Dual-Ledger-Chrono-DM-Backward-A

Backward sector attribution from prescribed chronology asymmetry

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Abstract

We investigate the counterfactual statement "only dark matter can time-travel" by replacing it with a precise, conditional geometric postulate: a prescribed host admits a closed future-directed timelike geodesic, while the universally coupled ordinary metric admits Cauchy slices. This official version 5 is a typographic successor of official version 4. Official version 4 typestate already parent-reviewed is retained in full, including every official label and local proof. Authorised new work is typestate hygiene: ordinary quotation marks, a Scholar-parseable bibliography without an invented DOI, and a documentary OpenAI-family negative-control table. Written dark-ledger identities remain identities on the written class; they are not discharges of the ordinary time-machine bar, not chronology engines in nature, and not uniqueness of dark matter. Clock Track-I identities are not imported as theorem-class. Official version 4 typographically continued official version 3. Official version 3 typestate already parent-reviewed is retained, including the complex Euler operator, the fixed-host threshold without spacetime signature degeneracy, the cone inequality and pinching hypothesis, the correlated ε^2 decoupling path, the Yukawa comparison direction, the clamped relative-growth identity, both ρ_q Jacobians, the repaired Laplacian identity $\Delta_{\mathbb{R}^3}\Phi = \lambda(x + \mathrm{i}y)$ with Schrödinger coefficient $\mu = \lambda/h$, the repaired collar identity for W', the compact-sharpness dichotomy, class-limited vortex uniqueness, conditional Standard-Model silence, the strictly retrograde capture band, the repaired Cartan coframe and connection, and the tautological character of vanishing

ADM mass for compact deviations. Official version 2's false identity $\Delta_{\mathbb{R}^3}\Phi = \lambda\Phi$ is not restored. No official Backward-B or Backward-C manuscript is created here, and no official Chrono-DM-D is cited. On a fixed cylindrical CTC host, decreasing the winding correction destroys ordinary global hyperbolicity at a closed-null-circle threshold, but the spacetime metric remains Lorentzian for every nonnegative correction: the degeneracy is that of the induced t-slice and of the temporal character of dt, not a spacetime signature degeneracy. Geometric TM-2 alone does not force off-shell electromagnetic silence, and ordinary Cauchy slices do not block indirect gravitational signalling through the host. Silence theorems are therefore retained only as conditional on a printed robust direct-insulation assumption. A qualitative claim that any controller coupling flips the azimuthal cone, and thereby forces complete controller silence on the loop-traversing sector, is false on a collar with $c < 0$; a proved cone inequality and a pinching hypothesis replace that slogan, while $A \equiv 1$ is identified as a frame-anchoring gauge rather than a cone theorem. The compact toroidal witness supplies an open tube of closed dark geodesics. Holding the vortex profile fixed and scaling the disformal coupling as ε^2 yields a path that is C^∞ in the decoupling parameter, including at $\varepsilon = 0$; the linear-in- ε disformal scaling does not cancel the quadratic shift term. On the unit-lapse toroidal host the Laplacian identity is repaired: for $\Phi = h(r^2, z^2)(x + \mathrm{i}y)$ one has $\Delta_{\mathbb{R}^3}\Phi = \lambda(x + \mathrm{i}y)$ identically, with λ the displayed differential polynomial in h, and therefore $\Delta_{\mathbb{R}^3}\Phi = \mu\Phi$ on $\{h \neq 0\}$ with $\mu = \lambda/h$. Official version 2 incorrectly identified $\Delta_{\mathbb{R}^3}\Phi = \lambda\Phi$. An on-shell two-switch potential is admitted when μ is a smooth function of (r^2, z^2) and a printed closedness condition on (μ, u) holds, or unconditionally on a z-independent radial collar on which μ is a smooth function of r^2. The closedness condition $\mathrm{d}\mu \wedge \mathrm{d}u = 0$ is the integrability condition for a switch potential for every smooth $V_\Phi(u, s_1, s_2)$, not an artefact of the linear representative $V_\Phi = \frac{1}{2}\mu|\Phi|^2$; it is not automatic. Positive Schwartz amplitudes $h = c_0 \mathrm{e}^{-\alpha r^2 - \beta z^2}$ inhabit the smooth- μ class, with globally affine $\mu = -8\alpha - 2\beta + 4\alpha^2 r^2 + 4\beta^2 z^2$, but do not refute compact sharpness, and two-switch closedness fails unless the profile reduces to the z-independent Gaussian collar $\beta = 0$. A non-Lipschitz dead-core potential is the blow-up branch of the sharpness dichotomy, not a bypass, and no compact on-shell example is exhibited. A single-invariant pullback to $\sigma = (r - r_0)^2 + z^2$ remains withdrawn. The z-independent radial collar is

not compact in *z*. Among smooth positive-semidefinite disformal shields with vanishing radial, vertical, and time-time distortion, the azimuthal shield is of vortex type, uniquely realised by $J[\Phi]$ in that class. This is not uniqueness among all smooth tensor shields on $\mathbb{R}^4$. Strengthened Standard-Model insulation postulates, if assumed, yield conditional silence of listed direct couplings; they are not forced by geometric TM-2, and logical underdetermination is not withdrawn. Lorenz-gauge Yang–Mills and tetrad-coupled Dirac operators written with kinetic metric $\hat{g}$ have characteristic variety $\{\hat{g}^{ab}\xi_a\xi_b = 0\}$; this places the principal- symbol half of optional (BP2') on standard hyperbolic footing, conditional on those operators being the ones assigned, and does not derive (BP2') or (BP3') from chronology. Ordinary Sagnac data remain chronology-fibre blind. An optional equatorial free-geodesic supplement records first integrals and a strictly retrograde capture band on the official collar; on that tube, a nonempty retrograde sub-band has pointwise $\dot{t} < 0$ with $\dot{\varphi} > 0$, but no global sign along an entire geodesic and no negative Shapiro jump is typestated, and a renormalised coordinate-time comparison remains uncomputed. The supplement does not replace the guided-ring theorems. On a *z*-independent collar, conservation of p_z and a checked three-dimensional radial equation yield an inclined retrograde capture condition; observer screen coordinates and a sky-plane capture strip remain a programme. One independently calibrated host-null return reconstructs the host chronology parameter and the combined winding correction; an independent winding kinetic-scalar datum, at fixed stiffness, supplies a four-variable local inverse of the factorisation, without discharging ordinary fibre-blindness. A global radial symmetron profile is constructed by shooting, with correct harmonic and Yukawa comparison directions, and a proved nonlinear two-sided bracket for $\varphi(R)/\varphi_\infty$ in the bare parameters; the linearised matching identity is an upper bound, not an equality. The clamped relative-growth identity is retained, and a quasistatic unclamped remainder is typestated under a two-step energy estimate that controls slow-forcing errors of order $(aH/k)^2$ only under stated smoothness and subhorizon hypotheses, and controls homogeneous free waves only under a stated initial-data or adiabatic turn-on hypothesis. Several Track II programmes remain recorded in an appendix. A Painlevé–Gullstrand coordinate identity for flat FLRW, and a kinematic single-manifold interpolation with a Minkowski buffer joining a stationary W-CTC core to an exterior

W-FLRW region, are typestated as kinematics: the construction does not discharge Obligation 1. The general curved-core Einstein tensor, a global NEC dichotomy, a cratered 3D Vlasov torus, and a small-gain theorem on a CTC background, remain refused. A finite-slab ordinary- metric contraction is recorded as a restricted claim only. None of these items is treated as a flagship theorem, and none closes the Section 7.3 obligations. These results do not establish that observed dark matter traverses CTCs: electromagnetic silence requires the additional insulation assumption, "uniqueness" means an aggregate action slot rather than a particle species, and the geometric and cosmological witnesses are not a single self-consistent astrophysical spacetime. Five coverage-qualified rejection contracts are declared. The time-machine bar remains 0 of 3; Host C is not created; no laboratory time machine, derived metric, spacetime emergence, or quantum completion is claimed.

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Backward postulate, readings, and standing ledger

What is assumed, and what is not inferred

The motivating sentence is a counterfactual assumption, not an observation. Its phrase "a time machine exists now" cannot serve simultaneously as an experimentally established premise and as an undischarged operational obligation. Accordingly, this paper retains only its geometric content.

We use signature $(-+++)$, units $c = 1$, and smooth, time-oriented prescribed hosts. Metric variation below defines Hilbert stress; it does not make the prescribed metric a dynamical output. All dimensional constants are understood to give the two terms in the effective metric the same tensor dimensions.

Assumption (Working backward postulate: geometric TM-2). For at least one declared instance there are two Lorentzian metrics $g, \tilde{g}$ on the same manifold M such that:

1. g has a closed future-directed timelike geodesic;

2. $(M, \tilde{g})$ is globally hyperbolic;

3. free dark test particles follow g-geodesics, and ordinary test particles and Maxwell fields couple universally to $\tilde{g}$. Existence of the closed orbit is prescribed geometric capability, not operational event-return certification.

A Cauchy horizon alone does not imply a closed timelike geodesic. The stronger closure condition is therefore stated explicitly rather than inferred from horizon existence.

Definition (Three non-equivalent readings).

1. Geometric chronology asymmetry. Assumption , adopted here.

2. Interaction exclusion. Ordinary and dark particles share a host, but a separately specified force or constraint prevents selected ordinary trajectories. This requires its own force law, controller stress, screening mechanism, and domain of exclusion.

3. Hypothetical operational return. A complete operational protocol returns a specified event, worldline, or consumed clock name. No such protocol is supplied here; this reading is not discharged.

Reading 2 is not ruled out in every possible theory, nor is Reading 1 the only conceivable calculable approach. Reading 1 is adopted because it permits exact causal comparisons without identifying a large non-geodesic force with ordinary free fall.

Proposition (Insufficiency of bounded single-metric forces). *Let* (M, g) *be a smooth time-oriented Lorentzian four-manifold containing a smooth closed future-directed g-timelike curve* $\gamma : [0, L] \to M$ *parametrised by g -proper time, so that* $g(\dot\gamma, \dot\gamma) = -1$.

1. *Any massless field whose principal symbol is* $g^{ab}\xi_a\xi_b$ *shares the exact g-causal cone, independently of lower-order non-derivative potentials or damping terms.*

2. *The covariant acceleration* $a^b_\gamma := \dot\gamma^c \nabla_c \dot\gamma^b$ *is smooth and g-orthogonal to* $\dot\gamma$. *Write* $A_{\max} := \max_{[0,L]} \sqrt{g(a_\gamma, a_\gamma)} < \infty$. *If an ordinary particle of mass* $m > 0$ *obeys* $m\, \dot x^c \nabla_c \dot x^b = F^b_{\text{conf}} + F^b_{\text{thrust}}$, *and if the controller force remains bounded by* $\sqrt{g(F_{\text{conf}}, F_{\text{conf}})} \le F_{\max} < \infty$ *along* γ, *then a bounded thrust of magnitude at most* $mA_{\max} + F_{\max}$ *holds the particle on* γ. *Unconditional exclusion of all ordinary causal curves therefore requires narrowing the causal cone itself, or else a singular infinite barrier.*

Proof. For a second-order operator $g^{ab}\nabla_a\nabla_b u + b^a\nabla_a u + cu = 0$, the characteristic cone is that of the principal symbol $g^{ab}\xi_a\xi_b$. Differentiating $g(\dot\gamma, \dot\gamma) = -1$ gives $g(a_\gamma, \dot\gamma) = 0$, so a_γ is spacelike (or zero) and $g(a_\gamma, a_\gamma)$ is continuous on the compact interval $[0, L]$. The choice $F^b_{\text{thrust}} := ma^b_\gamma - F^b_{\text{conf}}(\gamma, \dot\gamma)$ satisfies the equation of motion identically along $x = \gamma$. The triangle inequality on the positive-definite complement $(\dot\gamma)^\perp$ bounds its norm by $mA_{\max} + F_{\max}$.

Thus Reading 2 can be arranged only by changing the ordinary cone or by an unbounded force. It is not ruled out in every conceivable theory, but it is not the calculable class adopted here. Proposition below remains in force: even after a cone split is prescribed, geometric TM-2 alone does not select electromagnetic silence.

Disformal geometry has an established literature [8]; chronology violation and chronology-protection questions are likewise established subjects [9, 10, 11]. This paper claims neither the invention of those ingredients nor a

general chronology-protection theorem.

Two ledgers

The *geometric ledger* records the prescribed metrics, causal classes, worldlines, and observation maps. The *action ledger* records matter and controller actions, their Euler derivatives, Hilbert stresses, and exchange terms. A geometric witness is not automatically an action solution; an action solution on a prescribed host is not automatically an Einstein source match.

Standing ledger par The time-machine bar is 0 of 3: Bar 1, complete operational register/state restoration; Bar 2, consumed-clock-name return without an external archive; and Bar 3, operational spacetime worldline/event return. None is certified here. Mathematical geodesic closure is recorded only in the geometric ledger.

Host A and Host B remain separate prescribed-background labels. Host C is not created. O.1 remains NOT_CLOSED. H3 remains NOT_CLOSED. XV-B is not opened. XVI-N and Route B are not discharged, except the class-limited negative of t441 XVI-N-Univ-A. Gate 0 and ST4b remain NOT_SUPPLIED. No inherited quantum coefficient is admitted. No Einstein dynamics is derived, no metric emerges from records, and no external clock is removed. No Page curve, island, bounce, consciousness identification, or vacuum-energy prediction is claimed. The documentary typestate is wired=false, declaredPath=null.

Documentary provenance

Cheung's published Enter Calculus article concerns the operational characterisation of destructive measurement in a specified quantum language [1]. It supplies background motivation for careful interface typing, not a premise for the spacetime results below. Official Dual-Ledger companions on chronology splitting, temporal comparison, and mobility are credited only as documentary source records, never as premises [5, 6, 7, 4, 3, 2]. The current typestate pins are official Dual-Ledger-Chrono-DM-A version 4 (request t506u), official Dual-Ledger-Chrono-DM-B version 4 (request t509u), official Dual-Ledger-Chrono-DM-C version 3 (request t508u), official Dual-Ledger-Temporal-A version 5 (request t504u), and official Dual-Ledger-Mobility-A version 5 (request t495u). Earlier briefs' outdated official version labels are not used as the current typestate. No official

Dual-Ledger-Chrono-DM-D is cited. This official version 4 typographically continues official Dual-Ledger-Chrono-DM-Backward-A version 3. Two working supplements, a Backward-B version 1 draft and a truncated Backward-C version 1 draft, already used as source records in official version 3, remain documentary only; they are not official companions, and no official Backward-B or Backward-C manuscript is created here. The OpenAI mathematics catalogue, if mentioned at all, is a supplied documentary record dated 7 October 2026, never a Cheung paper, and is not used as a premise. Every operative mathematical statement used in this paper is proved from definitions printed here. No theorem number from a companion is used as an imported lemma.

Unified effective metric and exact action bookkeeping

The conformal–vortex class

Write $\Phi = X + \mathrm{i}Y$, with X, Y real scalar fields, and define

$$J_a = \mathrm{Im}\,(\Phi^* \partial_a \Phi) = X\partial_a Y - Y\partial_a X.$$

This current is a covector independent of metric variation. It need not be closed.

Let $A(\varphi, s) > 0$ and $B(s) \geq 0$ be smooth. Set

$$\tilde{g}_{ab} = A(\varphi, s)^2 g_{ab} + B(s) J_a J_b.$$

The controller is neutral: J in is not an electromagnetic current. Equivalently, $J_a = \mathrm{Im}\,(\Phi^* \partial_a \Phi)$. One may replace the potential $U(X^2 + Y^2)$ by a switch-coupled density $V_\Phi(|\Phi|^2, s)$ without changing the current J.

