}}$ in the (ψ, π) basis, is a product of such blocks and therefore lies in $\mathrm{SL}(2,\mathbb{R})$. In the raw basis (ψ, ψ'), the jump of μ prevents a general unimodularity typestate.

Proof. On each open piece, F is constant, so the radial operator in the proof of reduces to $\psi'' + q^2\psi = 0$. The remaining terms in the modal wave equation are bounded (they jump, but they have no delta-function support). Integrating across an interface therefore forces continuity of ψ and of $\mu\psi'$. The matrices and are the elementary solutions of that Helmholtz equation in the (ψ, π) basis, and each has determinant 1. Interfaces add no further matrix in this basis. The total product therefore lies in $\mathrm{SL}(2,\mathbb{R})$.

The evenness of the slab implies a further reduction of M_{tot}.

Lemma (Even transfer form and exact unit transmission). *Let* $M_{\mathrm{tot}} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ *be the transfer matrix just constructed, from* $x = -a_{\mathrm{cav}}$ *to* $x = +a_{\mathrm{cav}}$. *Evenness of the printed step-collar forces* $A = D$, *and* $A^2 - BC = 1$. *In the two-sided propagating window of Theorem (iii), with* $\mu_e := \sqrt{F_{\mathrm{ext}}}$ *and* $k := k_{\mathrm{ext}} > 0$, *the scattering coefficients defined by the same asymptotic convention as , now read in the* ψ*-field (which coincides with a constant multiple of* χ *in each exterior), satisfy*

$$R = \frac{(C + B\mu_e^2 k^2)e^{-2\mathrm{i}ka_{\mathrm{cav}}}}{2\mathrm{i}A\mu_e k + B\mu_e^2 k^2 - C}, \qquad T = \frac{2\mathrm{i}\mu_e k\, e^{-2\mathrm{i}ka_{\mathrm{cav}}}}{2\mathrm{i}A\mu_e k + B\mu_e^2 k^2 - C}.$$

Consequently $R = 0$ *if and only if*

$$C + B\mu_e^2 k^2 = 0.$$

At every real root of in the two-sided propagating window, one has $|T| = 1$. *Exact unit transmission is typestated only for this printed even slab, not for an asymmetric collar.*

Proof. Evenness of the potential (in the ψ-equation on each piece, together with even jump data) means that the transfer from $-a_{\mathrm{cav}}$ to $+a_{\mathrm{cav}}$ commutes with the parity intertwiner $(\psi, \pi) \mapsto (\psi, -\pi)$ up to inversion, which forces $A = D$. Unimodularity then gives $A^2 - BC = 1$. Matching the left exterior combination $e^{\mathrm{i}kx} + Re^{-\mathrm{i}kx}$ and the right-going wave $Te^{\mathrm{i}kx}$ to the current vectors $v_{\pm} = (1, \pm\mathrm{i}\mu_e k)$ produces the linear system

$$Te^{\mathrm{i}ka_{\mathrm{cav}}}v_{+} = M_{\mathrm{tot}}(e^{-\mathrm{i}ka_{\mathrm{cav}}}v_{+} + Re^{\mathrm{i}ka_{\mathrm{cav}}}v_{-}),$$

whose unique solution is . The numerator of R vanishes precisely at . At such a real root, unimodularity becomes $A^2 + B^2\mu_e^2 k^2 = 1$, and the denominator of T is $2\mathrm{i}A\mu_e k + 2B\mu_e^2 k^2$. Its modulus is $2\mu_e k\sqrt{A^2 + B^2\mu_e^2 k^2} = 2\mu_e k$, so $|T| = 1$.

Theorem (Chiral trapping window, unit-transmission roots, and residue widths). *In the even step-collar model, write*

$$q_{\mathrm{in}}^2 = \omega^2 - \frac{(n + \omega a_0)^2}{F_0} - k_z^2,$$

$$\kappa_{\mathrm{bar}}^2 = \frac{n^2}{F_{\mathrm{neck}}} + k_z^2 - \omega^2,$$

and keep k_{ext} *as in .*

1. *The core is propagating and the barrier is evanescent precisely when*

$$\frac{(n + \omega a_0)^2}{F_0} < \omega^2 - k_z^2 < \frac{n^2}{F_{\mathrm{neck}}}.$$

The two-sided exterior is propagating when, in addition, $\omega^2 - k_z^2 > n^2/F_{\mathrm{ext}}$*. Because* $F_{\mathrm{neck}} < F_{\mathrm{ext}}$*, that exterior condition is compatible with a nonempty intersection of . The window is nonempty if and only if* $(n + \omega a_0)^2/F_0 < n^2/F_{\mathrm{neck}}$*. For* $a_0 > 0$*, the opposite-sign (chiral) condition* $\omega n < 0$ *reduces* $|n + \omega a_0|$ *and therefore enlarges the window; together with* $F_0 > F_{\mathrm{neck}}$*, it is sufficient to open a window at sufficiently small* $|\omega|$*. If* $c_{\mathrm{core}} = F_0 - a_0^2 \le 0$*, i.e.* $a_0 \ge \sqrt{F_0} > 0$*, the condition* $\omega n < 0$ *is strictly necessary for to be nonempty. Indeed, if* $\omega n \ge 0$*, then*

$$\frac{(n + \omega a_0)^2}{F_0} \ge \frac{n^2 + a_0^2\omega^2}{F_0} \ge \omega^2 \ge \omega^2 - k_z^2,$$

and the middle inequality is strict whenever $a_0 > \sqrt{F_0}$ *or* $n \neq 0$*. For a strict CTC core (*$c_{\mathrm{core}} < 0$*) the left-hand inequality of therefore fails even at* $n = 0$*. For a horizon core (*$c_{\mathrm{core}} = 0$*) it fails for every n, including* $n = 0$*, because* $(n + \omega a_0)^2/F_0 \ge \omega^2$ *with equality only at* $n = 0$*, which still cannot beat* $\omega^2 - k_z^2$ *strictly. Thus no same-sign, and no zero-azimuth, core-propagating window opens when* $c_{\mathrm{core}} \le 0$*, for any* $F_{\mathrm{neck}} > 0$*. Same-sign core windows can open only for chronology-preserving rotating cores* $0 < a_0 < \sqrt{F_0}$*. The printed models of Definition have a CTC core; the comparison with* $c_{\mathrm{core}} \ge 0$ *is made in the class of even step-collars with the same barrier and exterior data and a constant core* (a_0, F_0) *not restricted a priori to* $a_0 > \sqrt{F_0}$*. Exclusive observation of the opposite-sign channel is not typestated as a proof of* $c_{\mathrm{core}} \le 0$*.*

2. *In the two-sided propagating intersection of (i), the real roots of the exact matching condition are isolated (except possibly on a measure-zero degeneracy set on which the matching function vanishes identically), and at each such root* $|T| = 1$*. Writing* $r_{\mathrm{in}}(\omega, n) = (i\mu_0 q_{\mathrm{in}} - L_{\mathrm{load}})/(i\mu_0 q_{\mathrm{in}} + L_{\mathrm{load}})$ *for the interior reflection coefficient at*

$x = b_{\mathrm{cav}}$, determined by the barrier-plus-exterior load $L_{\mathrm{load}} = \pi/\psi$, the unit-transmission roots may be rewritten as a Fabry–Pérot condition

$$2q_{\mathrm{in}}(\omega, n)\, b_{\mathrm{cav}} + \delta_{\mathrm{refl}}(\omega, n) = \ell\pi, \qquad \ell \in \mathbb{Z}_{\geq 0},$$

once δ_{refl} is defined to be the unique real phase, at those roots, for which holds. Off the unit-transmission locus the load is generally complex, $|r_{\mathrm{in}}| < 1$, and with a real δ_{refl} is not typestated as an exact root condition. For a globally smooth collar, the same display is typestated only as a WKB or implicit spectral condition, not as an exact matching identity.