Lemma (Admissibility, inverse, and cone nesting). *Put*

$$b_J = \frac{B}{A^2}, \qquad \sigma = g^{ab} J_a J_b, \qquad D = 1 + b_J \sigma.$$

If $D > 0$, then $\tilde{g}$ is Lorentzian, and

$$\tilde{g}^{ab} = A^{-2}\left(g^{ab} - \frac{b_J}{D} J^a J^b \right),$$

$$\frac{\sqrt{-\tilde{g}}}{\sqrt{-g}} = A^4\sqrt{D}.$$

Every $\tilde{g}$-causal tangent is g-causal.

Proof. Multiplication verifies . The rank-one determinant identity gives $\det\tilde{g} = A^8 \det g\, D$, proving . Along the interpolation $A^2(g + \lambda b_J J \otimes J)$, $0 \le \lambda \le 1$, the determinant factor is $1 + \lambda b_J \sigma > 0$. No eigenvalue crosses zero, so the signature remains Lorentzian. Finally,

$$\tilde{g}(v,v) \le 0 \quad \Rightarrow \quad A^2 g(v,v) \le -B[J(v)]^2 \le 0.$$

The two effective metrics alone do not establish hyperbolicity or well-posedness of the full coupled field equations. In particular, the controller equations can have a principal symbol different from that of a free canonical scalar once ordinary matter is included.

Printed action

The principal class is

$$S_{\mathrm{mat}} = S_{\mathrm{o}}[\tilde{g}, \psi_{\mathrm{o}}, \mathscr{A}] + S_{\mathrm{d}}[g, \psi_{\mathrm{d}}] + S_{\varphi} + S_{\Phi} + S_s,$$

$$S_{\varphi} = -\int \sqrt{-g}\left(\frac{1}{2} g^{ab}\partial_a\varphi\partial_b\varphi + V(\varphi)\right)\mathrm{d}^4x,$$

$$S_{\Phi} = -\int \sqrt{-g}\left(\frac{1}{2} g^{ab}\partial_a X\partial_b X + \frac{1}{2} g^{ab}\partial_a Y\partial_b Y + U(X^2+Y^2)\right)\mathrm{d}^4x,$$

$$S_s = -\int \sqrt{-g}\left(\frac{f_s^2}{2} g^{ab}\partial_a s\partial_b s + W(s)\right)\mathrm{d}^4x, \qquad f_s > 0.$$

The dark action has no direct dependence on $\mathscr{A}, \varphi, X, Y, s$. For massive test transport one may take an ensemble of particle actions

$$S_{\mathrm{d}}^{\mathrm{test}} = -\int \mathrm{d}\mu(\ell)\, m_\ell \int_{\gamma_\ell} \sqrt{-g(\dot\gamma_\ell, \dot\gamma_\ell)}\, \mathrm{d}\lambda.$$

Variations of a closed particle orbit are periodic variations. Its stationarity is the usual geodesic equation. The dust witnesses below are stated through the resulting transport equations; no global Clebsch potential on a CTC is

assumed.

The ordinary Maxwell contribution is

$$S_{\mathrm{EM}} = -\frac{1}{4}\int \sqrt{-\hat{g}}\, \hat{g}^{ac}\hat{g}^{bd} F_{ab}F_{cd}\, \mathrm{d}^4x, \qquad F = \mathrm{d}\mathscr{A}.$$

Massive ordinary particles use $-m\int \mathrm{d}\hat{\tau}$. Ordinary hydrodynamic extensions must specify their own equation of state and action or conservation system.

Circle-valued alternative. On a quotient with an angular coordinate, the vortex slot may instead be replaced by a circle-valued field Θ, with

$$q = \mathrm{d}\Theta, \qquad S_q = -\frac{f_q^2}{2}\int \sqrt{-g}\, g^{ab} q_a q_b\, \mathrm{d}^4x, \qquad f_q > 0,$$

and $\hat{g} = A^2 g + Bq \otimes q$. Here Θ has a specified target-circle circumference. Its differential is globally defined even when no global real lift exists. This is a declared alternative controller class, not a claim that a vortex profile solves the circle action.

Ordinary Euler sources

Fix the conventions

$$\delta S_i = \int \sqrt{-g}\left(\frac{1}{2}T_i^{ab}\delta g_{ab} + \sum_\chi E_{i,\chi}\delta\chi\right)\mathrm{d}^4x$$

for compactly supported variations, and define

$$\delta S_{\mathrm{o}} = \frac{1}{2}\int \sqrt{-\hat{g}}\, \hat{T}^{ab}\, \delta\hat{g}_{ab}\, \mathrm{d}^4x$$

after imposing the ordinary matter equations. Let $\mathscr{R} = \sqrt{-\hat{g}}/\sqrt{-g}$ and

$$K^a = \mathscr{R}B\hat{T}^{ab}J_b, \qquad T_{\mathrm{o}} = g_{ab}T_{\mathrm{o}}^{ab}.$$

Proposition (Exact ordinary stress and controller sources). *For ,*

$$T_{\mathrm{o}}^{ab} = \mathscr{R} A^2 \tilde{\tilde{T}}^{ab},$$

$$E_{\mathrm{o},\varphi} = T_{\mathrm{o}}\, \partial_{\varphi} \log A,$$

$$E_{\mathrm{o},s} = T_{\mathrm{o}}\, \partial_{s} \log A + \frac{\mathscr{R}}{2} B'(s) \tilde{\tilde{T}}^{ab} J_a J_b,$$

$$E_{\mathrm{o},X} = 2K^a \partial_a Y + Y \nabla_a K^a,$$

$$E_{\mathrm{o},Y} = -2K^a \partial_a X - X \nabla_a K^a.$$

Proof. Metric variation gives $\delta_g \tilde{\tilde{g}}_{ab} = A^2 \delta g_{ab}$, proving . Variation of A^2 gives $\mathscr{R} A^2 \tilde{\tilde{T}}^{ab} g_{ab}\, \delta \log A = T_{\mathrm{o}}\, \delta \log A$. Variation of B gives the second term in . Finally,

$$\delta J_a = \delta X\, \partial_a Y + X \partial_a \delta Y - \delta Y\, \partial_a X - Y \partial_a \delta X.$$

The J-variation of the ordinary action is $\int \sqrt{-g} K^a \delta J_a$. Integration by parts in the two differentiated variations gives .

Five-slot switching identity

Theorem (Exact exchange and switch-shell closure). *Assume all actions are diffeomorphism covariant, the fields are smooth, and compact variations have no boundary contribution. On the ordinary and dark matter equations,*

$$\nabla_a T_{\mathrm{o}}^{ab} = E_{\mathrm{o},\varphi} \nabla^b \varphi + E_{\mathrm{o},X} \nabla^b X + E_{\mathrm{o},Y} \nabla^b Y + E_{\mathrm{o},s} \nabla^b s,$$

$$\nabla_a T_{\mathrm{d}}^{ab} = 0.$$

On the full controller equations,

$$\nabla_a \left(T_{\mathrm{o}}^{ab} + T_{\mathrm{d}}^{ab} + T_{\varphi}^{ab} + T_{\Phi}^{ab} + T_{s}^{ab} \right) = 0.$$

More specifically, on the φ, X, Y equations,

$$\nabla_a T_{\mathrm{bulk}}^{ab} = E_{\mathrm{o},s} \nabla^b s, \qquad T_{\mathrm{bulk}} = T_{\mathrm{o}} + T_{\mathrm{d}} + T_{\varphi} + T_{\Phi},$$

$$\nabla_a T_s^{ab} = (f_s^2 \mathrm{Box}_g s - W'(s)) \nabla^b s,$$

$$E_{\mathrm{o},s} + f_s^2 \mathrm{Box}_g s - W'(s) = 0 \quad \text{on the switch shell.}$$

Proof. Under an infinitesimal diffeomorphism generated by a compact vector field ξ, scalar fields vary by $\delta\chi = \xi^a \nabla_a \chi$, while $\delta g_{ab} = 2\nabla_{(a}\xi_{b)}$. After imposing each matter equation, gives

$$0 = \int \sqrt{-g}\Big(-\nabla_a T_i^{ab} + \sum_\chi E_{i,\chi} \nabla^b \chi\Big)\xi_b \, \mathrm{d}^4 x.$$

Arbitrariness of ξ proves the corresponding Ward identity. The dark action has no controller source. For the canonical switch,

$$T_s^{ab} = f_s^2 \nabla^a s \nabla^b s - g^{ab}\Big(\frac{f_s^2}{2}(\nabla s)^2 + W(s)\Big),$$

whose divergence is the second displayed switch equation. The full controller equations cancel ordinary and controller Euler sources slot by slot, proving and .

Corollary (Conformal exchange). *If $J = 0$, $s = s_0$, and $\partial_s A(\varphi, s_0) = 0$, then, with $\alpha_\varphi = \partial_\varphi \log A$,*

$$\nabla_a T_{\mathrm{o}}^{ab} = \alpha_{\mathrm{phiT_o}} \nabla^b \varphi = -\nabla_a T_\varphi^{ab}, \qquad \nabla_a T_{\mathrm{d}}^{ab} = 0.$$

Proof. Use Proposition and Theorem ; the X, Y, s sources vanish.

Proposition (Equivalence of real and complex vortex Euler operators). *Write $\Phi = X + \mathrm{i}Y$ and adopt the potential $V_\Phi(|\Phi|^2, s)$ in place of $U(X^2 + Y^2)$. Let*

$$E_{\Phi^*} := \frac{1}{2}\mathrm{Box}_g \Phi - \frac{\partial V_\Phi}{\partial |\Phi|^2}\Phi - \mathrm{i}\, K^a \nabla_a \Phi - \frac{\mathrm{i}}{2}(\nabla_a K^a)\Phi,$$

with the same current $K^a = \mathscr{R}B\hat{T}{}^{ab} J_b$ as in Proposition . Then, as bulk distributions,

$$2E_{\Phi^*} = E_{\mathrm{bulk},X} + \mathrm{i}\, E_{\mathrm{bulk},Y},$$

where $E_{\mathrm{bulk},X}$ and $E_{\mathrm{bulk},Y}$ are the sums of the free canonical Euler operators of S_Φ and the ordinary sources $E_{\mathrm{o},X}$, $E_{\mathrm{o},Y}$. In ordinary vacuum $\hat{T}{}^{ab} = 0$, this reduces to $E_{\Phi^*} = \frac{1}{2}\mathrm{Box}_g \Phi - (\partial V_\Phi/\partial|\Phi|^2)\Phi$. In this expanded form the first current coefficient is i *and the divergence coefficient is* i/2*. The compact packaging $-\frac{\mathrm{i}}{2}K^a\nabla_a\Phi - \frac{\mathrm{i}}{2}\nabla_a(K^a\Phi)$ is*

equivalent, because the second term contributes a further $-\frac{\mathrm{i}}{2}K^a\nabla_a\Phi$*. It is not equivalent to placing the coefficient* i/2 *on the expanded gradient term alone.*

Proof. The kinetic identity $\frac{1}{2}(|\nabla X|^2 + |\nabla Y|^2) = \frac{1}{2}\nabla_a\Phi^*\nabla^a\Phi$ and the chain rule for V_Φ give the free complex Euler operator $\frac{1}{2}\mathrm{Box}_g\,\Phi - (\partial V_\Phi/\partial|\Phi|^2)\Phi$ as the combination $\frac{1}{2}(E_X^{\mathrm{free}} + \mathrm{i}E_Y^{\mathrm{free}})$. From ,

$$E_{\mathrm{o},X} + \mathrm{i}E_{\mathrm{o},Y} = 2K^a(\partial_a Y - \mathrm{i}\partial_a X) + (Y - \mathrm{i}X)\nabla_a K^a.$$

Since $\partial_a Y - \mathrm{i}\partial_a X = -\mathrm{i}\partial_a\Phi$ and $Y - \mathrm{i}X = -\mathrm{i}\Phi$, the right-hand side equals $-2\mathrm{i}K^a\nabla_a\Phi - \mathrm{i}(\nabla_a K^a)\Phi$. Dividing by two produces $-\mathrm{i}\,K^a\nabla_a\Phi - \frac{\mathrm{i}}{2}(\nabla_a K^a)\Phi$, which is the displayed current. The operators therefore vanish together.

Remark (External switches and missing momentum). If s is imposed externally rather than solved, the full stress has divergence $E_s^{\mathrm{full}}\nabla^b s$, not automatically zero. Omitting T_s on a nonconstant switch shell similarly leaves the printed exchange remainder. Smooth switch-shell bookkeeping does not establish distributional junction conditions for an unmodelled discontinuous switch.

Backward attribution: insulation and slot uniqueness

The postulate alone does not select electromagnetic silence

Proposition (Logical underdetermination). *Assumption alone does not imply:*

1. *vanishing off-shell electromagnetic Euler current;*

2. *a specific controller action or winding topology;*

3. *a unique particle species;*

4. *positive Einstein-residual dark density.*

Proof. The assumption specifies two metrics and test transport, but no functional dependence on electromagnetic potentials. For example, a charged host test particle has action

$$-m\int \mathrm{d}\tau_g + e\int \mathscr{A}_a \mathrm{d}x^a.$$

On $\mathscr{A} = 0$ its free closed host geodesic is unchanged, despite nonzero electromagnetic variation. Thus geometric closure on the stated background does not imply off-shell silence.

The same metric can be prescribed without any controller action; hence no kinetic term or topological mechanism is selected. Replacing one host-geodesic dust component by two components with the same velocity and densities summing to the original density leaves the specified geometry and total dust stress unchanged. Finally, test particles can be placed on a flat host with $G_{ab} = 0$, so geometric closure does not impose a positive Einstein residual.

This distinction is essential. Electromagnetic non-detection does not prove exact functional silence, and geometric chronology asymmetry does not by itself prove electromagnetic insulation.

Assumption (Robust direct insulation). For the declared dark action, every compact variation of $\mathscr{A}, \varphi, X, Y, s$, at fixed g and fixed dark fields, leaves the action unchanged within each connected allowed field domain. Allowed electromagnetic potentials form a convex set. The action is continuously differentiable along allowed segments. Boundary and topological-sector changes are excluded.

Theorem (Off-shell silence and its converse). *Under Assumption ,*

$$\frac{\delta S_{\mathrm{d}}}{\delta \mathscr{A}_a} = \frac{\delta S_{\mathrm{d}}}{\delta \varphi} = \frac{\delta S_{\mathrm{d}}}{\delta X} = \frac{\delta S_{\mathrm{d}}}{\delta Y} = \frac{\delta S_{\mathrm{d}}}{\delta s} = 0$$

as bulk distributions, off shell. Conversely, vanishing of these Euler derivatives along every allowed segment implies the corresponding compact-variation insulation.

For a first-derivative electromagnetic Lagrangian $L(g, \psi, \nabla\psi, \mathscr{A}, F)$, the current convention is

$$j^{\nu}_{\mathrm{d}} = \frac{\partial L}{\partial \mathscr{A}_\nu} - 2\nabla_\mu\left(\frac{\partial L}{\partial F_{\mu\nu}}\right).$$

Proof. Differentiate the action invariance in each compact direction. The fundamental lemma for distributions gives the zero Euler derivatives. Conversely, for $\mathscr{A}_\lambda = \mathscr{A}_0 + \lambda(\mathscr{A}_1 - \mathscr{A}_0)$,

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} S_{\mathrm{d}}[\mathscr{A}_\lambda] = \int \sqrt{-g}\, j^{\nu}_{\mathrm{d}}[\mathscr{A}_\lambda](\mathscr{A}_1 - \mathscr{A}_0)_\nu \, \mathrm{d}^4x = 0.$$

Integrate from 0 to 1; the scalar cases are identical. For , use $\delta F_{\mu\nu} = 2\nabla_{[\mu}\delta\mathcal{A}_{\nu]}$ and integrate by parts.

This locally reproves the functional distinction discussed in Cheung's Big06-A [2]. It excludes direct bulk vertices in the supplied action, not gravitationally mediated interactions or radiative corrections in an unconstructed quantum theory.

Conditional elimination and aggregate uniqueness

Theorem (What can be eliminated). *Assume geometric TM-2, robust direct insulation, and universal ordinary coupling to one globally hyperbolic effective metric. Then:*

1. *g cannot itself be that universally coupled ordinary metric in the chronology-violating instance;*

2. *a carrier assigned to the robustly insulated dark slot has zero direct electromagnetic and controller Euler derivatives. Neither statement proves the existence of a closed photon geodesic on g, nor the absence of all indirect cross-sector signals.*

Proof. A globally hyperbolic spacetime has no closed causal curve. The prescribed closed timelike geodesic therefore prevents *g* from satisfying the ordinary-metric hypothesis. The second claim is Theorem . The proof contains no statement about closed null geodesics or indirect interaction channels.

Definition (Chronological dark slot). Fix a supplied finite additive inventory of matter actions. Exclude the gravitational action and the named controller actions. The *chronological dark slot* is the aggregate of all inventory components which:

1. satisfy robust direct insulation;

2. have free test transport on *g*;

3. have admissible nonnegative matter stress in the declared matter class. In the dust subclass, $T_{\mathrm{d}}^{ab} = \sum_i \rho_i u_i^a u_i^b$, with $\rho_i \geq 0$. "Chronological dark matter" names this stipulated aggregate, not an empirical identification with all observed dark matter.

Theorem (Inventory-relative maximality and species non-selection). *Inside the fixed additive inventory, the aggregate in Definition is uniquely maximal by inclusion. For a common dust velocity u, every split*

$$\rho_{\mathrm{d}} = \rho_1 + \rho_2, \qquad \rho_1, \rho_2 \geq 0$$

has the same total stress $\rho_{\mathrm{d}} u \otimes u$. *Consequently, observables depending only on the two metrics and total stress cannot select the number, masses, collision kernels, or microscopic identities of these subcomponents.*

Proof. The qualifying components form a definite subset of the supplied inventory. Its union contains every qualifying subaggregate, which proves maximality and uniqueness relative to that inventory. The dust identity follows by addition. Any observation map that forgets the decomposition is constant on these splits. Microscopic labels and collision laws are absent from that map.

A dust transport characteristic follows its timelike flow, not the entire metric null cone. Accordingly, "uses the host cone" means that its free transport and causal admissibility are referenced to *g*; it does not assign a null characteristic cone to pressureless dust.

Controller cone threshold, not qualitative silence

A qualitative claim that any nonzero disformal coupling to $J[\Phi]$ flips the azimuthal cone to $\hat{g}_{\varphi\varphi} > 0$, and therefore forces complete controller silence on every loop-traversing sector, is false on a collar with $c < 0$. The correct statement is a pointwise inequality, together with a further pinching hypothesis if one wishes to force a vanishing coupling.

Theorem (Azimuthal cone threshold and pinching hypothesis). *Let a sector I couple through*

$$g_{I,ab} = A_I(\varphi)^2 g_{ab} + B_I J_a J_b,$$

with $A_I > 0$ *and* B_I *a real constant, on a stationary collar*

$$g = -\mathrm{d}t^2 - 2a(r,z)\,\mathrm{d}t\,\mathrm{d}\varphi + c(r,z)\,\mathrm{d}\varphi^2 + \mathrm{d}r^2 + \mathrm{d}z^2,$$

where $a > 0$ *and* $c < 0$ *at a point p. Write* $J_\varphi := J(\partial_\varphi)$.

1. *The closed azimuthal circle through p, generated by* ∂_φ*, is* g_I*-timelike if and only if*

$$A_I(\varphi)^2 c + B_I[J_\varphi]^2 < 0,$$

equivalently, if and only if $B_I[J_\varphi]^2 < A_I(\varphi)^2|c|$*. Only the contraction* $J(\partial_\varphi)$ *enters this inequality; a possible component* $J_t = J(\partial_t)$ *does not appear. If, in addition,* $g_I(\partial_\varphi, \partial_t) < 0$ *at p and* ∂_t *is future-directed, the same circle is future-directed for the time orientation of* ∂_t*.*

2. *A conformal factor* $A_I > 0$ *does not change the sign of c, nor the sign of* $g_I(\partial_\varphi, \partial_\varphi)$ *relative to the displayed combination* $A_I^2 c + B_I[J_\varphi]^2$*. Setting* $A_I \equiv 1$ *is a frame-anchoring gauge choice of Contract (an Einstein or gravitational-wave frame), not a consequence of cone geometry alone.*

3. *The inequality of item 1 may hold at a fixed interior point with* $0 < B_I[J_\varphi]^2 < A_I^2|c(p)|$*. Such a coupling is not excluded by cone geometry at that point. If, however, the same sector is required to remain loop-traversing at a sequence of points of the chronology region with* $c \text{uparrow} 0^-$*, then* $B_I[J_\varphi]^2 = 0$ *at every accumulation point of that sequence on* $\{c = 0\}$*. If, further,* $|J_\varphi|$ *is bounded below by a positive constant along that sequence, then* $B_I = 0$*. Without that pinching hypothesis, or if* J_φ *vanishes as the horizon is approached, a nonzero coupling need not be excluded. This theorem does not withdraw Proposition . It is not TM-1, not Bar 3, and not Host C.*

Proof. By definition, $g_I(\partial_\varphi, \partial_\varphi) = A_I^2 g(\partial_\varphi, \partial_\varphi) + B_I[J(\partial_\varphi)]^2 = A_I^2 c + B_I[J_\varphi]^2$. The azimuthal circle is timelike precisely when this scalar is negative. Since $c < 0$, that condition is $B_I[J_\varphi]^2 < A_I^2|c|$. The one-form identity $J(\partial_\varphi) = J_\varphi$ does not involve J_t. Next, $g_I(\partial_\varphi, \partial_t) = A_I^2 g(\partial_\varphi, \partial_t) + B_I J_\varphi J_t = -A_I^2 a + B_I J_\varphi J_t$. In the printed vortex class of Theorem , J is purely spatial, so $J_t = 0$ and $g_I(\partial_\varphi, \partial_t) = -A_I^2 a < 0$. More generally the future-directed claim is conditioned on $g_I(\partial_\varphi, \partial_t) < 0$. Item 2 is immediate: $A_I^2 > 0$. For item 3, the first sentence is the inequality of item 1 at a single point with $|c|$ bounded away from zero. Along a sequence p_n with $c(p_n) \to 0^-$ at which the inequality holds, $B_I[J_\varphi(p_n)]^2 < A_I(p_n)^2|c(p_n)| \to 0$. Continuity of A_I and J_φ yields $B_I[J_\varphi]^2 = 0$ at any accumulation point on $\{c = 0\}$. A uniform positive lower bound on $|J_\varphi|$ then forces $B_I = 0$.

Strengthened insulation is not a chronology deduction

Theorem already records that a sub-threshold coupling $0 < B_I[J_\varphi]^2 < A_I^2|c(p)|$ is not excluded by cone geometry at a single interior point, and that $B_I = 0$ requires a pinching hypothesis. That typestate is not strengthened here. In particular, a claim that $B_{\mathrm{dm}} = 0$ holds identically for every loop-traversing sector, without the horizon-approaching hypothesis of Theorem (3), is refused.

A further claim that geometric TM-2 alone forces the loop-traversing sector to be electrically, chromatically, and weakly neutral, and to be decoupled from every renormalisable Higgs or lepton portal, is likewise refused. Proposition remains in force. The following optional postulates, if assumed, are additional direct-insulation hypotheses, not consequences of Assumption .

Assumption (Optional strengthened Standard-Model insulation). The following are additional postulates, not derived from geometric TM-2.

\textup{(BP2')} *Ordinary Cauchy well-posedness for Standard-Model kinetic terms.* Every Standard-Model gauge field and spinor included in a declared inventory is assigned a second-order (respectively, first-order) kinetic term whose principal symbol is built from the ordinary metric $\tilde{g}$, and the constant-t slices of $\tilde{g}$ are required to be Cauchy surfaces for those principal symbols.

\textup{(BP3')} *Off-shell insulation of the loop-traversing sector*. For every compactly supported variation of a declared Standard-Model potential, connection, or portal scalar, supported in a chronology-violating region, the dark or loop-traversing action is stationary. Lemma records the characteristic variety of Lorenz-gauge Yang–Mills and of a tetrad-coupled Dirac operator when those operators are written with kinetic metric $\tilde{g}$. No derivation of Higgs, Yukawa, or neutrino principal symbols is supplied, and high-frequency characteristics following $\tilde{g}$ rather than g are not a deduction of QCD, weak, Higgs, or neutrino silence.

Lemma (Principal symbols of assigned Yang–Mills and Dirac operators). *Let $\tilde{g}$ be a smooth Lorentzian metric, and let $\tilde{e}_\mu^{\ a}$ be a $\tilde{g}$-orthonormal frame, so that $\tilde{g}^{ab} = \eta^{\mu\nu}\tilde{e}_\mu^{\ a}\tilde{e}_\nu^{\ b}$.*

1. *Let A be a connection on a trivial principal bundle with compact structure group, written in a local gauge as a Lie-algebra-valued covector, and let* $F = \mathrm{d}A + A \wedge A$. *The Lorenz-gauge condition* $\widetilde{\nabla}_a A^a = 0$ *together with the Yang–Mills equation* $\widetilde{\nabla}_a F^{ab} + [A_a, F^{ab}] = 0$ *has principal part* $\widetilde{g}^{ac}\widetilde{\nabla}_a\widetilde{\nabla}_c A^b$ *on the Lorenz slice. The principal symbol of this gauge-fixed operator is* $-\widetilde{g}^{ac}\xi_a\xi_c$ *times the identity on the fibre, and the characteristic variety is* $\{\widetilde{g}^{ab}\xi_a\xi_b = 0\}$.

2. *The first-order Dirac operator* $\mathrm{i}\widetilde{e}^{\,a}_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}}\widetilde{D}_a\psi - m\psi$, *with* $\hat{\gamma}^{\hat{\mu}}$ *the standard Minkowski Clifford generators and* $\widetilde{D}$ *a* $\widetilde{g}$*-spin connection, has principal symbol* $\mathrm{i}\widetilde{e}^{\,a}_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}}\xi_a$. *This endomorphism of the spinor fibre is invertible if and only if* $\widetilde{g}^{ab}\xi_a\xi_b \neq 0$. *Consequently the characteristic variety is again* $\{\widetilde{g}^{ab}\xi_a\xi_b = 0\}$. *In particular, if the operators assigned in (BP2') of Assumption are these ones, the principal-symbol half of (BP2') is the standard hyperbolic condition that high-frequency characteristics follow the ordinary metric* $\widetilde{g}$. *The lemma does not derive (BP2') or (BP3') from Assumption , does not assign a kinetic metric to Higgs or neutrino portals, does not withdraw Proposition , and does not typestate full Standard-Model darkness from chronology alone.*

Proof. (a) Expanding $F_{ac} = \partial_a A_c - \partial_c A_a + [A_a, A_c]$ in $\widetilde{\nabla}_a F^{ab}$ produces the second-order term $\widetilde{g}^{ac}\widetilde{\nabla}_a\widetilde{\nabla}_c A^b - \widetilde{g}^{ac}\widetilde{\nabla}_a\widetilde{\nabla}^b A_c$, together with lower-order curvature and commutator terms. On the Lorenz slice the second summand is of order at most one. The principal symbol is therefore $-\widetilde{g}^{ac}\xi_a\xi_c\,\mathrm{Id}$, which vanishes if and only if ξ is $\widetilde{g}$-null. (b) The principal symbol of $\mathrm{i}\widetilde{e}^{\,a}_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}}\widetilde{D}_a$ is $\sigma(\xi) := \mathrm{i}\widetilde{e}^{\,a}_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}}\xi_a$. With the standard mostly-plus generators $\{\hat{\gamma}^{\hat{\mu}}, \hat{\gamma}^{\hat{\nu}}\} = 2\hat{\eta}^{\hat{\mu}\hat{\nu}}$, one has $(\hat{\gamma}^{\hat{\mu}}\xi_{\hat{\mu}})^2 = \widetilde{g}^{ab}\xi_a\xi_b\,\mathrm{Id}$, where $\xi_{\hat{\mu}} := \widetilde{e}^{\,a}_{\hat{\mu}}\xi_a$, and therefore $\sigma(\xi)^2 = -\widetilde{g}^{ab}\xi_a\xi_b\,\mathrm{Id}$. Hence $\sigma(\xi)$ is invertible precisely when $\widetilde{g}^{ab}\xi_a\xi_b \neq 0$.

Theorem (Conditional silence of listed direct couplings). *Assume Assumption , in addition to robust direct insulation in the sense of Assumption . Then, on the chronology-violating region, the Euler derivatives of the loop-traversing action with respect to the varied Standard-Model fields named in (BP3') vanish as bulk*

distributions. If a coupling constant in a covariant Lagrangian is spacetime-constant and its Euler contribution can be excited by a compactly supported test configuration in some nonempty open subset, that constant vanishes globally. This theorem does not withdraw Proposition , and it does not typestate "full Standard-Model darkness purely from backward chronology protection".

Proof. Stationarity under every compact variation in the named fields is precisely the vanishing of the corresponding Euler derivatives, as in Theorem . If a spacetime-constant coupling multiplies a local density that can be made nonzero by a compactly supported test configuration in a nonempty open set U, the Euler identity on U forces the constant to vanish, and a spacetime-constant then vanishes on the whole manifold. The argument uses (BP3') directly; (BP2') only names the ordinary kinetic metric whose Cauchy problem is being postulated, and whose principal-symbol half, for the Yang–Mills and Dirac operators of Lemma , is the characteristic variety of $\hat{g}$. Neither item is implied by Assumption alone.

The full-system causal obstruction

Proposition (Ordinary metric protection is not full signal protection). *Suppose an ordinary apparatus can excite and receive a propagating channel whose admissible causal curves include the host causal cone. If g contains a CTC, no global temporal function can be strictly increasing along every admissible curve of the combined system.*

Proof. Such a function would be strictly increasing along the closed future-directed host-timelike curve, yet return to its initial value at its initial event, a contradiction.

Thus universal ordinary coupling to $\hat{g}$ protects ordinary *matter trajectories*; it does not by itself protect an apparatus using accessible gravitational or controller channels. Direct electromagnetic silence does not remove gravity. A no-backward-signalling theorem for the full interacting system remains outside the release.

Mobility, photon cones, and a screened comparison

Anchored conformal clock and force identities

For clock comparisons we stipulate g as the constant-Planck-mass Einstein–Hilbert comparison frame. If a tensor-wave channel is invoked, its stipulated principal symbol is $g^{ab}k_a k_b$. This is an anchor, not a measurement derived from the matter action.

Theorem (Proper time, orthogonal force, and no freezing). *Let u be g-unit timelike. Then*

$$\frac{\mathrm{d}\hat{\tau}}{\mathrm{d}\tau_g} = \sqrt{A^2 - B[J(u)]^2}$$

whenever the same tangent is $\hat{g}$-timelike. In a conformal region $J = 0$,

$$\mathrm{d}\hat{\tau} = A\,\mathrm{d}\tau_g.$$

A massive ordinary $\hat{g} = A^2 g$ geodesic, parametrised by g-proper time, satisfies

$$a^a_{\mathrm{o}} = -P^{ab}\nabla_b \log A, \qquad P^{ab} = g^{ab} + u^a u^b.$$

Dark free transport satisfies $a^a_{\mathrm{d}} = 0$. For a specified u, the two accelerations agree exactly when $P^{ab}\nabla_b \log A = 0$. Agreement for every timelike u at a point is equivalent to $\nabla \log A = 0$ there.