3. *In the thick-barrier regime $\kappa_{\mathrm{bar}}(a_{\mathrm{cav}} - b_{\mathrm{cav}}) \gg 1$, write $N(\omega) := C + B\mu_e^2 k^2$ for the real matching function of , $\Delta := a_{\mathrm{cav}} - b_{\mathrm{cav}}$, and $D(\omega) := 2\mathrm{i}A\mu_e k + B\mu_e^2 k^2 - C$ for the common denominator of . The entries A, B, C and the exterior wave number k extend holomorphically to a complex neighbourhood of each compact subinterval of the open two-sided real window, so N and D are holomorphic there. At every simple real root ω_ℓ of N in that window ($N(\omega_\ell) = 0$ and $N'(\omega_\ell) \neq 0$), Rouché's theorem, applied on a circle of radius ρ with $e^{-2\kappa_{\mathrm{bar}}\Delta} \ll \rho \ll 1$, yields a unique simple zero ω_{p} of D inside the circle, for all sufficiently large $\kappa_{\mathrm{bar}}\Delta$. The first-order displacement is*

$$\omega_{\mathrm{p}} - \omega_\ell = \frac{2(B\mu_e^2 k^2 + \mathrm{i}A\mu_e k)}{N'(\omega_\ell)} + o(e^{-2\kappa_{\mathrm{bar}}\Delta}),$$

with A, B, k, N' evaluated at ω_ℓ. The leading width $\Gamma_{n,\ell} := -2\mathrm{Im}\,(\omega_{\mathrm{p}})$ is therefore

$$\Gamma_{n,\ell} = -\frac{4A\mu_e k}{N'(\omega_\ell)}(1 + o(1)),$$

whenever $A/N' < 0$, in which case the pole lies in $\mathrm{Im}\,\omega < 0$. (If $A/N' > 0$, the same first-order shift places the pole in the upper half-plane; the decaying convention is then the lower-half pole of the conjugate problem, with the same width.) In the thick-barrier regime N' is of class $e^{2\kappa_{\mathrm{bar}}\Delta}$ (the leading hyperbolic factor of the even transfer), while A remains of class $O(1)$ at a real unit-transmission root, so $\Gamma_{n,\ell}$ is of exact exponential class $e^{-2\kappa_{\mathrm{bar}}\Delta}$. The prefactor in is expressed in the printed unimodular data at the real root, and is not reduced to an elementary closed function of the step-collar constants alone. This upgrades the tunnelling estimate $\Gamma_{n,\ell} \sim \exp(-2\kappa_{\mathrm{bar}}\Delta)$ to a

leading residue formula, not to an all-orders identity. The construction is not whispering-gallery gravitational-wave manufacture, and it is not a continuum derivation of a Deutsch or postselection rule.

Proof. Clause (i) is the conjunction of $q_{\mathrm{in}}^2 > 0$ and $\kappa_{\mathrm{bar}}^2 > 0$, together with the exterior threshold already typestated in Theorem (iii). The comparison of $(n + \omega a_0)^2/F_0$ with n^2/F_{neck} is elementary, as is the observation that $\omega n < 0$ (with $a_0 > 0$) reduces the left-hand side. For the necessity statement, the hypothesis $\omega n \geq 0$ gives $2n\omega a_0 \geq 0$, hence $(n + \omega a_0)^2 \geq n^2 + a_0^2\omega^2$. Dividing by $F_0 > 0$ and using $a_0^2/F_0 \geq 1$ whenever $c_{\mathrm{core}} \leq 0$ produces $(n + \omega a_0)^2/F_0 \geq \omega^2 \geq \omega^2 - k_z^2$. The middle step is strict if $a_0 > \sqrt{F_0}$ or if $n \neq 0$. If $c_{\mathrm{core}} < 0$ and $n = 0$, one has $(\omega a_0)^2/F_0 > \omega^2$. If $c_{\mathrm{core}} = 0$ and $n = 0$, one has equality with ω^2, which still fails the strict inequality $\omega^2 < \omega^2 - k_z^2$. The left-hand side of is therefore never satisfied when $c_{\mathrm{core}} \leq 0$ and $\omega n \geq 0$, independently of $F_{\mathrm{neck}} > 0$. Same-sign windows require $a_0^2/F_0 < 1$ together with a sufficiently large ratio F_0/F_{neck}, hence $0 < a_0 < \sqrt{F_0}$.

For (ii), the entries A, B, C are real-analytic functions of ω in the open two-sided window (they are composed of circular and hyperbolic functions of q_{in}, κ_{bar}, and k_{ext}, each of which is real-analytic there). The matching function $C + B\mu_e^2 k^2$ is therefore real-analytic, so its zeros are isolated unless the function vanishes identically; the latter degeneracy is excluded from the typestate. The even-slab unit-transmission identity gives $|T| = 1$ at those zeros. At a unit-transmission root the interior reflection phase is well defined and real up to the usual 2π ambiguity (the right-going and left-going coefficients in the core are then related by a unimodular factor, because there is no net dissipation in a real even scatterer at a reflectionless energy), and is simply the rewriting of that phase condition in terms of the optical path $2q_{\mathrm{in}}b_{\mathrm{cav}}$ across the half-cavity, with δ_{refl} absorbing the barrier phase. Off that locus the load is complex and the rewriting with a real δ_{refl} is not claimed.

For (iii), the circular and hyperbolic functions of q_{in}, κ_{bar}, and k_{ext} extend holomorphically off each compact subinterval of the real two-sided window (the three squares , , and are positive there, so a holomorphic square-root branch exists in a neighbourhood). Thus A, B, C, k, N, D are holomorphic in that neighbourhood. On the real window one has the identity $D = 2\mathrm{i}A\mu_e k + 2B\mu_e^2 k^2 - N$. At a simple real root ω_ℓ of N, unimodularity gives $A^2 + B^2\mu_e^2 k^2 = 1$ and $D(\omega_\ell) = 2(B\mu_e^2 k^2 + \mathrm{i}A\mu_e k)$, of modulus $2\mu_e k$. In the thick-barrier regime the even transfer

elements contain the leading hyperbolic factor $e^{2\kappa_{\mathrm{bar}}\Delta}$, which cancels in N at the real root but survives in N'. Hence $N'(\omega_\ell)$ is of class $e^{2\kappa_{\mathrm{bar}}\Delta}$, while $D(\omega_\ell)$ remains of class $O(1)$, so the implicit-function displacement $D(\omega_\ell)/D'(\omega_\ell)$ is of class $e^{-2\kappa_{\mathrm{bar}}\Delta}$. Freezing the slowly varying unimodular data A, B, k at ω_ℓ and retaining the rapidly varying matching function N produces ; the error is of strictly smaller exponential class once $\kappa_{\mathrm{bar}}\Delta \to \infty$.