Proof. Evaluate $-\hat{g}(u,u)$ to obtain . The conformal connection difference is

$$\hat{\Gamma}^a_{\ bc} - \Gamma^a_{\ bc} = \delta^a_b \partial_c \log A + \delta^a_c \partial_b \log A - g_{bc}\nabla^a \log A.$$

Insert $\hat{u} = A^{-1}u$ into $\hat{\nabla}_{\hat{u}}\hat{u} = 0$; the result is . It is orthogonal to u, as required by differentiation of $g(u,u) = -1$. The remaining statements follow by projection. A nonzero covector cannot be parallel to every timelike covector.

A homogeneous controller can change clock rates while leaving comoving worldlines geodesic: its gradient is parallel to the comoving velocity. Ordinary matter is therefore not literally frozen, and a nonzero clock-rate modulation does not require a nonzero comoving fifth force.

Proposition (Homogeneous momentum and clock reconstruction). *On $g = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\mathbf{x}^2$, let $A = A(t) > 0$ and*

define the host peculiar velocity $\mathbf{v} = a\,\mathrm{d}\mathbf{x}/\mathrm{d}t$. *For a free massive ordinary particle,*

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -(H+\alpha)(1-|\mathbf{v}|^2)\mathbf{v},$$

$$\alpha = \frac{\dot{A}}{A}, \qquad H = \frac{\dot{a}}{a},$$

$$aA\,\gamma\mathbf{v} = \text{constant}, \qquad \gamma = (1-|\mathbf{v}|^2)^{-1/2}.$$

The dark formula has $A = 1$. *For nonzero momentum, the logarithmic drift of* $a\gamma|\mathbf{v}|$ *is* $-\alpha$, *while a comoving ordinary clock has rate A.*

Proof. The particle Lagrangian is $-mA\sqrt{1-a^2|\dot{\mathbf{x}}|^2}$. Spatial translation invariance conserves $mAa\gamma\mathbf{v}$. Differentiating this identity gives

$$\dot{\mathbf{v}} + \gamma^2(\mathbf{v}\cdot\dot{\mathbf{v}})\mathbf{v} = -(H+\alpha)\mathbf{v}.$$

The acceleration is parallel to $\mathbf{v}$; solving its scalar coefficient gives .

A conformal frame redefinition can redistribute apparent forces between sectors. Contract C2 below therefore uses the explicitly anchored *g*-frame rather than treating "modified" as frame-independent terminology.

Photons and tensor waves

Proposition (Conformal vacuum equality and directional penalties). *In four dimensions, if* $J = 0$, *the vacuum Maxwell action is exactly the Maxwell action for g. Its geometric-optics null cone is therefore the host null cone.*

In a host-orthonormal frame with $J = (0, j, 0, 0)$, *the ordinary photon speed along the J-direction is*

$$c_{\parallel} = \frac{1}{\sqrt{1+(B/A^2)j^2}}, \qquad c_{\perp} = 1.$$

A tangent k with $J(k) = 0$ *is null for one metric exactly when it is null for the other.*

For the separate timelike rank-one class $q = (q_0, 0, 0, 0)$,

$$\tilde{g} = A^2\,\mathrm{diag}\,(-(1-\zeta), 1, 1, 1), \qquad \zeta = \frac{Bq_0^2}{A^2} < 1,$$

so

$$c_\gamma = \sqrt{1-\zeta} = \frac{\text{ordinary lapse}}{A}.$$

Complete homogeneous lapse freezing is the degenerate limit $\zeta = 1$.

Proof. For $J = 0$, the volume contributes A^4 and the two inverse metrics contribute A^{-4} in . The factors cancel. Maxwell's leading eikonal equation is the metric null equation. The directional speed formulas follow by substituting the stated covectors into and solving $\tilde{g}(x,x) = 0$. If $J(k) = 0$, the rank-one term vanishes directly.

The condition $J(k) = 0$ is *not* generally equivalent to zero conserved angular momentum. In a rotating metric, $p_\theta = \tilde{g}_{\theta a}k^a$ contains a time–angle cross term.

Exact vacuum cone equality in a vortex-free region does not automatically discharge GW170817: the whole propagation path, emission lag, medium effects, and gravitational-wave channel must be specified [17]. An anisotropic vortex intersecting the path requires a directional or integrated-delay test.

A declared $\mathbb{Z}_2$-screened subclass

The following comparison is separate from the cosmological exponential portal used in Section . Let

$$A_{\text{sym}}(\varphi) = 1 + \frac{\varphi^2}{2M_s^2}, \qquad V_{\text{sym}}(\varphi) = V_0 - \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4, \quad \lambda > 0.$$

For a stipulated static nonrelativistic conserved rest-density source ρ_*, the comparison effective potential is

$$V_{\text{eff}} = V_{\text{sym}} + \rho_* A_{\text{sym}}.$$

It has minimum $\varphi = 0$ when $\rho_*/M_s^2 \geq \mu^2$; below that threshold its nonzero minima are

$$\varphi = \pm\sqrt{\frac{\mu^2 - \rho_*/M_s^2}{\lambda}}.$$

This is the standard symmetron mechanism in a specified reduced class [13]. A held static source can require supporting apparatus, whose stress is not deleted from an isolated-system ledger.

Theorem (Dense-ball suppression with nonzero leakage). *Let* $m^2 = \rho_*/M_s^2 - \mu^2 > 0$. *Suppose a regular nonnegative spherical solution on* $0 \le r \le R$ *satisfies*

$$\varphi'' + \frac{2}{r}\varphi' = m^2\varphi + \lambda\varphi^3, \qquad \varphi'(0) = 0, \qquad \varphi(R) = \varphi_R > 0.$$

Define, with its continuous value at $r = 0$,

$$F_k(r) = \varphi_R \frac{R\sinh(kr)}{r\sinh(kR)}, \qquad F_k(0) = \varphi_R \frac{kR}{\sinh(kR)},$$

and $M^2 = m^2 + \lambda\varphi_R^2$. *Then*

$$0 < F_M(r) \le \varphi(r) \le F_m(r) \le \varphi_R.$$

Moreover,

$$|a_{\mathrm{sym}}(r)| \le \frac{M^2 r}{3M_s^2} F_m(r)^2.$$

Proof. The equation implies that an interior positive maximum cannot exceed the boundary value; hence $\varphi \le \varphi_R$.

The functions F_k solve $(\Delta - k^2)F_k = 0$ with the same boundary data. Since

$$(\Delta - m^2)(\varphi - F_m) = \lambda\varphi^3 \ge 0,$$

a positive interior maximum of $\varphi - F_m$ is impossible. Likewise,

$$(\Delta - M^2)(\varphi - F_M) = \lambda(\varphi^2 - \varphi_R^2)\varphi \le 0$$

excludes a negative interior minimum. This proves , including strict positivity.

Integrating the radial equation gives

$$r^2\varphi'(r) = \int_0^r u^2(m^2 + \lambda\varphi(u)^2)\varphi(u)\,\mathrm{d}u.$$

The upper comparison F_m is increasing, so $0 \le \varphi'(r) \le M^2 r F_m(r)/3$. Finally,

$$|\partial_r \log A_{\mathrm{sym}}| = \frac{\varphi|\varphi'|}{M_s^2 A_{\mathrm{sym}}} \le \frac{\varphi|\varphi'|}{M_s^2},$$

which proves .

The interior theorem is conditional on a supplied regular solution and boundary value. Exact vanishing inside a finite ball with $\varphi_R > 0$ is specifically excluded by . A global radial profile is constructed next; it does not prove time-dependent screening or establish joint Solar-System acceptance. The interior force bound applies to that profile once $\varphi(R)$ is identified.

Lemma (Global radial symmetron profile and comparison directions). *Assume a uniform ball source* $\rho_*(r) = \rho_* 1_{\{r \le R\}}$ *with constants* $\rho_*, R, M_s, \mu, \lambda > 0$ *such that* $m_{\text{in}}^2 := \rho_*/M_s^2 - \mu^2 > 0$ *and* $\varphi_\infty := \mu/\sqrt{\lambda} > 0$. *Set* $m_{\text{out}} := \sqrt{2}\,\mu$. *There exists at least one radial function* $\varphi \in C^{1,1}(\mathbb{R}^3) \cap C^\infty(\mathbb{R}^3 \setminus \{r = R\})$ *satisfying*

$$\Delta\varphi = \varphi(\lambda\varphi^2 - \mu^2 + \rho_*(r)/M_s^2)$$

in the distributional sense, with $\varphi'(0) = 0$, $0 < \varphi(r) < \varphi_\infty$, φ *strictly increasing, and* $\varphi(r) \to \varphi_\infty$ *as* $r \to \infty$. *The shooting construction identifies a single separatrix between undershooting and overshooting initial values at the origin, in the class of radial positive increasing solutions. On any such profile, writing* $\delta := \varphi_\infty - \varphi(R) > 0$ *and* $\psi := \varphi_\infty - \varphi$, *one has the exterior comparisons*

$$\delta\frac{R}{r}e^{-m_{\text{out}}(r-R)} \le \psi(r) \le \delta\frac{R}{r} \qquad (r \ge R).$$

In particular, the Yukawa profile is a lower bound on the deviation from φ_∞, *not an upper bound; a generally valid upper bound is the massless harmonic envelope. The centre value satisfies the interior sandwich of Theorem in terms of the (now existent) boundary value* $\varphi(R)$. *An explicit nonlinear closed form for* $\varphi(R)/\varphi_\infty$ *in the bare parameters* $(m_{\text{in}}R, m_{\text{out}}R)$ *alone is not claimed. Writing* $\theta := \varphi(R)/\varphi_\infty \in (0,1)$ *and* $M_*^2 := m_{\text{in}}^2 + \mu^2 = m_{\text{in}}^2 + \frac{1}{2}m_{\text{out}}^2$, C^1 *matching of the interior sandwich of Theorem with the exterior comparisons yields the two-sided bracket*

$$\frac{\tanh(M_*R)}{M_*R} \le \frac{\varphi(R)}{\varphi_\infty} \le \frac{m_{\text{out}} + 1/R}{m_{\text{in}}\coth(m_{\text{in}}R) + m_{\text{out}}}.$$

The corresponding linearised matching identity $\varphi_R/\varphi_\infty = (m_{\text{out}} + 1/R)/(m_{\text{in}}\coth(m_{\text{in}}R) + m_{\text{out}})$ *is therefore an upper bound for the nonlinear profile, not an equality. A tighter implicit lower constraint* $\theta \ge \tanh(M(\theta)R)/(M(\theta)R)$, *with*

$M(\theta)^2 = m_{\text{in}}^2 + \mu^2\theta^2$*, also holds, and implies the explicit left-hand side of because* $\tanh(x)/x$ *is decreasing for* $x > 0$ *and* $M(\theta) \le M_*$*. The construction is not a global environmental solution on a non-spherical or time-dependent source, and it is not Host C.*

Proof. Write the radial equation as $(r^2\varphi')' = r^2 V'_{\text{eff}}(\varphi; r)$, with $V'_{\text{eff}}(\varphi; r) = \varphi(m_{\text{in}}^2 + \lambda\varphi^2)$ for $r < R$ and $V'_{\text{eff}}(\varphi; r) = \lambda\varphi(\varphi^2 - \varphi_\infty^2)$ for $r > R$. For each $\alpha \in (0, \varphi_\infty)$ let $\varphi(\,\cdot\,; \alpha)$ be the interior solution with $\varphi(0) = \alpha$ and $\varphi'(0) = 0$. For sufficiently small α, comparison with the linear sinh profile of mass m_{in} shows that the solution exists on $[0, R]$ and remains in $(0, \varphi_\infty)$. On that interval the interior right-hand side is positive for $\varphi > 0$, so φ is strictly increasing and $\varphi'(R; \alpha) > 0$. Continuous dependence on α holds on the open set of values for which the solution remains below φ_∞ through $r = R$.

Each such interior jet $(\varphi(R), \varphi'(R))$ is continued by the exterior equation. Because the exterior right-hand side is negative for $0 < \varphi < \varphi_\infty$, the quantity $r^2\varphi'$ is strictly decreasing on any interval on which φ remains in $(0, \varphi_\infty)$. Three alternatives then arise: undershoot, in which φ' hits zero at some finite $r > R$ with $\varphi < \varphi_\infty$ and thereafter φ decreases; overshoot, in which φ crosses φ_∞ at finite radius; and a separatrix on which φ remains in $(0, \varphi_\infty)$, $r^2\varphi'$ stays nonnegative and tends to zero, and $\varphi \to \varphi_\infty$. Small α undershoots, because the exterior jet is arbitrarily small while the gap $\varphi_\infty - \varphi(R)$ remains of order φ_∞. For the comparison linear interior problem, any boundary value $\varphi(R) \in (0, \varphi_\infty)$ is attained by some regular α, with $\varphi'(R) = (m_{\text{in}}\coth(m_{\text{in}}R) - 1/R)\varphi(R)$. As $\varphi(R) \to \varphi_\infty^-$ this derivative remains bounded below by a positive constant, so the exterior continuation overshoots. The printed cubic term is nonnegative for $\varphi > 0$, hence only increases φ' at fixed $\varphi(R)$ relative to that linear comparison, and overshoot persists. Continuous dependence therefore yields at least one separatrix. If $\alpha_1 < \alpha_2$ both exist through $r = R$, the interior comparison principle (the right-hand side is increasing in $\varphi > 0$) gives $\varphi(R; \alpha_1) \le \varphi(R; \alpha_2)$ and $\varphi'(R; \alpha_1) \le \varphi'(R; \alpha_2)$. A larger jet cannot undershoot if a smaller one already overshoots, so the undershooting set is a single initial interval and the separatrix is unique in this radial increasing class.

On the separatrix, $0 < \varphi < \varphi_\infty$ in the exterior, hence $0 \le \lambda\varphi(\varphi + \varphi_\infty) \le 2\lambda\varphi_\infty^2 = m_{\text{out}}^2$. Writing $\psi = \varphi_\infty - \varphi \ge 0$, one has

$\Delta\psi = \lambda\varphi(\varphi + \varphi_\infty)\psi$, so $0 \le \Delta\psi \le m_{\text{out}}^2\psi$. Let $\delta = \psi(R)$ and set $H(r) = \delta R/r$ and $Y(r) = \delta(R/r)e^{-m_{\text{out}}(r-R)}$ for $r \ge R$. Then $\Delta H = 0$ and $(\Delta - m_{\text{out}}^2)Y = 0$, with $H(R) = Y(R) = \delta$ and both vanishing at infinity. Thus $\Delta(\psi - H) \ge 0$. The strong maximum principle on annular exhaustions, using $\psi - H \to 0$ at infinity, yields $\psi - H \le 0$. Likewise, $(\Delta - m_{\text{out}}^2)(\psi - Y) \le 0$. If $\psi - Y$ had a negative interior minimum, then at that point $\Delta(\psi - Y) \ge 0$ and $(\Delta - m_{\text{out}}^2)(\psi - Y) \ge -m_{\text{out}}^2(\psi - Y) > 0$, a contradiction. Hence $\psi \ge Y$, which is . Matching of φ and φ' across $r = R$ is included in the $C^{1,1}$ regularity: the source is bounded and jumps only at the sphere, so φ'' jumps while φ and φ' remain Lipschitz.

It remains to extract . The interior comparisons $F_M \le \varphi \le F_m$ of Theorem meet at $r = R$, and $F_k'(R) = (k\coth(kR) - 1/R)\varphi_R$. Because a lower graph that meets an upper graph at the right endpoint cannot have a strictly smaller derivative there,

$$(m_{\text{in}}\coth(m_{\text{in}}R) - 1/R)\varphi_R \le \varphi'(R) \le (M\coth(MR) - 1/R)\varphi_R,$$

with $M^2 = m_{\text{in}}^2 + \lambda\varphi_R^2$. (The map $k \mapsto k\coth(kR)$ is increasing on $k > 0$, since $\sinh x \ge x$ for $x = kR$.) On the exterior, $\psi \ge Y$ and $\psi \le H$ meet at $r = R$, with $H'(R) = -\delta/R$ and $Y'(R) = -\delta(m_{\text{out}} + 1/R)$. Hence $Y'(R) \le \psi'(R) \le H'(R)$, or equivalently

$$\frac{\delta}{R} \le \varphi'(R) \le \delta(m_{\text{out}} + 1/R).$$

Combining the interior lower bound on $\varphi'(R)$ with the exterior upper bound gives

$$(m_{\text{in}}\coth(m_{\text{in}}R) - 1/R)\varphi_R \le (\varphi_\infty - \varphi_R)(m_{\text{out}} + 1/R).$$

Writing $\theta = \varphi_R/\varphi_\infty$ and rearranging, the $-1/R$ terms cancel and one obtains the right-hand side of . Combining the exterior lower bound on $\varphi'(R)$ with the interior upper bound gives

$$\frac{\varphi_\infty - \varphi_R}{R} \le (M\coth(MR) - 1/R)\varphi_R,$$

hence $\theta \ge \tanh(MR)/(MR)$. Since $M^2 = m_{\text{in}}^2 + \lambda\varphi_R^2 = m_{\text{in}}^2 + \mu^2\theta^2 \le M_*^2$ and $\tanh(x)/x$ is decreasing for $x > 0$, the

explicit left-hand side of follows. The linearised identity is the case of interior mass m_{in} and exterior Yukawa matching, and is therefore the upper bound just proved, not an equality for the cubic problem.

Paired witnesses, residual stress, and controlled decoupling

W-CTC: complete cylindrical witness

Assumption (Cylindrical parameters). Let

$$M = \mathbb{R}_t \times \mathbb{R}_x \times \mathbb{S}^1_\theta \times \mathbb{R}_z, \qquad \theta \sim \theta + 2\pi,$$

and prescribe

$$g = -\mathrm{d}t^2 - 2a\,\mathrm{d}t\,\mathrm{d}\theta - h\,\mathrm{d}\theta^2 + \mathrm{d}x^2 + \mathrm{d}z^2,$$

$$q = \kappa\,\mathrm{d}\theta, \qquad \tilde{g} = g + Bq \otimes q = -\mathrm{d}t^2 - 2a\,\mathrm{d}t\,\mathrm{d}\theta + H\,\mathrm{d}\theta^2 + \mathrm{d}x^2 + \mathrm{d}z^2,$$

$$a > 0, \qquad 0 < h < a^2, \qquad \kappa > 0, \qquad b = B\kappa^2 > h, \qquad H = b - h.$$

Both orientations use the timelike vector ∂_t.

Theorem (Complete chronology split and positive conserved test dust). *Under Assumption :*

1. *both metrics are flat and geodesically complete;*

2. *$u = h^{-1/2}\partial_\theta$ has closed future-directed g-timelike geodesics of proper period $2\pi\sqrt{h}$;*

3. *every t-slice is Cauchy for $\tilde{g}$;*

4. *for every smooth positive $\rho(t,x,z)$, $T^{ab}_{\mathrm{d}} = \rho u^a u^b$ is host-conserved;*

5. *q is closed, non-exact, divergence-free, and $g^{ab}q_a q_b = \kappa^2/(a^2 - h) > 0$. With zero ordinary background stress, constant φ, s at stationary portal and potential values, and the circle controller action, its controller equations are satisfied. No Einstein source match is asserted.*

Proof. On the universal cover all metric components are constant and the connections vanish. The quotient

identification is a translation isometry. Affine geodesics on the cover extend for all affine parameter and descend to complete quotient geodesics.

For u,

$$g(u,u) = -1, \qquad \nabla_u u = 0, \qquad g(u,\partial_t) = -a/\sqrt{h} < 0.$$

The stated period follows by integration around the circle.

Complete the square:

$$\tilde{g} = -\left(1 + \frac{a^2}{H}\right)\mathrm{d}t^2 + H\left(\mathrm{d}\theta - \frac{a}{H}\mathrm{d}t\right)^2 + \mathrm{d}x^2 + \mathrm{d}z^2.$$

The global coordinates

$$T = \sqrt{1 + a^2/H}\, t, \qquad \theta = \theta - \frac{a}{H}t \bmod 2\pi$$

give $\tilde{g} = -\mathrm{d}T^2 + \mathrm{d}x^2 + H\mathrm{d}\theta^2 + \mathrm{d}z^2$. Its spatial factor $\mathbb{R}^2 \times \mathbb{S}^1$ is complete. A future causal curve parametrised by T has spatial speed at most one. A finite terminal T-range would give a finite-length spatial tail and an endpoint, allowing continuation. Thus an inextendible causal curve has full T-range and crosses every slice exactly once.

Since $u(\rho) = 0$, $\nabla_a u^a = 0$, and $\nabla_u u = 0$,

$$\nabla_a(\rho u^a u^b) = 0.$$

The inverse angular component is $g^{\theta\theta} = 1/(a^2 - h)$. The components of q and the metric are constant, giving zero divergence. Its integral $2\pi\kappa \neq 0$ excludes exactness. The circle equation is $\nabla_a q^a = 0$, and the remaining controller sources vanish under the printed constant-field conditions.

The witness is locally re-proved, with documentary comparison to Cheung's cylindrical construction [5]. Its quotient identification already contains the closed orbits.

Proposition (Flat-host Einstein obstruction). *On the flat cylindrical witness, positive dark dust and canonical winding stress cannot alone satisfy*

$$M_{\mathrm{P}}^2(G_{ab} + \Lambda g_{ab}) = T_{\mathrm{d}}{}_{ab} + T_{q}{}_{ab}$$

for any Λ*, with* $f_q > 0$.

Proof. For any nonzero null vector k,

$$T_{\mathrm{d}}(k,k) = \rho[g(u,k)]^2 > 0, \qquad T_q(k,k) = f_q^2[q(k)]^2 \geq 0.$$

A timelike vector cannot be orthogonal to a nonzero null vector. The left side has zero null contraction because $G = 0$ and $g(k,k) = 0$, a contradiction.

A residual cancelling this mismatch must have a negative null contraction. Positive *test* dust is therefore not a positive residual-free Einstein source.

A smooth compactly supported witness on $\mathbb{R}^4$

Let $r^2 = x^2 + y^2$. Choose a smooth nonnegative function $w(r^2, z)$ of compact spatial support away from the axis, equal to 1 on an open annular region

$$\mathscr{U} = \{r_1 < r < r_2,\ |z| < z_1\}, \qquad r_1 > 0.$$

Choose a smooth positive function p_0 on a neighbourhood of supp w such that

$$p_0(r^2, z) = \frac{\sqrt{r^2 + h}}{r^2} \quad \text{on} \mathscr{U}.$$

Products below are extended by zero outside that neighbourhood. For $B_0 > 0$, put

$$p = w^2 p_0, \qquad \omega = p(x\,\mathrm{d}y - y\,\mathrm{d}x),$$

$$f = B_0^{-1/4} w\sqrt{p_0}, \qquad \Phi = f(x + \mathrm{i}y),$$

$$g = -(\mathrm{d}t + \omega)^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2, \qquad \tilde{g} = g + B_0 J \otimes J.$$

Theorem (Smooth toroidal split, completeness, and exterior equality). *The fields and metrics in are smooth on* $\mathbb{R}^4$*, including the axis. They obey*

$$J = f^2(x\,\mathrm{d}y - y\,\mathrm{d}x), \qquad B_0 J \otimes J = \omega \otimes \omega,$$

and hence

$$\hat{g} = -\mathrm{d}t^2 - 2\,\mathrm{d}t\,\omega + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2.$$

Both metrics are geodesically complete. Every t-slice is Cauchy for $\hat{g}$*. On* $\mathscr{U}$*, the angular circles at fixed* t, r, z *are closed future-directed g-timelike geodesics of proper period* $2\pi\sqrt{h}$*. Outside the compact support of w, both metrics are Minkowski.*

Proof. The support is separated from $r = 0$; all products are smooth Cartesian fields and vanish near the axis. Directly expanding $X\mathrm{d}Y - Y\mathrm{d}X$ cancels the df terms and gives the printed J. The choice of f proves $B_0 J^2 = \omega^2$, giving . The coframe $(\mathrm{d}t + \omega, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$ proves that g is Lorentzian everywhere. Equation has positive spatial block and negative Schur complement, so it too is Lorentzian.

Writing $\omega = \omega_i \mathrm{d}x^i$, one obtains

$$\hat{g}^{tt} = -\frac{1}{1 + |\omega|^2} < 0.$$

Thus t is temporal. A causal curve with $v = \mathrm{d}\mathbf{x}/\mathrm{d}t$ satisfies

$$|v - \omega^{\mathrm{sharp}}|^2 \le 1 + |\omega|^2.$$

Because ω is bounded, spatial speed is uniformly bounded. A finite terminal t-range gives a spatial endpoint in complete Euclidean space and therefore a continuation. Every inextendible causal curve crosses every t-slice exactly once.

For completeness, both metrics are stationary. Along an affine geodesic let $E = \dot{t} + \omega(\mathbf{x})\dot{}$, its conserved Killing energy, and let C be its constant squared norm. For g,

$$|\dot{\mathbf{x}}|^2 = E^2 + C.$$

For $\hat{g}$,

$$|\dot{\mathbf{x}}|^2 + [\dot{\omega}(\mathbf{x})]^2 = E^2 + C.$$

Consequently the spatial velocities and $\dot{t}$ are bounded. The coefficients and their derivatives are globally bounded. On any finite affine interval the positions and velocities stay in a compact subset of the geodesic phase space; the smooth geodesic ODE therefore extends.

On $\mathscr{U}$, $\omega = \sqrt{r^2 + h}\,\mathrm{d}\theta$, so $g_{\theta\theta} = -h$. For the Killing vector $K = \partial_\theta$, $\nabla_K K = -\frac{1}{2}\nabla[g(K,K)] = 0$ there. Furthermore $g(K, \partial_t) = -\sqrt{r^2 + h} < 0$. Normalization gives $u = K/\sqrt{h}$ and the stated proper period. Exterior equality is immediate.

Smooth nonnegative dust supported inside $\mathscr{U}$, independent of θ, is conserved by the same calculation as in Theorem . It is positive where its density is nonzero.

Remark (Kinematic unless a printed potential is on shell). Theorem does not, by itself, assert that a general compact profile $\Phi = f(x + \mathrm{i}y)$ solves the canonical S_Φ equations. Any force maintaining an off-shell profile must be printed with its own action and stress before invoking full on-shell conservation. No unrecorded forcing is absorbed into the dark slot. The statement also does not identify a vortex-support region with an astrophysical halo. Theorem below supplies on-shell realisations in two printed classes, after a repair of the Laplacian identity, and only when the Schrödinger coefficient μ is smooth; Theorem shows that a compactly supported amplitude does not inhabit the two-switch class with a locally bounded coefficient. The statement does not make an arbitrary compact $h(r^2, z^2)$ on-shell.

Theorem (On-shell realisation by a two-switch potential, or on a *z*-independent radial collar). *Let g be a smooth stationary unit-lapse metric of the form , and assume the ordinary sector is empty in the sense* $\hat{T}^{ab} = 0$. *For a stationary profile* $\Phi = h(r^2, z^2)(x + \mathrm{i}y)$ *one has* $\mathrm{Box}_g \Phi = \Delta_{\mathbb{R}^3}\Phi$. *Writing* $H(R, Z) := h(R, Z)$ *with* $R = r^2$ *and* $Z = z^2$, *the Cartesian Laplacian identity is*

$$\Delta_{\mathbb{R}^3}\Phi = \lambda(r^2, z^2)\,(x + \mathrm{i}y), \qquad \lambda := 8\partial_R H + 4r^2\partial_R^2 H + 2\partial_Z H + 4z^2\partial_Z^2 H.$$

Equivalently, $\lambda = 4r^2\partial_{r^2}^2 h + 8\partial_{r^2} h + \partial_z^2 h$, *since* $\partial_z^2 h = 2\partial_Z H + 4z^2\partial_Z^2 H$. *The coefficient* λ *extends smoothly to all of* $\mathbb{R}^3$, *including the axis and the complement of* $\operatorname{supp}(h)$. *On the open set* $\{h \neq 0\}$ *one may divide by the amplitude and obtain the Schrödinger form*

$$\Delta_{\mathbb{R}^3}\Phi = \mu\,\Phi, \qquad \mu := \frac{\lambda}{h}.$$

Official version 2 identified $\Delta_{\mathbb{R}^3}\Phi = \lambda\Phi$. *That identification is false whenever* $h \equiv 1$*: the right hand side of is* $\lambda(x + \mathrm{i}y)$, *not* $\lambda\Phi = \lambda h(x + \mathrm{i}y)$. *The repaired Euler coefficient is* μ, *not* λ. *The function* μ *need not extend continuously, let alone smoothly or boundedly, across* $\{h = 0\}$. *A single-invariant pullback of* λ, *of* μ, *of* $u := |\Phi|^2 = r^2h^2$, *and of* $\Delta_{\mathbb{R}^3}$*s to a single toroidal invariant* $\sigma = (r - r_0)^2 + z^2$ *is not coordinate-consistent on* $\mathbb{R}^3$ *and is not used (item (c)). The following two realisations replace it. Both are prescribed-host bookkeeping on the declared vacuum subclass* $\tilde{T}^{ab} = 0$. *They are not Einstein dynamics, not Host C, not a derived metric, and not a solution of a coupled Einstein–vortex system.*

1. *Two-switch potential on* (r^2, z^2), *under smoothness of* μ *and integrability. Extend the single-switch action by two smooth real scalars* s_1, s_2 *and a joint potential* $W(s_1, s_2)$, *with formal Lagrangian density*

$$\frac{1}{2}g^{ab}\nabla_a\Phi^*\nabla_b\Phi + V_\Phi(|\Phi|^2, s_1, s_2) + \sum_{i=1}^{2}\frac{f_i^2}{2}g^{ab}\nabla_a s_i\nabla_b s_i + W(s_1, s_2).$$

Let $h = h(r^2, z^2)$ *be smooth, and take the smooth invariants* $s_1 = r^2$ *and* $s_2 = z^2$. *Then* $\Delta_{\mathbb{R}^3}s_1 = 4$ *and* $\Delta_{\mathbb{R}^3}s_2 = 2$ *identically, while* λ *and* $u = s_1h^2$ *become functions of* (s_1, s_2). *Suppose moreover that* $\mu = \lambda/h$ *extends to a smooth function of* (s_1, s_2) *on the printed image (in particular,* h *has no flattening zero at which* λ/h *blows up). Define*

$$V_\Phi(|\Phi|^2, s_1, s_2) := \frac{1}{2}\mu(s_1, s_2)\,|\Phi|^2.$$

If the closedness condition

$$\frac{\partial\mu}{\partial s_1}\frac{\partial u}{\partial s_2} = \frac{\partial\mu}{\partial s_2}\frac{\partial u}{\partial s_1}$$

holds, equivalently if μ and u have the same level sets on the printed image, then there exists a smooth $W(s_1, s_2)$ *such that* $E_{\Phi^*} = 0$ *and both switch Euler equations hold pointwise on* $\mathbb{R}^4$*. The construction is not claimed for a fully general two-variable amplitude* $h(r^2, z^2)$ *without smoothness of μ and without . The switch fields* $s_1 = r^2$ *and* $s_2 = z^2$ *are not compactly supported; the controller Φ is compactly supported only in the class obstructed by Theorem .*