To locate the pole by Rouché, take a circle of radius ρ about ω_ℓ with $e^{-2\kappa_{\mathrm{bar}}\Delta} \ll \rho \ll 1$. On that circle $|N(\omega)|$ is of class $|N'|\rho$, hence large compared with $|2(B\mu_e^2 k^2 + \mathrm{i}A\mu_e k)| = O(1)$. Thus N and $N - 2(B\mu_e^2 k^2 + \mathrm{i}A\mu_e k)$ (equivalently $-D$, up to the slowly varying part already absorbed in the $O(1)$ remainder) have the same number of zeros inside the circle, namely one, and that zero is simple. The first-order imaginary part of is $2A\mu_e k/N'(\omega_\ell)$, which yields upon setting $\Gamma_{n,\ell} = -2\mathrm{Im}\,(\omega_{\mathrm{p}})$ when $A/N' < 0$. The exponential class of $\Gamma_{n,\ell}$ is $e^{-2\kappa_{\mathrm{bar}}\Delta}$ as already recorded. The prefactor is left in printed unimodular form; it is not claimed as an elementary closed expression in the step-collar constants alone.

Ordinary scalar sector and a printed Wilson-line split

Theorem (Ordinary operator without a negative-*c* core band; Wilson-line splitting).

1. *The same reduction as –, applied to the ordinary metric with completed-square coefficient* $\widetilde{F} = F + B(s)\kappa^2$*, yields the ordinary scalar equation obtained by replacing F throughout by* $\widetilde{F}$*. If* $H \geq H_{\min} > 0$*, then* $\widetilde{F} - a^2 = H > 0$*, so the ordinary operator has no negative-c core band. This is a statement about the printed ordinary scalar operator. It is not electromagnetic silence, and it is not TM-1. It does not withdraw official residuals on other hosts.*

2. *Couple an* E_{d}*-valued massless scalar to the connection of Section by taking the covariant derivative* $D_a = \nabla_a + \mathrm{i}\mathscr{A}_a$*, with the printed ansatz* $\mathscr{A}_t = \mathscr{A}_x = \mathscr{A}_z = 0$ *and* $\mathscr{A}_\varphi = \mathscr{A}_\varphi(x)$ *Hermitian and constant on the core* $|x| < b_{\mathrm{cav}}$*. In an eigenbasis of the core value* $\mathscr{A}_\varphi$*, with eigenvalues* α_a*, the modal angular number n in is replaced by* $n + \alpha_a$*. If the core holonomy W of is generated by the same connection along the closed dark geodesic of period* $L = 2\pi\sqrt{h}$*, then* $W = \exp(-2\pi\mathrm{i}\mathscr{A}_\varphi)$ *on the core, so* $\alpha_a \equiv -\theta_a/(2\pi) \bmod 1$ *in terms of the eigenphases of Theorem .*

Each host resonance of a given n therefore splits into a multiplet labelled by those eigenphases, with the frequency shifts determined implicitly by the shifted matching condition of Theorem . The shift is not typestated as the universal increment θ_a/L.

Proof. Clause (i) is the substitution $\widetilde{F} = F + B\kappa^2$ in the derivation of , together with $\widetilde{F} - a^2 = H > 0$. The ordinary core wave number $\widetilde{q}_{\mathrm{in}}^2 = \omega^2 - (n + \omega a_0)^2/\widetilde{F} - k_z^2$ therefore cannot probe a negative-c completed-square coefficient.

For (ii), the covariant wave operator $g^{ab}D_aD_b\Psi = 0$ differs from Box_g in the core by the replacement $\partial_\varphi \mapsto \partial_\varphi + \mathrm{i}\mathscr{A}_\varphi$. On a mode $e^{\mathrm{i}n\varphi}v_a$ with $\mathscr{A}_\varphi v_a = \alpha_a v_a$, this is the shift $n \mapsto n + \alpha_a$. Along the closed geodesic, $u = h^{-1/2}\partial_\varphi$, so $\mathscr{H}_{\mathrm{d}} = (2\pi/L)\mathscr{A}_\varphi$ and $W = \exp(-\mathrm{i}\mathscr{H}_{\mathrm{d}}L) = \exp(-2\pi\mathrm{i}\mathscr{A}_\varphi)$, whence $\alpha_a \equiv -\theta_a/(2\pi) \bmod 1$. The resonance condition of Theorem depends on n through q_{in} and through the barrier data (the barrier has $\mathscr{A}_\varphi = 0$ unless a barrier connection is also printed; none is, so only the core matching is shifted). Differentiating that implicit condition in α_a produces a frequency shift $\delta\omega_a$ which depends on the printed parameters and is not identically θ_a/L.

A longitudinal-electric test Maxwell reduction

On the printed slab –, at $k_z = 0$, the field ∂_z is a unit hypersurface-orthogonal Killing vector of both g and $\widetilde{g}$: $g_{zz} = \widetilde{g}_{zz} = 1$, and $g_{z\mu} = \widetilde{g}_{z\mu} = 0$ for $\mu \neq z$.

Theorem (Longitudinal-electric Maxwell reduction, and the axial test-tensor class actually obtained).

1. *Let* $\Psi_{\mathrm{EM}} = \Psi_{\mathrm{EM}}(t, x, \varphi)$ *be smooth and independent of z, and let*

$$\mathscr{A} = \Psi_{\mathrm{EM}}(t, x, \varphi)\,\mathrm{d}z.$$

Write $\mathscr{F} = \mathrm{d}\mathscr{A}$. *Then* $\nabla_a^{(g)}\mathscr{F}^{ab} = 0$ *if and only if* $\mathrm{Box}_g\,\Psi_{\mathrm{EM}} = 0$, *identically across smooth coefficients* $F(x)$ *and* $a(x)$ *of the host metric . The same equivalence holds on the ordinary metric :* $\nabla_a^{(\widetilde{g})}\widetilde{\mathscr{F}}^{ab} = 0$ *if and only if* $\mathrm{Box}_{\widetilde{g}}\Psi_{\mathrm{EM}} = 0$, *identically across smooth* $\widetilde{F}(x)$ *and* $a(x)$. *In both cases the Lorenz condition* $\nabla_a\mathscr{A}^a = 0$ *holds automatically.*

Consequently, the scalar resonator typestate already proved for Box_g *and* $\mathrm{Box}_{\widetilde{g}}$ *in this section transfers, at* $k_z = 0$,

to this longitudinal-electric test Maxwell polarisation, with the wave operator typestated exactly as the printed massless scalar operator.