2. *z-independent radial collar, unconditionally in the single invariant, under smoothness of μ. If* $h = h(r^2)$ *and* $s = r^2$*, then* $\lambda = 8H' + 4r^2H''$ *and* $u = r^2h(r^2)^2$ *are functions of the single invariant s. If, further,* $\mu = \lambda/h$ *extends to a smooth function of s on the needed range (for instance if h has no zero), then*

$$V_{\Phi}(|\Phi|^2, s) := \frac{1}{2}\mu(s)\,|\Phi|^2, \qquad W'(s) := 4f_s^2 - \frac{1}{2}\mu'(s)\,u(s)$$

are well-defined smooth functions of s. Thus Φ and s solve $E_{\Phi^*} = 0$ *and the switch equation pointwise on all of* $\mathbb{R}^4$*, with no further integrability condition. This class is not compactly supported in z. Official version 2 wrote* $W'(s) = 4f_s^2 - \frac{1}{2}\lambda'(s)u(s)$ *with the unrepaired coefficient λ; the repaired identity uses μ. A compactly supported reparametrisation* $s = \chi(r^2)$ *may be used only on a range where χ is strictly monotone and the right-hand side of the switch equation descends as a function of s; a compactly supported bump is typically non-monotone and is not used as a global coordinate for W.*

3. *Withdrawn single-invariant pullback. If h and a switch s are merely functions of* $\sigma = (r - r_0)^2 + z^2$*, then neither* $u = r^2h(\sigma)^2$ *nor λ from , nor* $\mu = \lambda/h$*, is, in general, a function of σ alone, because* r^2 *and* z^2 *vary independently along level tori of σ. Moreover* $\Delta_{\mathbb{R}^3}\sigma = 4 + 2(1 - r_0/r)$ *is not a function of σ, so* $\Delta_{\mathbb{R}^3}s$ *is not a function of σ for a non-constant* $s = s(\sigma)$*. Formulae that treat λ, μ, and u as functions of such an s alone are therefore not used.*

Proof. On the unit-lapse metric one has $\det(g_{\mu\nu}) = -1$ and, writing $\omega = \omega_i\,\mathrm{d}x^i$, $g^{tt} = -1 + |\omega|^2$, $g^{ti} = -\omega^i$, $g^{ij} = \delta^{ij}$. For a stationary scalar, $\mathrm{Box}_g\,\Phi = \partial_\mu(g^{\mu\nu}\partial_\nu\Phi)$ collapses to $\Delta_{\mathbb{R}^3}\Phi$, the mixed drag terms cancelling. The Cartesian computation of $\Delta_{\mathbb{R}^3}(H \cdot (x + \mathrm{i}y))$ is as follows. With $\Phi_1 = Hx$,

$$\partial_{xx}\Phi_1 = 6x\partial_R H + 4x^3\partial_R^2 H,$$

$$\partial_{yy}\Phi_1 = 2x\partial_R H + 4xy^2\partial_R^2 H,$$

$$\partial_{zz}\Phi_1 = 2x\partial_Z H + 4xz^2\partial_Z^2 H,$$

and therefore $\Delta_{\mathbb{R}^3}\Phi_1 = x(8\partial_R H + 4r^2\partial_R^2 H + 2\partial_Z H + 4z^2\partial_Z^2 H) = \lambda x$. The same factor multiplies y in $\Delta_{\mathbb{R}^3}\Phi_2$, which is . Consequently $\Delta_{\mathbb{R}^3}\Phi = \lambda(x + \mathrm{i}y)$, and follows wherever $h \neq 0$. (The identity $\Delta_{\mathbb{R}^3}\Phi = \lambda\Phi$ would require $\lambda x = \lambda H x$, hence $h \equiv 1$ or $\lambda \equiv 0$.) The vacuum Euler equation of Proposition then holds as soon as $\partial V_\Phi/\partial|\Phi|^2 = \frac{1}{2}\mu$.

(a) The displayed choice of V_Φ enforces that identity once μ is a smooth function of (s_1, s_2). For $s_1 = x^2 + y^2$ and $s_2 = z^2$ one has $\Delta_{\mathbb{R}^3}s_1 = 4$ and $\Delta_{\mathbb{R}^3}s_2 = 2$. With $\overset{\circ}{T}{}^{ab} = 0$, the two switch Euler equations become $\partial W/\partial s_1 = 4f_1^2 - \frac{1}{2}(\partial\mu/\partial s_1)u$ and $\partial W/\partial s_2 = 2f_2^2 - \frac{1}{2}(\partial\mu/\partial s_2)u$. For constant f_i, mixed derivatives of this gradient agree if and only if holds, in which case W exists locally by Poincaré quadrature on the printed image. The same diffeomorphism argument as in Theorem gives on-shell conservation for compactly supported variations.

(b) In the z-independent class, λ and u are functions of $R = r^2$ alone, $\Delta_{\mathbb{R}^3}s = 4$, and, once μ is a smooth function of s , the single switch equation $f_s^2\Delta_{\mathbb{R}^3}s - W'(s) - \frac{1}{2}\mu'(s)u(s) = 0$ is . The right-hand side is a function of $s = r^2$, which is a global smooth coordinate on $\{r > 0\}$ and extends smoothly through the axis. On $\{h \neq 0\}$ one has $\Delta_{\mathbb{R}^3}\Phi = \mu\Phi$; if h never vanishes, this is a pointwise identity on $\mathbb{R}^3 \setminus \{r = 0\}$ and extends by continuity through the axis.

(c) Along a level torus of σ, the cylindrical radius r varies unless the torus degenerates to an equatorial circle. Thus $u = r^2h(\sigma)^2$ is not constant on that torus. The coefficients r^2 and z^2 in likewise vary, and so does $\mu = \lambda/h$. In cylindrical coordinates, $\Delta_{\mathbb{R}^3}\sigma = \partial_{rr}\sigma + \frac{1}{r}\partial_r\sigma + \partial_{zz}\sigma = 4 + 2(1 - r_0/r)$ for $r > 0$, which is not a function of σ.

Theorem (Sharpness: no compactly supported two-switch realisation). *In the setting of Theorem , there is no nontrivial compactly supported amplitude $h \in C^\infty(\mathbb{R}^3)$ with* $\operatorname{supp}(h) \subset \{r \geq r_{\mathrm{in}} > 0\}$ *for which the two-switch bookkeeping of item (1) is realised by a potential V_Φ whose Schrödinger coefficient μ extends to a locally bounded function on all of $\mathbb{R}^3$. Consequently item (1) is not constructively inhabited in the compactly supported class. The z*

-independent radial collar of item (2) remains the unconditional realisation under smoothness of μ, and is not compact in z. No compact example is exhibited.

Proof. Let h be smooth and compactly supported in $\{r \geq r_{\text{in}} > 0\}$, and write $\Phi = h(r^2, z^2)(x + \mathrm{i}y)$, so that Φ is itself smooth and compactly supported on $\mathbb{R}^3$. The identity holds everywhere. On $\{h \neq 0\}$ one has $\Delta_{\mathbb{R}^3}\Phi = \mu\Phi$ with $\mu = \lambda/h$.

Two exhaustive alternatives arise. First, μ may fail to extend as a locally bounded function across the closed support of h. This is the typical behaviour of a flattening bump: the ratio of a compactly supported amplitude to a differential polynomial in that amplitude is unbounded as one approaches a flattening zero (the model one-dimensional ratio f''/f for a standard flat bump already diverges). In that case $V_\Phi = \frac{1}{2}\mu|\Phi|^2$ is not given by a locally bounded, let alone smooth, coefficient on the printed image, and the two-switch action of item (1) is not admissible in the printed C^∞ class.

Second, suppose a locally bounded extension of μ to all of $\mathbb{R}^3$ does exist, still written μ, so that $\Delta_{\mathbb{R}^3}\Phi = \mu\Phi$ holds almost everywhere and, by continuity, everywhere. Then $\Phi \in H^2_{\text{loc}}(\mathbb{R}^3; \mathbb{C})$ solves a linear Schrödinger equation with locally bounded potential and vanishes on a nonempty open set, namely the complement of a large ball. The classical unique continuation property for second-order elliptic operators with locally bounded zero-order terms [12] forces $\Phi \equiv 0$ on the connected domain $\mathbb{R}^3$, hence $h \equiv 0$.

The same unique-continuation obstruction applies whether or not the closedness condition holds, and whether or not μ is a single-valued function of u: the linear equation uses the frozen coefficient $\mu(x)$. A failure of single-valuedness of μ as a function of u is therefore not a loophole yielding a compact example. No other compact example is constructed.

The kinematic remark stands, with the closedness condition now read as for the repaired coefficient μ, together with the sharpness bound of Theorem . A general compact profile remains kinematic. Unique continuation is used here as a classical elliptic fact, not as a companion premise, and not as a substitute for an Einstein–vortex

existence theorem.

Proposition (Universal form of switch closedness, and honest Schwartz classes). *In the setting of Theorem , let* $V_\Phi(u, s_1, s_2)$ *be an arbitrary smooth potential of* $u = |\Phi|^2$ *and of the two-switch coordinates* $(s_1, s_2) = (r^2, z^2)$*, and write* $\mu := 2\partial_u V_\Phi$*. Along a two-switch profile define* $\widetilde{W}(s_1, s_2) := V_\Phi(u(s), s_1, s_2)$*. The chain rule gives the identity*

$$(\partial_{s_i} V_\Phi)|_u \, \mathrm{d}s_i = \mathrm{d}\widetilde{W} - \tfrac{1}{2}\mu \, \mathrm{d}u.$$

The exterior derivative of the switch one-form on the left is $-\frac{1}{2}\mathrm{d}\mu \wedge \mathrm{d}u$*. Consequently the closedness condition* $\mathrm{d}\mu \wedge \mathrm{d}u = 0$*, equivalently , is the integrability condition for a switch potential* $W(s_1, s_2)$ *for every smooth* $V_\Phi(u, s_1, s_2)$*, not an artefact of the linear representative* $V_\Phi = \frac{1}{2}\mu|\Phi|^2$*. Closedness is not automatic for an arbitrary smooth potential; it remains an extra hypothesis, now shown to be universal in form. For the positive Schwartz amplitudes* $h = c_0 \exp(-\alpha r^2 - \beta z^2)$ *with* $c_0 > 0$*,* $\alpha > 0$*, and* $\beta \geq 0$*, the amplitude never vanishes, and*

$$\mu = -8\alpha - 2\beta + 4\alpha^2 r^2 + 4\beta^2 z^2$$

is a globally smooth affine function of (r^2, z^2)*. This profile inhabits the smooth-μ class of Theorem . It does not refute Theorem , because h is not compactly supported, and unique continuation does not force* $\Phi \equiv 0$*, because* Φ *vanishes on no nonempty open set. Already for this affine μ and* $u = r^2 h^2$*, the two-form* $\mathrm{d}\mu \wedge \mathrm{d}u$ *fails to vanish identically unless* $\beta = 0$*. The clean Schwartz realisation is therefore the z-independent Gaussian collar of Theorem (2), with* $\mu = -8\alpha + 4\alpha^2 r^2$*, not a compact two-switch example. If the ordinary and host metrics are built from this z -independent Gaussian in the official vortex convention of , namely* $\omega = \sqrt{B_0}\, J[\Phi]$ *(extended by the same formula, now with noncompact support) and* $g = -(\mathrm{d}t + \omega)^2 + \mathrm{d}\mathbf{x}^2$*,* $\widehat{g} = g + B_0 J \otimes J$*, then both metrics approach Minkowski exponentially fast at spatial infinity. A potential with* $V_\Phi \sim |\Phi|^{2p}$ *for* $\frac{1}{2} < p < 1$ *can make* $\partial V_\Phi / \partial \Phi^*$ *fail to be locally Lipschitz at* $\Phi = 0$*, and can make μ unbounded at a flattening zero. That alternative is already the first branch of Theorem , not a bypass of the theorem, and not an evasion of unique continuation on the locally bounded branch. The vortex ansatz* $\Phi = h(x + \mathrm{i}y)$ *would over-constrain λ against a monomial* $\kappa|\Phi|^{2p-2}$*. No compact dead-core*

on-shell profile is constructed; the dead-core idea remains a programme on the blow-up branch.

Proof. The differential of $\widetilde{W}$ is $\mathrm{d}\widetilde{W} = (\partial_u V_\Phi)|_u\,\mathrm{d}u + (\partial_{s_i} V_\Phi)|_u\,\mathrm{d}s_i$. The definition $\mu = 2\partial_u V_\Phi$ rearranges to . Exterior differentiation, using $\mathrm{d}^2\widetilde{W} = 0$, yields $\mathrm{d}((\partial_{s_i} V_\Phi)|_u\,\mathrm{d}s_i) = -\frac{1}{2}\,\mathrm{d}\mu \wedge \mathrm{d}u$. With $\tilde{T}^{ab} = 0$ and constant stiffnesses f_i, the switch Euler equations of Theorem (1) ask for a potential W whose one-form is $\sum_i (f_i^2 \Delta_{\mathbb{R}^3} s_i)\,\mathrm{d}s_i - (\partial_{s_i} V_\Phi)|_u\,\mathrm{d}s_i$. The first summand is $4f_1^2\,\mathrm{d}s_1 + 2f_2^2\,\mathrm{d}s_2$, which is closed. The second has exterior derivative $-\frac{1}{2}\,\mathrm{d}\mu \wedge \mathrm{d}u$, so W exists locally if and only if $\mathrm{d}\mu \wedge \mathrm{d}u = 0$, which is . For the linear representative $V_\Phi = \frac{1}{2}\mu u$ one recovers the printed source terms $\frac{1}{2}(\partial_{s_i}\mu)u$.

For $H(R, Z) = c_0 \exp(-\alpha R - \beta Z)$ one has $\partial_R H = -\alpha H$, $\partial_R^2 H = \alpha^2 H$, $\partial_Z H = -\beta H$, and $\partial_Z^2 H = \beta^2 H$. Substitution into gives $\lambda = H(-8\alpha - 2\beta + 4\alpha^2 r^2 + 4\beta^2 z^2)$, hence . In particular μ extends smoothly through the axis and to spatial infinity. Writing $s_1 = r^2$, $s_2 = z^2$, one has $\mathrm{d}\mu = 4\alpha^2\,\mathrm{d}s_1 + 4\beta^2\,\mathrm{d}s_2$ and, on $\{s_1 > 0\}$, $\mathrm{d}u = u((1/s_1) - 2\alpha)\,\mathrm{d}s_1 - 2\beta u\,\mathrm{d}s_2$. Therefore

$$\mathrm{d}\mu \wedge \mathrm{d}u = -4\beta u(2\alpha^2 + \beta(\tfrac{1}{s_1} - 2\alpha))\,\mathrm{d}s_1 \wedge \mathrm{d}s_2.$$

The prefactor vanishes identically on the printed image if and only if $\beta = 0$. Thus two-switch bookkeeping is not admitted for a genuinely two-dimensional Gaussian, while the z-independent Gaussian collar $\beta = 0$ falls under Theorem (2), with $\mu = -8\alpha + 4\alpha^2 r^2$ a smooth function of $s = r^2$.

In the official vortex convention, $J[\Phi] = h^2(x\,\mathrm{d}y - y\,\mathrm{d}x)$ and $|J| = h^2 r$. For the z-independent Gaussian, $h^2 r = c_0^2 r \exp(-2\alpha r^2)$ and all Cartesian components of $\omega = \sqrt{B_0}J$, together with their derivatives, are Schwartz functions on $\mathbb{R}^3$. Hence g and $\tilde{g}$ differ from Minkowski by Schwartz tensors, and the difference (together with every derivative) decays faster than any inverse power, in particular exponentially fast.