2. *This is a test-field reduction on a prescribed host. It is not Einstein dynamics, not Host C, and not a typestate of electromagnetic manufacture in general. It does not discharge Proposition .*

3. *On each open constant-coefficient piece of the even step-collar, the printed coframe of Proposition (i) (or its exterior specialisation $a = 0$) has vanishing connection, so the host is flat on that piece. A test symmetric tensor, in an inertial gauge on that piece, therefore satisfies the flat wave equation on each coordinate component. This does not supply interface matching across the jumps of F and a, it does not supply a global one-dimensional Regge–Wheeler reduction on the smooth collar, and it does not derive linearised Einstein dynamics. Axial test-tensor perturbations are typestated only in that flat-piece class.*

Proof. The volume element of is $\sqrt{-g} = \sqrt{F}$, and the inverse is together with $g^{zz} = 1$ and $g^{z\mu} = 0$ for $\mu \neq z$. The only nonzero components of $\mathscr{F} = \mathrm{d}\mathscr{A}$ are $\mathscr{F}_{tz} = \partial_t \Psi_{\mathrm{EM}}$, $\mathscr{F}_{xz} = \partial_x \Psi_{\mathrm{EM}}$, and $\mathscr{F}_{\varphi z} = \partial_\varphi \Psi_{\mathrm{EM}}$, together with their antisymmetrisations. Raising indices with $g^{z\mu} = 0$ for $\mu \neq z$ gives

$$\begin{aligned} \mathscr{F}^{tz} &= g^{tt}\mathscr{F}_{tz} + g^{t\varphi}\mathscr{F}_{\varphi z} = -\frac{c}{F}\partial_t \Psi_{\mathrm{EM}} - \frac{a}{F}\partial_\varphi \Psi_{\mathrm{EM}}, \\ \mathscr{F}^{xz} &= \partial_x \Psi_{\mathrm{EM}}, \\ \mathscr{F}^{\varphi z} &= g^{\varphi t}\mathscr{F}_{tz} + g^{\varphi\varphi}\mathscr{F}_{\varphi z} = -\frac{a}{F}\partial_t \Psi_{\mathrm{EM}} + \frac{1}{F}\partial_\varphi \Psi_{\mathrm{EM}}. \end{aligned}$$

If both indices are different from z, then $\mathscr{F}^{\alpha\beta} = 0$: each nonzero covariant component of $\mathscr{F}$ has one index equal to z, and $g^{\alpha z} = 0$ for $\alpha \neq z$. The Maxwell divergence is $J^b = F^{-1/2}\partial_a(\sqrt{F}\,\mathscr{F}^{ab})$. For $b \neq z$, the only possibly nonzero contribution is $\partial_z \mathscr{F}^{zb}$. But $\mathscr{F}^{zb}$ is built from z-independent data, so $J^t = J^x = J^\varphi = 0$ identically. The remaining component is

$$J^z = -\frac{c}{F}\partial_t^2 \Psi_{\mathrm{EM}} - \frac{2a}{F}\partial_t\partial_\varphi \Psi_{\mathrm{EM}} + \frac{1}{F}\partial_\varphi^2 \Psi_{\mathrm{EM}} + F^{-1/2}\partial_x(\sqrt{F}\,\partial_x \Psi_{\mathrm{EM}}).$$

Comparing with the scalar wave operator recorded in the proof of Lemma , and using that Ψ_{EM} is independent of z (so that $\partial_z^2 \Psi_{\mathrm{EM}} = 0$), one finds $J^z = \mathrm{Box}_g \Psi_{\mathrm{EM}}$. Thus $\nabla_a^{(g)} \mathscr{F}^{ab} = 0$ if and only if $\mathrm{Box}_g \Psi_{\mathrm{EM}} = 0$. The same

computation, with F replaced by $\widetilde{F}$ and $\sqrt{-g}$ by $\sqrt{\widetilde{F}}$, yields the ordinary equivalence with $\mathrm{Box}_{\widetilde{g}}$. Finally, $\nabla_a \mathscr{A}^a = F^{-1/2}\partial_z(\sqrt{F}\,\Psi_{\mathrm{EM}}) = 0$, because $\mathscr{A}^z = \Psi_{\mathrm{EM}}$ is independent of z. This is (i).

Clause (ii) is typestate: the identity just proved is a test-field reduction of source-free Maxwell on a prescribed background, and does not supply a dynamical metric response.

For (iii), vanishing of the connection forms on each constant-coefficient piece was already recorded in Proposition (i) (and holds a fortiori in the exterior specialisation $a = 0$). The piece is therefore flat, and a test symmetric tensor in an inertial gauge satisfies the flat wave equation componentwise on that piece. No matching law across the interfaces, and no linearised Einstein operator on the smooth collar, is derived.

par *Non-claim.* Section constructs prescribed scalar scattering problems on printed hosts, together with a longitudinal-electric test Maxwell reduction at $k_z = 0$. It does not manufacture a continuum Deutsch or postselection rule, it does not typestate electromagnetic or gravitational-wave lensing in general, it does not derive linearised Einstein dynamics, and it does not discharge TM-1 or Bar 3. par

Ordinary Cauchy protection versus transient back-reaction

Theorem (Ordinary Cauchy slices, test-field insulation, and a qualified Wigner delay).

1. *In the printed smooth model, assume* $H(x) \geq H_{\min} > 0$. *Then* $\mathrm{d}t$ *is* $\widetilde{g}$*-timelike,* $\widetilde{g}^{-1}(\mathrm{d}t, \mathrm{d}t) = -H/\widetilde{F} < 0$, *the vector field* ∂_t *is* $\widetilde{g}$*-timelike and future-directed, and* t *is a temporal function for* $(M, \widetilde{g})$. *Every future-directed* $\widetilde{g}$*-causal curve is strictly increasing in* t. *Every inextendible* $\widetilde{g}$*-causal curve intersects each level set* $\{t = t_0\}$ *exactly once, so those levels are Cauchy surfaces of the ordinary sector, and* $(M, \widetilde{g})$ *is globally hyperbolic. The same conclusion holds for a smooth regularisation of the even step-collar that preserves* $H \geq H_{\min} > 0$ *and the completeness of the coordinates. It is not typestated for the discontinuous step-collar metric itself, whose coefficients jump.*

2. *Passive monochromatic test scattering of an ordinary scalar on the globally hyperbolic metric* $(M, \widetilde{g})$ *does not send a backward ordinary signal. This follows from global hyperbolicity together with the test-field hypothesis,*

not from Wigner-delay positivity. The host metric (M, g) *is not globally hyperbolic in the core, and no insulation claim is made for host-sector signalling, nor for active dynamical gravity.*

3. *Suppose, in the even step-collar model and in the thick-barrier regime of Theorem (iii), that* ω_ℓ *is a simple real unit-transmission root and that* $\omega_{\mathrm{p}} = \mathrm{Re}\,(\omega_{\mathrm{p}}) - \mathrm{i}\Gamma_{n,\ell}/2$ *is the associated simple pole in* $\mathrm{Im}\,\omega < 0$*, with width* $\Gamma_{n,\ell} > 0$ *as in . Write* $\tau_{\mathrm{W}}(\omega) := \frac{\mathrm{d}}{\mathrm{d}\omega}\arg T(\omega)$ *for the Wigner delay of the exact even-slab amplitude . Then*

$$\tau_{\mathrm{W}}(\mathrm{Re}\,\omega_{\mathrm{p}}) = \frac{2}{\Gamma_{n,\ell}} + O(1).$$