Finally, if $V_\Phi = (\kappa/(2p))u^p$ with $\frac{1}{2} < p < 1$, then $\mu = \kappa u^{p-1} = \kappa r^{2p-2} h^{2p-2}$ on $\{h \neq 0\}$, which is unbounded as $h \to 0$ at fixed $r > 0$. The map $\Phi \mapsto \partial V_\Phi / \partial \Phi^*$ fails to be locally Lipschitz at the origin, since it scales as $|\Phi|^{2p-1}$ with $0 < 2p - 1 < 1$. This is the first alternative of Theorem . The vortex ansatz would moreover require $\lambda = \kappa r^{2p-2} h^{2p-1}$

, a nonlinear constraint on the linear differential polynomial λ, which is not solved here. No compact example is exhibited.

Why a real scalar gradient is insufficient for this loop

Proposition (Closed-orbit Rolle obstruction and Stokes dichotomy). *Let γ be a smooth closed g-timelike curve and F a globally single-valued real scalar. For finite $B \geq 0$ and positive A, the metric $A^2 g + B\,\mathrm{d}F \otimes \mathrm{d}F$ cannot make every tangent of γ non-timelike: the periodic function $F \circ \gamma$ has a stationary point, at which the corrected norm equals $A^2 g(\dot\gamma, \dot\gamma) < 0$.*

Moreover, suppose $\tilde{g} = A^2 g + B\,\alpha \otimes \alpha$ makes a coordinate t a $\tilde{g}$-temporal function on a stationary collar in which γ is a constant-t closed g-timelike circle, with α a smooth global one-form. Then $\alpha(\dot\gamma) \neq 0$ everywhere on γ, so $\oint_\gamma \alpha \neq 0$. If $\mathrm{d}\alpha = 0$, then $[\alpha] \neq 0$ in $H^1(M;\mathbb{R})$, which is impossible for $M = \mathbb{R}^4$. If $\gamma = \partial\Sigma$ bounds a smooth embedded disk, Stokes' theorem forces $\int_\Sigma \mathrm{d}\alpha = \oint_\gamma \alpha \neq 0$. For the printed current , one has $\mathrm{d}J = 2\,\mathrm{d}X \wedge \mathrm{d}Y$.

Proof. The first claim is Rolle's theorem on $\mathbb{S}^1$. If t is $\tilde{g}$-temporal and $\mathrm{d}t(\dot\gamma) = 0$, then $\dot\gamma$ lies in the spacelike complement of $\nabla^{\tilde{g}} t$, so $\tilde{g}(\dot\gamma, \dot\gamma) > 0$. Combined with $A^2 g(\dot\gamma, \dot\gamma) < 0$ and $B \geq 0$, this forces $\alpha(\dot\gamma) \neq 0$ at every point of the connected circle, hence a strictly signed period. The cohomological and Stokes alternatives are then immediate, and the identity for $\mathrm{d}J$ is the exterior derivative of .

This excludes uniform exclusion of the same loop by one exact real gradient. It does not classify every mechanism, and it does not prove that the complex vortex Φ is the unique minimal distortion among all smooth tensor constructions on $\mathbb{R}^4$. A rank-two sum of exact real gradients may remain non-vanishing along a circle; winding controllers and purely azimuthal currents are other printed mechanisms (Appendix). The smooth J of Theorem is generally not closed, so no non-exact closed one-form on $\mathbb{R}^4$ is required.

Class-limited uniqueness of the vortex shield

Proposition does not classify every shielding mechanism. In particular, a sum of exact real gradients need not

vanish along a circle. The following statement isolates the zero-radial, zero-vertical, and zero time-time distortion class, and claims uniqueness of $J[\Phi]$ only inside that class. It does not assert uniqueness among all smooth tensor shields on $\mathbb{R}^4$. Official winding controllers, and disformal stresses with a nonzero radial, vertical, or time-time component, are not eliminated.

Definition (Zero-distortion azimuthal class). Let Δg_{ab} be a smooth symmetric positive-semidefinite $(0,2)$-tensor on $\mathbb{R}^4$. It belongs to the *zero-distortion azimuthal class* if

\textup{(U1)} $\Delta g(\partial_r, \partial_r) \equiv \Delta g(\partial_z, \partial_z) \equiv \Delta g(\partial_t, \partial_t) \equiv 0$ on $\{r > 0\}$;

\textup{(U2)} at some circle γ_{r_0} of radius $r_0 > 0$ lying in a chronology-violating collar, $\Delta g(\partial_\varphi, \partial_\varphi)|_{\gamma_{r_0}} > 0$.

Lemma (Vanishing of mixed components). *If Δg is positive-semidefinite and $\Delta g(X, X) = 0$ at a point, then $\Delta g(X, Y) = 0$ at that point for every tangent vector Y.*

Proof. For every real λ, $\Delta g(Y + \lambda X, Y + \lambda X) = \Delta g(Y, Y) + 2\lambda \Delta g(X, Y) + \lambda^2 \Delta g(X, X) = \Delta g(Y, Y) + 2\lambda \Delta g(X, Y) \geq 0$. A linear polynomial in λ that is bounded from below must have vanishing slope, so $\Delta g(X, Y) = 0$.

Theorem (Vortex representation in the zero-distortion class). *Let Δg_{ab} belong to the class of Definition . Then, on $\{r > 0\}$,*

$$\Delta g = \Delta g_{\varphi\varphi}\, \mathrm{d}\varphi \otimes \mathrm{d}\varphi,$$

with $\Delta g_{\varphi\varphi} \geq 0$, and every other cylindrical component vanishes. Moreover there is a unique smooth function $\beta_{\mathrm{vort}} \geq 0$ on $\mathbb{R}^4$ such that

$$\Delta g = \beta_{\mathrm{vort}}\, (x\, \mathrm{d}y - y\, \mathrm{d}x) \otimes (x\, \mathrm{d}y - y\, \mathrm{d}x)$$

globally, Δg vanishes to order $O(r^2)$ in Cartesian components at the axis, and Δg has algebraic rank at most one. Writing $\Phi = \beta_{\mathrm{vort}}^{1/4}(x + \mathrm{i}y)$ on $\{\beta_{\mathrm{vort}} > 0\}$, and extending by zero where $\beta_{\mathrm{vort}} = 0$ (or factorising a smooth bump weight into $B(s)$), one has $\Delta g = B\, J[\Phi] \otimes J[\Phi]$ in the printed current convention of . This realisation is unique within the class (U1)–(U2), up to the usual reparametrisation of the pair (B, Φ) that leaves the rank-one tensor $BJ \otimes J$

invariant. It is not unique among all smooth positive-semidefinite tensors on $\mathbb{R}^4$.

Proof. Lemma applied to $X \in \{\partial_t, \partial_r, \partial_z\}$ on $\{r > 0\}$ forces every mixed component involving one of these vectors to vanish. The only surviving cylindrical component is therefore $\Delta g_{\varphi\varphi} \geq 0$, which is strictly positive on γ_{r_0} by (U2). Since $x\,\mathrm{d}y - y\,\mathrm{d}x = r^2\,\mathrm{d}\varphi$, one has

$$\Delta g = \frac{\Delta g_{\varphi\varphi}}{r^4}(x\,\mathrm{d}y - y\,\mathrm{d}x) \otimes (x\,\mathrm{d}y - y\,\mathrm{d}x)$$

on $\{r > 0\}$. The Cartesian components are $\Delta g_{xx} = y^2\beta$, $\Delta g_{yy} = x^2\beta$, $\Delta g_{xy} = -xy\beta$, and $\Delta g_{xx} + \Delta g_{yy} = r^2\beta$, where $\beta := \Delta g_{\varphi\varphi}/r^4$. Smoothness of Δg_{ab} on $\mathbb{R}^4$, including the axis, implies that $\Delta g_{xx} + \Delta g_{yy}$ is smooth and vanishes at $r = 0$. Along the x-axis one has $\Delta g_{yy} = x^2\beta$ and $\Delta g_{xx} = 0$; along the y-axis, $\Delta g_{xx} = y^2\beta$ and $\Delta g_{yy} = 0$. Thus β remains bounded as $r \to 0$, and the standard division argument in two variables applies: a smooth function on $\mathbb{R}^2$ whose restriction to every line through the origin is $O(r^2)$, and whose associated tensor components $x^2\beta$, $y^2\beta$, and $xy\beta$ are smooth, is itself smooth. (If the 2-jet of $r^2\beta$ at the origin contained a linear part or a non-isotropic quadratic part, at least one of $x^2\beta$, $y^2\beta$, $xy\beta$ would fail to be C^2.) Hence $\beta_{\mathrm{vort}} := \beta$, extended by continuity through the axis, is smooth and nonnegative, which is . Algebraic rank at most one is immediate. The current of $\Phi = \beta_{\mathrm{vort}}^{1/4}(x + \mathrm{i}y)$ is $J = \beta_{\mathrm{vort}}^{1/2}(x\,\mathrm{d}y - y\,\mathrm{d}x)$ on $\{\beta_{\mathrm{vort}} > 0\}$, and the claimed factorisation follows. Any other pair (B', Φ') with the same rank-one tensor differs by a pointwise rescaling leaving $BJ \otimes J$ fixed. A shield with a nonzero radial, vertical, or time-time component lies outside the class and is not claimed to be of this form.

Proposition (Radial distortion of exact real-gradient shields). *Let* $\Delta g = \sum_{I=1}^{k} B_I\,\mathrm{d}\psi_I \otimes \mathrm{d}\psi_I$ *with single-valued real scalars* $\psi_I \in C^1(\mathbb{R}^4)$ *and weights* $B_I \geq 0$. *Suppose the weights are strictly positive throughout a closed disk* D_{r_0} *of radius* $r_0 > 0$ *whose boundary circle lies in a chronology-violating collar, and suppose* $\Delta g(\partial_\varphi, \partial_\varphi) > 0$ *at some point of that circle. Then the radial distortion* $\mathscr{E}_{rr} := \Delta g(\partial_r, \partial_r)$ *cannot vanish identically on* D_{r_0}.

Proof. If $\mathscr{E}_{rr} \equiv 0$ on the disk, then $\sum_I B_I(\partial_r\psi_I)^2 \equiv 0$, hence $\partial_r\psi_I \equiv 0$ on D_{r_0} because each $B_I > 0$. Each single-valued C^1 function is therefore constant along radii. At the origin it is independent of angle, so each ψ_I is constant on the

disk, $\partial_\varphi \psi_I \equiv 0$ on the boundary, and $\Delta g(\partial_\varphi, \partial_\varphi) \equiv 0$, a contradiction.

The same conclusion holds, with the same proof, if each $\{B_I > 0\}$ merely contains a radial segment from the origin to the boundary. No quantitative Fourier constant, no Sobolev cutoff, and no axis birefringence "even where $g = \eta$ " is typestated. A complete linearised gravitational or optical calculation of a monopole defect is not written, and is not claimed.

Gödel-type source comparison with explicit winding residual

For a local comparison, let $\beta > 0$, set

$$D(x) = \frac{e^{\beta x}}{\beta}, \qquad e^0 = \mathrm{d}t + \sqrt{2}D(x)\mathrm{d}y, \quad e^1 = \mathrm{d}x, \quad e^2 = D(x)\mathrm{d}y, \quad e^3 = \mathrm{d}z,$$

and prescribe $g = -(e^0)^2 + (e^1)^2 + (e^2)^2 + (e^3)^2$. The coordinate y may be periodic for a winding comparison.

Proposition (Locally recomputed dust match and azimuthal residual). *In the displayed orthonormal coframe,*

$$G_{\hat{a}\hat{b}} = \frac{\beta^2}{2}\mathrm{diag}\,(1, 1, 1, 1).$$

Thus, with $u = \partial_t$,

$$M_{\mathrm{P}}^2(G + \Lambda g) = \rho_{\mathrm{d}} u^{\mathrm{flat}} \otimes u^{\mathrm{flat}}, \qquad \rho_{\mathrm{d}} = M_{\mathrm{P}}^2\beta^2, \qquad \Lambda = -\frac{\beta^2}{2}.$$

If canonical winding $q = \kappa \mathrm{d}y$ *is adjoined without changing the metric, dust, or* Λ*, then*

$$T_{\mathrm{res}} = -T_q, \qquad T_q{}_{\hat{2}\hat{2}} - T_q{}_{\hat{1}\hat{1}} = f_q^2\frac{\kappa^2}{D^2} > 0.$$

No adjustment of only comoving dust density and Λ *removes this anisotropic mismatch on this metric.*

Proof. Put $\Omega = \beta/\sqrt{2}$ and $m = \beta$. Then

$$\mathrm{d}e^0 = 2\Omega e^1 \wedge e^2, \qquad \mathrm{d}e^2 = m e^1 \wedge e^2.$$

The nonzero connection forms can be taken as

$$\omega^{0}{}_{1} = \Omega e^{2}, \qquad \omega^{0}{}_{2} = -\Omega e^{1}, \qquad \omega^{1}{}_{2} = \Omega e^{0} - m e^{2},$$

with the remaining forms fixed by metric antisymmetry. Cartan's curvature equation gives

$$\mathscr{R}^{0}{}_{1} = -\Omega^{2} e^{0} \wedge e^{1}, \qquad \mathscr{R}^{0}{}_{2} = -\Omega^{2} e^{0} \wedge e^{2}, \qquad \mathscr{R}^{1}{}_{2} = (3\Omega^{2} - m^{2}) e^{1} \wedge e^{2}.$$

Contraction yields $R_{\hat{0}\hat{0}} = 2\Omega^{2} = \beta^{2}$, $R_{\hat{1}\hat{1}} = R_{\hat{2}\hat{2}} = 2\Omega^{2} - m^{2} = 0$, $R_{\hat{3}\hat{3}} = 0$, and $R = -\beta^{2}$. The Einstein tensor and dust match follow.

Write $q = Qe^{2}$, where $Q = \kappa/D$. Canonical stress is

$$T_{q}{}^{\hat{a}\hat{b}} = f_{q}^{2}\Big(q^{\hat{a}} q^{\hat{b}} - \tfrac{1}{2} Q^{2} \eta^{\hat{a}\hat{b}} \Big).$$

Its 22 − 11 difference is $f_{q}^{2} Q^{2}$. The same difference vanishes for the geometric left side, the cosmological term, and comoving dust. Therefore the mismatch cannot be cancelled within that source class.

This credits the classical Gödel solution [9] and the companion residual comparisons [5, 6] without transferring their proofs. The periodic y-circles here are not geodesics: $g_{yy} = -D^{2}$ has nonzero x-gradient. The closed-geodesic witness remains the cylinder or the toroid.

W-FLRW: positive residual density and active exchange

Choose a separate conformal instance of :

$$\Phi = 0, \quad s = s_{0}, \quad A_{\eta}(\varphi) = e^{\eta\varphi/M_{\mathrm{P}}}, \quad V(\varphi) = V_{0} + \frac{m_{\varphi}^{2}}{2}\varphi^{2}, \quad m_{\varphi}^{2} \geq 0,$$

with $U(0) = W(s_{0}) = 0$, $W'(s_{0}) = 0$, and stationary s-portal derivatives. Let $g = -\mathrm{d}t^{2} + a(t)^{2}\mathrm{d}\mathbf{x}^{2}$. Ordinary dust, dark dust, and radiation have host-frame densities $\rho_{\mathrm{o}}, \rho_{\mathrm{d}}, \rho_{\mathrm{r}}$. For ordinary dust, $\rho_{\mathrm{o}} = A^{4}\hat{\rho}_{\mathrm{o}}$.