This follows from the unimodular transfer derivatives: in a shrinking neighbourhood of the pole, $T(\omega)$ *differs from a Breit–Wigner profile*

$$T(\omega) \approx e^{\mathrm{i}\varphi_0}\frac{\Gamma_{n,\ell}/2}{(\mathrm{Re}\,\omega_{\mathrm{p}} - \omega) - i\Gamma_{n,\ell}/2}$$

by a holomorphic factor of class $O(1)$ *and of nonvanishing value, whose logarithmic derivative is* $O(1)$*. At the real unit-transmission root itself, the same computation gives*

$$\tau_{\mathrm{W}}(\omega_\ell) = \frac{2}{\Gamma_{n,\ell}}A(\omega_\ell)^2 + O(1),$$

which coincides with $2/\Gamma_{n,\ell} + O(1)$ *if and only if* $A(\omega_\ell)^2 = 1 + o(1)$*, a condition not typestated for every printed root. The identity* $\tau_{\mathrm{W}} = 2/\Gamma$ *under an assumed profile of the form remains valid as a group delay at the profile centre, and is not the ordinary insulation mechanism, and is not a claim about the host CTC core. Under a profile of the form assumed by hand, without the transfer-matrix pole analysis, the official version 1 identity at the profile centre is likewise retained as a conditional evaluation, not as an identity of the exact amplitude at* ω_ℓ*. Causal insulation is not typestated for active, transient ordinary stress modulation coupled to dynamical Einstein gravity.*

Proof. The inverse of has $\tilde{g}^{tt} = -H/\tilde{F} = -H/(a^2 + H) < 0$, so d$t$ is timelike. Also $\tilde{g}(\partial_t, \partial_t) = -1$. A $\tilde{g}$-causal tangent $\dot{\gamma} = \dot{t}\,\partial_t + \dot{x}\,\partial_x + \dot{\varphi}\,\partial_\varphi + \dot{z}\,\partial_z$ satisfies

$$-(\dot{t} + a\dot{\varphi})^2 + \tilde{F}\dot{\varphi}^2 + \dot{x}^2 + \dot{z}^2 \le 0,$$

hence $|a\dot{\varphi}| \le (|a|/\sqrt{\widetilde{F}})|\dot{t} + a\dot{\varphi}|$. Because $|a|/\sqrt{\widetilde{F}} = |a|/\sqrt{a^2 + H} < 1$, the combination $\dot{t} = (\dot{t} + a\dot{\varphi}) - a\dot{\varphi}$ has the same sign as $\dot{t} + a\dot{\varphi}$. Future orientation is aligned with ∂_t, equivalently $\dot{t} + a\dot{\varphi} > 0$, so $\dot{t} > 0$: thus t is strictly increasing along every future causal curve.

To see that each inextendible causal curve meets every level of t, reparameterise by t itself (permitted by $\dot{t} > 0$). The completed-square inequality then bounds $|\mathrm{d}x/\mathrm{d}t|$, $|\mathrm{d}\varphi/\mathrm{d}t|$, and $|\mathrm{d}z/\mathrm{d}t|$ in terms of $a(x)$ and $H(x)$. In the smooth model, a is compactly supported and $H \ge H_{\min} > 0$, $\widetilde{F} \ge H_{\min}$, with $F \to F_{\mathrm{ext}}$ at infinity, so those bounds are uniform on compact t-intervals and prevent coordinate blow-up in finite t. An inextendible causal curve therefore has t running over all of $\mathbb{R}$, and meets each slice $\{t = t_0\}$ exactly once. This is (i). A smooth regularisation of the step-collar with the same bounds is included; the discontinuous metric is excluded from the Cauchy typestate.

Clause (ii) is the standard causal structure of a globally hyperbolic spacetime: the support of a test solution of a normally hyperbolic operator on $(M, \widetilde{g})$ cannot leave the ordinary causal future of its data, and a monochromatic test mode is a spectral constituent of such a solution in the stationary regime. No property of $\arg T$ is used.

For (iii), first recall the evaluation under an assumed profile centred at a real frequency ω_0. Writing $T(\omega) = e^{\mathrm{i}\varphi_0}(\Gamma/2)/f(\omega)$ with $f(\omega) = (\omega_0 - \omega) - i\Gamma/2$, one has $\frac{\mathrm{d}}{\mathrm{d}\omega}\arg T = -\mathrm{Im}\,(\dot{f}/f)$. At $\omega = \omega_0$ one has $f = -i\Gamma/2$ and $\dot{f} = -1$, so $\dot{f}/f = -2i/\Gamma$ and $-\mathrm{Im}\,(\dot{f}/f) = 2/\Gamma$. This is the official version 1 identity at the profile centre.

For the exact even-slab amplitude, Theorem (iii) supplies a unique simple pole $\omega_{\mathrm{p}} = \omega_\ell + \Delta_R - i\Gamma_{n,\ell}/2$ in a shrinking neighbourhood of each simple real unit-transmission root, with $\Delta_R = 2B\mu_e^2 k^2/N'(\omega_\ell) + o(\Gamma_{n,\ell})$ and $\Gamma_{n,\ell}$ as in . The denominator $D(\omega)$ of therefore differs from a nonvanishing slowly varying holomorphic factor times $(\omega - \omega_{\mathrm{p}})$ by a remainder of strictly smaller exponential class. The numerator $2\mathrm{i}\mu_e k\, e^{-2\mathrm{i}ka_{\mathrm{cav}}}$ is holomorphic and nonvanishing on the real window. Hence $\tau_{\mathrm{W}}(\omega) = -\mathrm{Im}\,(D'(\omega)/D(\omega)) + O(1)$. At $\omega = \mathrm{Re}\,\omega_{\mathrm{p}}$ the leading term is $2/\Gamma_{n,\ell}$, which is .

At the real unit-transmission root, the real displacement Δ_R need not be $o(\Gamma_{n,\ell})$. The first-order ratio is

$\Delta_R/(\Gamma_{n,\ell}/2) = B\mu_e k/(-A)$ to leading order, and unimodularity $A^2 + B^2\mu_e^2 k^2 = 1$ then gives

$$\tau_{\mathrm{W}}(\omega_\ell) = \frac{\Gamma_{n,\ell}/2}{\Delta_R^2 + (\Gamma_{n,\ell}/2)^2} + O(1) = \frac{2}{\Gamma_{n,\ell}} A(\omega_\ell)^2 + O(1),$$

which is .

Proposition (Active transient metric signalling remains outside the insulation typestate). *If an ordinary source actively modulates a perturbation* $\mathrm{deltaT}_{\mathrm{o}}(t,x)$ *in the time domain so as to source dynamical Einstein perturbations* δg_{ab} *that traverse the core* $|x| < b_{\mathrm{cav}}$*, then fixed-host decoupling no longer applies, and the TM-1 typestate remains exact (undischarged). Official Chrono-DM-A's documentary Proposition 5.7 (*prop : leakage*), a formal first-order holonomy increment on the cover under a declared cover kernel, is neither upgraded to a completed dynamical Einstein theorem nor withdrawn as a documentary neighbour [2] Documentary source record; not a theorem premise.. Official Proposition 5.8 (*prop : images*) likewise stands as a documentary record that the naive periodized retarded sum is not locally finite. Electromagnetic silence, in the sense of official Definition 5.3 and Proposition 5.4, excludes only a declared direct channel and does not by itself yield TM-1 isolation on a dynamically coupled host.*

Proof. The insulation statement of Theorem (ii) uses a fixed ordinary metric $\tilde{g}$ and a test field. If the ordinary stress is allowed to source a dynamical metric perturbation that enters the core, the host is no longer a fixed background, and that hypothesis is withdrawn. No substitute well-posed dynamical Einstein theory is supplied on the chronology-violating quotient, so TM-1 is not discharged. The documentary comparisons are named only as such.

A declared chiral contract R − FP − lens

Theorem (Chirality of the spectrum, a sign-of-c_{core} test, and a declared rejection contract).

1. *Whenever* $a_0 \neq 0$*, the effective potential of is strictly chiral in the azimuthal number:*

$$V_{\mathrm{eff}}(x;\omega,-n) - V_{\mathrm{eff}}(x;\omega,+n) = -\frac{4n\omega a(x)}{F(x)}.$$

Thus $V_{\mathrm{eff}}(x;\omega,-n) \neq V_{\mathrm{eff}}(x;\omega,+n)$ *as soon as* $n\omega a$ *is not identically zero. In particular, on the printed core of either model, with* $a_0 \neq 0$ *and* $n \neq 0$*, the potentials at opposite azimuthal numbers differ wherever* $\omega \neq 0$*. The resonant spectra of Theorem are therefore chiral.*

2. *Unique inversion of the triple* $(a_0, F_0, b_{\mathrm{cav}})$*, and hence of* $c_{\mathrm{core}} = F_0 - a_0^2$*, from two adjacent resonance frequencies* $(\omega_{n,\ell}, \omega_{n,\ell+1})$ *together with a* $(+n,-n)$ *chiral splitting, from exterior asymptotic scattering data alone, is not typestated. The barrier parameters* F_{neck}*,* a_{cav}*,* F_{ext}*, the transverse momentum* k_z*, and the reflection phase* δ_{refl} *remain nuisance parameters of the matching condition unless they are declared calibrated. Even if the barrier parameters* $(F_{\mathrm{ext}}, F_{\mathrm{neck}}, a_{\mathrm{cav}}, k_z = 0)$ *are declared calibrated, and even if a named triple of co-rotating roots such as* $(\omega_{-m,\ell}, \omega_{-m,\ell+1}, \omega_{-2m,\ell'})$ *with* $m \geq 1$ *is supplied, no explicit non-vanishing* 3×3 *Jacobian of a complete local inverse is computed in this sheet. That calibration is therefore recorded as a programme, not as an inverse.*

3. *The declared contract* R − FP − lens *is the following rejection and reconstruction typestate, and not a uniqueness theorem. The even step-collar class in this clause includes the comparison class of Theorem (i), in which the core constants* (a_0, F_0) *are not restricted a priori to a CTC core.*

• *Data. A finite list of exterior resonant frequencies in the two-sided propagating window, including at least one pair of opposite azimuthal numbers* $\pm n$*, together with the associated transmission spectrum.*

• *Declared known parameters, if assumed. The transverse momentum* k_z *and the exterior coefficient* F_{ext} *may be declared known; the barrier data* F_{neck} *and* a_{cav} *may likewise be declared known, or left as nuisances.*

• *Rejection of* $a_0 \neq 0$*. If, at the stated tolerance, the measured* $(\pm n)$ *spectra coincide (no chiral splitting), the data reject* $a_0 \neq 0$ *in the even step-collar class. This remains a chirality test, not a uniqueness theorem.*

• *Falsification of* $c_{\mathrm{core}} \leq 0$*. At* $\omega > 0$*, observation of a trapped core resonance in a co-propagating channel* $n = +|n|$ *(the same sign as* ωa_0 *when* $a_0 > 0$*) falsifies* $c_{\mathrm{core}} \leq 0$ *in that class, by the necessity statement of Theorem (i).*

• *One-chirality is not a characterisation. Strict one-chirality (only the opposite-sign channel, in a band where both signs are observationally accessible) is compatible with a CTC or horizon core, but does not by itself characterise*

or prove $c_{\mathrm{core}} \le 0$*, because a chronology-preserving core may simply fail to open a same-sign window in the observed band.*

• *Reconstruction. Observation of a nonzero chiral splitting is compatible with* $a_0 \neq 0$ *in that class, but does not, on the evidence proved here, uniquely invert* $(a_0, F_0, b_{\mathrm{cav}})$*. Official Chrono-DM-A's documentary Theorem 10.10 (* thm : jointID*) is a different, guided-ring Jacobian contract [2] Documentary source record; not a theorem premise.. The present contract does not complete it, discharge it, or replace it.*

Proof. The difference is the elementary expansion

$$\frac{(-n+\omega a)^2-(n+\omega a)^2}{F} = -\frac{4n\omega a}{F},$$

the Liouville potential and the term $k_z^2 - \omega^2$ being independent of the sign of n. This is (i). Clause (ii) is the typestate of a missing inverse: the exact root condition depends on the triple $(a_0, F_0, b_{\mathrm{cav}})$ and also on $(F_{\mathrm{neck}}, a_{\mathrm{cav}}, F_{\mathrm{ext}}, k_z)$ through the barrier and exterior blocks of M_{tot}. Three named data therefore cannot be claimed to invert three target parameters in the presence of those nuisances, unless a Jacobian on a declared known-parameter set is produced. Even after the barrier parameters are declared calibrated, no such Jacobian is computed in this sheet, so the calibration remains a programme. Clause (iii) records the residual contract that does follow from (i) and from Theorem (i): a chiral rejection test of $a_0 \neq 0$; a one-sided falsification of $c_{\mathrm{core}} \le 0$ by a co-propagating trapped resonance at $\omega > 0$; and an explicit refusal to treat exclusive opposite-sign data as a characterisation of a CTC or horizon core. Uniqueness of an inverse is withheld.

Master typestate of official Dual-Ledger-Chrono-DM-D

Theorem (Master release of official Dual-Ledger-Chrono-DM-D version 2). *The typestate of this official sheet is the conjunction of the admitted identities and the explicit refusals already proved.*

1. *Admitted. Printed closed dark geodesics on the three declared hosts of Proposition ; the scalar-age obstruction and unitary rank–entropy invariance of Proposition ; conjugacy-class memory of a* U(N) *monodromy, with an* SU(N) *traceless subclass, in Theorem ; order-insensitive two-kick traces, and order-sensitive three-kick traces, in*

Theorem ; Levi–Civita spinor holonomy on the three hosts, including trivial spin holonomy on the flat cylindrical core, the already repaired holonomy-shifted connection, and the co-rotating Darboux connection on the cratered ring, with elliptic invariant frequency $1/6$ *and holonomy angle* $\pi\sqrt{5}/3$*, in Proposition ; the spin lift , and the discrete holonomy-shifted locus on which the vectorial holonomy is +I while the spin lift is* $-I_4$*, likewise in Proposition ; an adiabatic Floquet–Berry split with a remainder bound of class* $C(\Delta_m, \|H\|_{C^2})\, L^{-1}$*, and a Wilczek–Zee matrix on a printed isolated subbundle, in Theorem ; a damped resolvent projecting onto* $\ker(W - I)$*, together with an exact unit-eigenvalue orbit within* $O(L^{-1})$ *of each simple adiabatic root under a written nondegeneracy hypothesis, in Theorem ; the massless-scalar reduction and Liouville potential –; horizon-regular flux conservation and two-sided unitarity in the smooth model, in Theorem ; an* $\mathrm{SL}(2,\mathbb{R})$ *transfer matrix and exact unit-transmission roots for the printed even step-collar, the strict necessity of* $\omega n < 0$ *on a CTC or horizon core, and a leading residue width in the thick-barrier regime, in Theorem ; absence of a negative-c ordinary core band under* $H \geq H_{\min} > 0$*, and a Wilson-line multiplet split that is not typestated as* θ_a/L*, in Theorem ; a longitudinal-electric test Maxwell reduction, at* $k_z = 0$*, with wave operator typestated exactly as* Box_g *or* Box^{ν}_{g}*, in Theorem ; ordinary Cauchy protection and test-field insulation, with a Wigner delay* $2/\Gamma + O(1)$ *at the real part of a thick-barrier pole, in Theorem ; and chirality of the effective potential, a falsification of* $c_{\mathrm{core}} \leq 0$ *by a co-propagating trapped resonance at* $\omega > 0$*, and the declared rejection contract* $\mathrm{R} - \mathrm{FP} - \mathrm{lens}$*, in Theorem .*

2. *Refused or not typestated. Unique inversion of* $(a_0, F_0, b_{\mathrm{cav}})$ *from two adjacent frequencies and a chiral splitting, including under a declared barrier calibration for which no non-vanishing Jacobian is computed; the slogan that exclusive opposite-sign data characterise a CTC or horizon core; a Wigner identity* $\tau_{\mathrm{W}}(\omega_\ell) = 2/\Gamma + O(1)$ *at every real unit-transmission root; gauge-invariant path-order memory from two Pauli kicks alone; identification of* $\mathrm{U}(N)$ *with the traceless* $\mathrm{SU}(N)$ *subclass in the absence of* $\mathrm{Tr}\,\mathscr{H}_{\mathrm{d}} = 0$*; identification of a globally* C^∞ *potential with a piecewise-constant step-collar; unitarity outside the two-sided propagating window; electromagnetic or linearised-gravity typestate of the scalar modes outside the printed longitudinal-electric class; a global Regge–Wheeler reduction, or dynamical gravitational-wave manufacture, for axial test tensors; inverse-scattering*

reconstruction of the full collar profiles; a non-Abelian multi-channel Wilson resonator; and three-dimensional whispering-gallery quasimodes on $\mathbb{R}^4$*, as recorded in Appendix ; causal insulation for active transient ordinary stress coupled to dynamical Einstein gravity; promotion of official* prop : leakage *to a completed dynamical Einstein theorem; discharge of TM-1, Bar 1, Bar 2, or Bar 3; creation of Host C; closure of O.1 or H3; supply of Gate 0 or ST4b; opening of XV-B; and the writing of any further companion package.*

3. *The operational time-machine bar remains* 0 *of* 3 *as the full three-part bar. Holonomy memory is not scalar clock return. Horizon-regular one-dimensional Sturm–Liouville theory is not dynamical CTC manufacture. Passive stationary lensing is not an unconditional dynamical gravitational firewall. A test-field Maxwell or tensor reduction is not Bar 1, not Bar 2, and not Bar 3. External geometric time remains supplied. The source is not companion-wired:* wired = false, *and* declaredPath *remains* null.

Proof. Each admitted clause is the corresponding theorem, proposition, or lemma already proved, and each refusal is the typestate recorded therein. No additional identity is claimed.

Official Version 3 retains the master typestate of official Version 2 unmoved. The identities recorded in Theorem remain typestated only on the written non-self-scrambling, portal-silent class. They are not typestated as operational hardware, as a laboratory time machine, or as a chronology engine in nature.

appendix

Debugging table

The following table is a typestate guard, not a list of new theorems. It records conflations that are not admitted.

toprule **Conflation to avoid Official typestate in this sheet**

midrule endhead SU(*N*) or U(*N*) holonomy memory is a scalar clock return, or discharges Bar 1 or Bar 2. Conjugacy classes and eigenphases are gauge-invariant loop data (Theorem). They are not a single-valued elapsed-time field (Proposition). The time-machine bar remains 0 of 3.

midrule Two localised Pauli kicks typestate gauge-invariant path-order memory. The two-kick trace is

order-insensitive . A based axis is order-dependent but not conjugation-invariant. Three non-commuting kicks distinguish orders by their traces .

midrule Closed dark geodesics inhabit unshifted official-B $g_{\alpha,\beta}$ for $\alpha > 1$, or an Einstein-manufactured official-C torus. They inhabit the holonomy-shifted ring in the restricted class $0 < \alpha < 1$, $A_0 > 0$, and the printed cratered unit-lapse ring. Unshifted azimuthal circles are not geodesics (Remark). Official C v3 withdrew the v2 single-invariant pullback. Host C is not created.

midrule The massless scalar resonator is electromagnetic lensing, or a host-null gravitational-wave polarisation, in general. The operative operator is Box_g on a scalar, except for the longitudinal-electric test Maxwell reduction of Theorem , whose wave operator is typestated exactly as Box_g or $\mathrm{Box}_{\tilde{g}}$. That reduction is a test field on a prescribed host, not electromagnetic manufacture in general. Axial test tensors are typestated only as flat-piece fields, not as a global Regge–Wheeler reduction and not as dynamical gravitational waves.

midrule A globally C^∞ potential and a piecewise-constant step-collar are one exact model. They are two models, as printed in Section . The Liouville potential is not classical at jumps. Matching is typestated for ψ and $\sqrt{F}\,\psi'$.

midrule The scattering matrix is unitary for evanescent incidence or unmatched exteriors. $|R|^2 + |T|^2 = 1$ is typestated only in the two-sided propagating window with matched exteriors (Theorem).

midrule Horizon-regular one-dimensional Sturm–Liouville theory manufactures a dynamical CTC, or discharges Bar 3. Self-adjointness and flux conservation are statements about a printed one-dimensional operator on a prescribed host. Prescribed hosts do not discharge Bar 3.

midrule A Wigner delay is the ordinary insulation mechanism, or insulates active dynamical gravity. Ordinary insulation follows from global hyperbolicity of $(M, \tilde{g})$ and the test-field hypothesis (Theorem (ii)). A delay $2/\Gamma + O(1)$ is typestated at the real part of a thick-barrier pole, as a group delay, and not as an identity at every real unit-transmission root (Theorem (iii)). TM-1 remains undischarged for active transients (Proposition).

midrule Two adjacent frequencies plus a chiral splitting uniquely invert $(a_0, F_0, b_{\mathrm{cav}})$, or complete official

thm : jointID; or exclusive opposite-sign data characterise a CTC core. Chirality is proved; a co-propagating trapped resonance at $\omega > 0$ falsifies $c_{\text{core}} \le 0$; exclusive opposite-sign data do not by themselves prove $c_{\text{core}} \le 0$; unique inversion is not typestated, even under a declared barrier calibration, because no Jacobian is computed (Theorem). Official Theorem 10.10 is a different documentary contract and is not completed, discharged, or replaced.

midrule The damped resolvent projects onto the adiabatic phase-quantisation set, or is a chronology firewall. It projects onto $\ker(W - I)$ (Theorem). Under a written nondegeneracy hypothesis an exact unit-eigenvalue orbit lies within $O(L^{-1})$ of each simple adiabatic root; the two loci are not identified. The identity is not TM-1.

midrule The cratered-ring vectorial holonomy is the commuting product of a half-turn and a rapidity $2\pi/3$. Those quantities are the integrals of the Cartesian pullbacks. The co-rotating generator is elliptic, with invariant frequency 1/6 and holonomy angle $\pi\sqrt{5}/3 \notin 2\pi\mathbb{Z}$ (Proposition (iii)).

midrule A Borg–Marchenko reconstruction of the full collar, a non-Abelian $2N \times 2N$ Wilson resonator, or a three-dimensional whispering-gallery quasimode on $\mathbb{R}^4$ is typestated in this sheet. Those programmes remain explicit refusals (Appendix). No companion package is written.

midrule The sheet opens XV-B, closes O.1 or H3, or supplies Gate 0 or ST4b. O.1 and H3 stay NOT_CLOSED. Gate 0 and ST4b stay NOT_SUPPLIED. XV-B is not opened. wired = false; declaredPath = null.

bottomrule

Track II programmes, with explicit refusals

The following programmes were proposed as structural breakthroughs. They are recorded here as typestate refusals, not as theorems, and not as a second official companion package. Mid-argument "check" language in any commissioning note is treated as scratch, and is not typestate.

Breakthrough 2 — two-winding Gel'fand–Levitan–Marchenko profile reconstruction. The proposal is that the complex reflection and transmission coefficients $\{R_n(\omega), T_n(\omega)\}_{n \in \{0,\pm1\}}$, across $\omega \in \mathbb{R}$ at $k_z = 0$, uniquely

reconstruct both smooth profile functions $a(x)/F(x)$ and $1/F(x)$, and therefore reconstruct $c(x) = F(x) - a(x)^2$ pointwise, without prior barrier assumptions, by a one-dimensional Borg–Marchenko inverse. No such uniqueness theorem is proved in this sheet. Barrier parameters therefore remain nuisances of finite-frequency root matching, as already recorded in Theorem (ii), and the reconstruction is refused.

Breakthrough 3 — non-Abelian 2N × 2N multi-channel Wilson resonator. The proposal is to couple an E_{d} -valued dark multiplet to a spatially non-commuting SU(N) connection $(\mathscr{A}_x(x), \mathscr{A}_\varphi(x,\varphi))$ encoding the three-kick path-ordered holonomy of Theorem , and to derive a block-symplectic $2N \times 2N$ transfer matrix whose path-ordering difference splits exterior resonance multiplets. Theorem (ii) restricts the printed internal connection to a single core-constant matrix $\mathscr{A}_\varphi$, which commutes with itself and drops out of the barrier. No non-commuting barrier connection, and no $\mathrm{Sp}(2N,\mathbb{R})$ or $\mathrm{U}(N,N)$ transfer matrix, is constructed. The programme is refused.

Breakthrough 4 — three-dimensional toroidal whispering-gallery quasimodes on $\mathbb{R}^4$. The proposal is to reduce $\mathrm{Box}_g \Psi = 0$ on the compactly supported cratered metric to a self-adjoint three-dimensional magnetic-Helmholtz operator, and to prove existence of chiral whispering-gallery resonances trapped about the stable cratered ring $(r_*, z_*) = (2, 0)$, with asymptotically flat scattering phase shifts at Minkowskian infinity. Proposition (iii) constructs that ring as a prescribed geodesic, but the scattering analysis of Section is typestated only on the translation-invariant cylinder $M = \mathbb{R}_t \times \mathbb{R}_x \times S^1_\varphi \times \mathbb{R}_z$. No Agmon estimate, and no three-dimensional quasimode, is proved on the cratered $\mathbb{R}^4$ host. The programme is refused.

par *Non-claim.* Appendix records refusals. It does not typestate an inverse-scattering reconstruction, a non-Abelian resonator, or a three-dimensional quasimode, and it does not discharge Bar 1, Bar 2, or Bar 3. par

Documentary negative controls

Table records a one-page documentary ledger of OpenAI catalogue families and refused slogans. The entries are unverified external catalogue items, cited only as documentary source records [9], and are isolated to this section. They are not theorem premises, not scheduler-derived cones, not typestate of a Page curve, island, bounce, or de

Sitter state, and not a discharge of the operational time-machine bar, which remains 0 of 3 for ordinary observers with Enter instruments. No retained official identity in this sheet depends on these entries. Written dark-ledger identities, where retained, remain on the written dark ledger and the printed class; they are not chronology engines in nature and not uniqueness of dark matter.

Conjugacy-class memory of a U(N) monodromy, the three-kick order-sensitive trace, the adiabatic Floquet–Berry split with an $O(L^{-1})$ remainder, the damped resolvent projecting onto $\ker(W - I)$, the $\mathrm{SL}(2,\mathbb{R})$ transfer-matrix unit-transmission roots, the strict necessity of $\omega n < 0$ on a CTC or horizon core, chirality of the effective potential, and the declared rejection contract R − FP − lens remain typestated only on the written non-self-scrambling, portal-silent class. They are not typestated as operational hardware, as a laboratory time machine, or as a chronology engine in nature.

References

[1] Cheung, N. (2026). Dual-Ledger-Chrono-DM-D: Loop Holonomy Memory, a Spectral Filter, and a Prescribed Bimetric Chronology Resonator. Typestate-Honest Completions of Official Monodromy and Resonator Programmes, with Explicit Refusals. *Enter Calculus Companion*, official version 2 (request t512u). Typographic parent of the present official Version 3; not a premise. *Source record; not a premise.*

[2] Cheung, N. (2026). Dual-Ledger-Chrono-DM-A: Winding, Chronology Asymmetry, and Operational Insulation on Prescribed Lorentzian Hosts. Exact Geometric Witnesses, Corrected No-Paradox Results, and Complete-Record Rejection Contracts. *Enter Calculus Companion*, official version 4 (request t506u). *Source record; not a premise.*

[3] Cheung, N. (2026). Dual-Ledger-Chrono-DM-B: Class-Limited Einstein Matches, a Closed Causal-Geodesic Obstruction, On-Shell Ordinary Shields, a Prescribed Misner Overlay, and One-Sided Sagnac Identifiability on Prescribed Hosts. *Enter Calculus Companion*, official version 4 (request t509u). *Source record; not a premise.*

[4] Cheung, N. (2026). Dual-Ledger-Chrono-DM-C: Continuum Dark-Field Self-Consistency, Detector Seam

Rules, and a Prescribed Toroidal Chronology Split on $\mathbb{R}^4$. *Enter Calculus Companion*, official version 3 (request t508u). *Source record; not a premise.*

[5] Cheung, N. (2026). Dual-Ledger-Temporal-A: Sector-Dependent Clock Rates and Transport on a Prescribed Lorentzian Host. Conservation, No-Freezing Results, Controlled Cosmological Witnesses, and Observational Rejection Contracts. *Enter Calculus Companion*, official version 5 (request t504u). *Source record; not a premise.*

[6] M. V. Berry, "Quantal phase factors accompanying adiabatic changes," *Proceedings of the Royal Society of London A* **392**, 45–57 (1984). doi:10.1098/rspa.1984.0023.

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[9] Documentary unverified external catalogue entries: Oai365-A (Family 365), Oai290-A (Family 290), Oai265-A/B, Oai276-A, Oai212-A, and refused Family 260. Unverified documentary negative controls only; isolated to Section and Table ; never premises. Family 365 is boundary-metric recovery, not a scheduler-derived cone or a time machine. Family 290 is not a decoder of an unread complement and cannot decode $r_{\mathrm{ord}} = 1$. Oai265-A/B is not a Page curve; Oai276-A is not a free clock; Oai212-A is not worldline return. Family 260 remains refused.