The supplied host-compatibility system is

$$\dot{a} = Ha, \qquad \dot{\varphi} = v,$$

$$\dot{H} = -\frac{\rho_{\mathrm{o}} + \rho_{\mathrm{d}} + \frac{4}{3}\rho_{\mathrm{r}} + v^2}{2M_{\mathrm{P}}^2},$$

$$\dot{v} = -3Hv - m_\varphi^2\varphi - \frac{\eta}{M_{\mathrm{P}}}\rho_{\mathrm{o}},$$

$$\dot{\rho}_{\mathrm{o}} = \left(-3H + \frac{\eta v}{M_{\mathrm{P}}}\right)\rho_{\mathrm{o}},$$

$$\dot{\rho}_{\mathrm{d}} = -3H\rho_{\mathrm{d}}, \qquad \dot{\rho}_{\mathrm{r}} = -4H\rho_{\mathrm{r}},$$

with constraint

$$3M_{\mathrm{P}}^2H^2 = M_{\mathrm{P}}^2\Lambda + V_0 + \rho_{\mathrm{o}} + \rho_{\mathrm{d}} + \rho_{\mathrm{r}} + \frac{1}{2}v^2 + \frac{1}{2}m_\varphi^2\varphi^2.$$

Theorem (Local positive-density host-compatible exchange witness). *For initial data with* $a_0 > 0$*, positive* $\rho_{\mathrm{o},0}, \rho_{\mathrm{d},0}, \rho_{\mathrm{r},0}$*, and a real positive* H_0 *satisfying , system has a unique smooth solution on some common neighbourhood of the initial time. The constraint is preserved, and*

$$M_{\mathrm{P}}^2(G + \Lambda g) - T_{\mathrm{o}} - T_{\mathrm{r}} - T_\varphi = \rho_{\mathrm{d}} n^{\mathrm{flat}} \otimes n^{\mathrm{flat}}, \qquad n = \partial_t, \qquad \rho_{\mathrm{d}} > 0.$$

If $\eta v_0 > 0$*, ordinary–controller exchange is active initially. This witness is globally hyperbolic on its time interval and has no CTC.*

Proof. The right side of is smooth. Choose a closed state-space ball about the initial data, inside $a > 0$, with vector-field bound C and Lipschitz constant L. On continuous curves in that ball, the integral map

$$y(t) \mathrm{longmapsto} y_0 + \int_{t_0}^{t} F(y(u))\,\mathrm{d}u$$

maps the ball into itself when TC is smaller than its radius and is a contraction when $TL < 1$. Iteration yields a unique solution; differentiation and the smooth vector field give smoothness.

The density equations integrate to

$$\rho_{\mathrm{d}} = \rho_{\mathrm{d},0}(a_0/a)^3, \qquad \rho_{\mathrm{r}} = \rho_{\mathrm{r},0}(a_0/a)^4, \qquad \rho_{\mathrm{o}} = \rho_{\mathrm{o},0}(a_0/a)^3 e^{\eta(\varphi - \varphi_0)/M_{\mathrm{P}}},$$

so positivity persists. Differentiate the difference of the two sides of . Substitution of cancels the scalar–ordinary

exchange terms and all expansion terms, giving zero. The 00 and spatial Einstein equations are then precisely the preserved constraint and the H equation. The scalar equation is the Euler equation from $S_\varphi + S_{\mathrm{o}}$.

The ordinary energy source is $\alpha\rho_{\mathrm{o}}$, with $\alpha = \eta v/M_{\mathrm{P}}$; the scalar energy source is its negative. Finally, on every compact time subinterval, a and A are bounded above and below by positive constants. The usual finite-speed endpoint argument on complete spatial $\mathbb{R}^3$ makes the constant-time slices Cauchy for both metrics.

This is a local witness of the type discussed in [4, 3]. The Einstein equation is *supplied and checked*, not derived.

Quadratic cosmological decoupling

Theorem (Finite-interval $O(\eta^2)$ approach to a specified baseline). *Fix baseline densities, a_0, and $\Lambda_{\mathrm{eff}} = \Lambda + V_0/M_{\mathrm{P}}^2$. Set*

$$\varphi_\eta(t_0) = \eta p_0, \qquad v_\eta(t_0) = \eta q_0,$$

and choose $H_\eta(t_0) > 0$ from . On a sufficiently short interval independent of small η,

$$\|\varphi_\eta\| + \|v_\eta\| = O(|\eta|),$$

and

$$\|a_\eta - a_0^{\mathrm{base}}\| + \|H_\eta - H_0^{\mathrm{base}}\| + \sum_{i\,=\,\mathrm{o,\,d,\,r}} \|\rho_{i,\eta} - \rho_{i,0}^{\mathrm{base}}\| + \|A_\eta - 1\| = O(\eta^2).$$

The baseline is flat with ordinary dust and radiation. If $p_0 q_0 > 0$, then for $\eta \neq 0$ the initial clock factor is nontrivial and $\alpha(t_0) > 0$.

Proof. The contraction construction can be made uniform for parameters in a compact neighbourhood of 0. The scalar subsystem is linear in (φ, v) with bounded coefficients and forcing $O(|\eta|)$. Its integral inequality gives $\|(\varphi, v)\| \le C|\eta|e^{CT}$. Therefore $\eta v/M_{\mathrm{P}} = O(\eta^2)$, $v^2 + m_\varphi^2\varphi^2 = O(\eta^2)$, and $A_\eta - 1 = O(\eta^2)$. The initial Friedmann root differs from the baseline root by $O(\eta^2)$. Subtracting the remaining ODEs from their baseline equations produces a Lipschitz difference inequality with $O(\eta^2)$ forcing. Integration gives . Initially, $A_\eta = e^{\eta^2 p_0/M_{\mathrm{P}}}$ and $\alpha = \eta^2 q_0/M_{\mathrm{P}}$,

proving the last assertion.

Proposition (What the decoupling theorem does and does not cover). *At $A = 1$, $J = 0$, and controllers at stationary values, the action reduces to minimally coupled ordinary matter, the stipulated dark matter action, and a constant vacuum label. Thus a supplied cold-dark-dust and ordinary hydrodynamic class reduces exactly to its GR+ comparison model.*

On a fixed finite interval, suppose an additional declared linear observable system is

$$\dot{Z}_\eta = \mathscr{M}_\eta(t) Z_\eta + f_\eta(t),$$

with uniformly bounded matrices and

$$\|\mathscr{M}_\eta - \mathscr{M}_0\| + \|f_\eta - f_0\| + |Z_\eta(t_0) - Z_0(t_0)| = O(\eta^2).$$

Then $Z_\eta - Z_0 = O(\eta^2)$ on that interval. For a fixed acoustic integration window, if

$$R_b = \frac{3\rho_{\mathrm{o}}}{4\rho_{\mathrm{r}}}, \qquad c_s^2 = \frac{1}{3(1+R_b)}, \qquad r_s = \int \frac{c_s}{a}\,\mathrm{d}t,$$

then $r_{s,\eta} - r_{s,0} = O(\eta^2)$.

Proof. Substitution gives the exact action reduction. For the linear system, the difference satisfies an integral inequality with bounded homogeneous coefficient and $O(\eta^2)$ forcing; its exponential majorant proves the claim. For the acoustic functional, positivity and boundedness of the densities on the fixed window make c_s/a a smooth function of the variables in . Integrating its $O(\eta^2)$ difference proves the final assertion.

The proposition does not construct an Einstein–Boltzmann hierarchy, a recombination model, or a nonlinear hydrodynamic convergence theorem. In particular, moving last-scattering endpoints, damping, primordial scalar modes, and data likelihoods remain additional inputs. At exact decoupling the standard late-time dust growth equation is

$$\ddot{D} + 2H\dot{D} - \frac{\rho_{\mathrm{o}} + \rho_{\mathrm{d}}}{2M_{\mathrm{P}}^2} D = 0;$$

a nonzero-coupling growth prediction requires its own perturbation equations. The next proposition supplies the clamped identity and a named quasistatic unclamped remainder on the printed conformal background; it is not an observational fit and not a derivation.

Proposition (Clamped relative contrast and quasistatic unclamped remainder). *On the printed conformal FLRW class of Section , write* $\alpha := \eta v/M_{\mathrm{p}} = \partial_t \log A$ *and* $\alpha_\varphi := \eta/M_{\mathrm{p}} = \partial_\varphi \log A$*. Work in a common weak-field spatial potential* Ψ *on flat slices, with pressureless number-density contrasts* $\delta_{\mathrm{o}}, \delta_{\mathrm{d}}$ *and peculiar velocities obeying the linearised continuity equations* $\dot{\delta}_i = -a^{-1}\nabla \cdot v_i$*.*

1. *Clamped controller. If* $\delta\varphi = 0$*, and if the linearised Euler equations are*

$$\dot{v}_{\mathrm{o}} + (H + \alpha)v_{\mathrm{o}} = -a^{-1}\nabla\Psi, \qquad \dot{v}_{\mathrm{d}} + Hv_{\mathrm{d}} = -a^{-1}\nabla\Psi,$$

then $D := \delta_{\mathrm{o}} - \delta_{\mathrm{d}}$ *satisfies*

$$\frac{\mathrm{d}}{\mathrm{d}t}(a^2\dot{D}) = -a^2\alpha\dot{\delta}_{\mathrm{o}}.$$

2. *Unclamped quasistatic remainder. If* $\delta\varphi$ *is retained, the ordinary Euler equation acquires the additional fifth-force term* $-a^{-1}\nabla(\alpha_\varphi \delta\varphi)$*, while the dark Euler equation is unchanged at this order. Linearising the printed scalar equation* $\mathrm{Box}_g\,\varphi - m_\varphi^2\varphi + (\eta/M_{\mathrm{p}})T_{\mathrm{o}} = 0$ *about the homogeneous background yields, on each Fourier mode, the forced oscillator*

$$\ddot{y} + 3H\dot{y} + \omega^2(t)\, y = S(t),$$

where $y = \delta\varphi_k$*,* $\omega^2(t) := k^2/a^2 + m_{\mathrm{eff}}^2$*,* m_{eff}^2 *is the background second derivative of the printed effective potential, and* S *contains the leading ordinary-dust source* $(\eta/M_{\mathrm{p}})\bar{\rho}_{\mathrm{o}}\delta_{\mathrm{o}}$ *together with linearised trace-coupling corrections. Write* $y = y_{\mathrm{p}} + y_{\mathrm{h}}$*, with particular solution* $y_{\mathrm{p}} := S/\omega^2$ *and* y_{h} *the associated homogeneous remainder. Under the stated smoothness and subhorizon slow-variation hypotheses that, on a declared time interval,* $|\partial_t^n S| \le C_S H^n |S|$ *and* $|\partial_t^n \omega| \le C_\omega H^n \omega$ *for* $n = 1, 2$*, together with* $\omega^2 \ge k^2/a^2$*, the slow-forcing defect satisfies* $|\ddot{y}_{\mathrm{p}} + 3H\dot{y}_{\mathrm{p}}| \le C(aH/k)^2|S|$*. The*

homogeneous free-wave remainder y_{h} *is not controlled by* $(aH/k)^2$*; it is controlled only under a stated initial-data bound, or under an adiabatic turn-on hypothesis that S and* $\dot{y}_{\mathrm{h}}$ *vanish in the far past, not by chronology. When both remainders are so controlled, one obtains*

$$(k^2/a^2 + m_{\mathrm{eff}}^2)\delta\varphi = -\frac{\eta}{M_{\mathrm{p}}}\bar{\rho}_{\mathrm{o}}\delta_{\mathrm{o}} + R_{\mathrm{QS}},$$

with $R_{\mathrm{QS}} = R_{\mathrm{slow}} + R_{\mathrm{hom}} + R_{\mathrm{trace}}$*, where* R_{slow} *is of order* $(aH/k)^2$ *times the leading source,* $R_{\mathrm{hom}} = \omega^2 y_{\mathrm{h}}$ *is the homogeneous contribution, and* R_{trace} *collects the linearised trace-coupling corrections not retained in the leading dust source. The named mode-by-mode bound*

$$|\partial_t^2\delta\varphi| + H|\partial_t\delta\varphi| \le \varepsilon_{\mathrm{QS}}(k^2/a^2 + m_{\mathrm{eff}}^2)|\delta\varphi|$$

holds for the slow-forced particular solution under the hypotheses above, with $\varepsilon_{\mathrm{QS}} = O((aH/k)^2)$*; it is not an unconditional property of every solution of , and it is not typestated as a universal replacement of* $\varepsilon_{\mathrm{QS}}$ *that silently drops homogeneous waves. In particular, the brief prefactor commonly written as a closed* $Y_k(a)$ *multiplier of* $8\pi G\bar{\rho}_{\mathrm{o}}\delta_{\mathrm{o}}$ *is not typestated as an exact identity of the printed system. Contract therefore tests both the clamped Hubble-drag identity of item 1 and this named two-step remainder, on a declared quasistatic domain including the stated homogeneous-wave hypothesis.*

Proof. (a) Continuity and the clamped Euler equations give $\ddot{\delta}_{\mathrm{o}} + (2H + \alpha)\dot{\delta}_{\mathrm{o}} = a^{-2}\Delta\Psi$ and $\ddot{\delta}_{\mathrm{d}} + 2H\dot{\delta}_{\mathrm{d}} = a^{-2}\Delta\Psi$. Subtracting and multiplying by a^2 cancels the common potential and yields the displayed identity.

(b) The additional ordinary force term follows from the orthogonal conformal force of Theorem upon linearizing $\log A$ at fixed host metric. Dark test dust couples to g, not to A. For the scalar, the background equation is the v -equation of . At linear order, the spatial Laplacian contribution to $\mathrm{Box}_g\,\delta\varphi$ is $a^{-2}\Delta\delta\varphi$, and the mode equation is . For the particular solution $y_{\mathrm{p}} = S/\omega^2$, direct differentiation gives

$$\dot{y}_{\mathrm{p}} = \frac{\dot{S}}{\omega^2} - \frac{2S\dot{\omega}}{\omega^3},$$

and a further derivative produces $\ddot{y}_{\mathrm{p}}$ as a linear combination of $\ddot{S}/\omega^2$, $\dot{S}\dot{\omega}/\omega^3$, $S\ddot{\omega}/\omega^3$, and $S\dot{\omega}^2/\omega^4$. Under the stated bounds $|\partial_t^n S| \le C_S H^n |S|$ and $|\partial_t^n \omega| \le C_\omega H^n \omega$ for $n = 1, 2$, each term in $\ddot{y}_{\mathrm{p}} + 3H\dot{y}_{\mathrm{p}}$ is $O(H^2/\omega^2)|S|$. The subhorizon comparison $\omega^2 \ge k^2/a^2$ upgrades this to $O((aH/k)^2)|S|$, which is the slow-forcing defect. The difference $y_{\mathrm{h}} = y - y_{\mathrm{p}}$ solves the homogeneous equation with a forcing equal to minus that defect; the associated homogeneous free data at a fixed initial time (or the absence of such data under an adiabatic turn-on in the far past) are additional hypotheses, not consequences of Assumption . The Helmholtz form follows by rearranging , and the remainder splits as displayed. Algebraic elimination of $\delta\varphi$ therefore holds only up to that named two-step remainder, and no closed Y_k prefactor is asserted.

Debugged fixed-host and correlated limits

Proposition (The fixed-host threshold is not signature degeneracy). *In , decrease $b = B\kappa^2$ at fixed a, h. At $b = h$, angular circles become closed null curves and ordinary global hyperbolicity fails. For $0 \le b < h$, those circles are timelike. Nevertheless, the metric remains Lorentzian and nondegenerate for every $b \ge 0$, since*

$$\det(\tilde{g}|_{t,\theta}) = -(a^2 + b - h) < 0.$$

Thus controller removal on this fixed CTC host is not a globally hyperbolic limit.

Proof. The angular squared norm is $H = b - h$. At $H = 0$, ∂_θ is future null because $\tilde{g}(\partial_\theta, \partial_t) = -a < 0$. For $H < 0$ it is future timelike. The determinant formula and $a^2 > h$ prove the signature claim.

This corrects the proposed assertion of a Lorentzian signature degeneracy at $B\kappa^2 = h$. The degeneracy is that of the induced t-slice metric and the temporal character of dt, not of the spacetime metric.

Theorem (Correlated geometric removal with C^∞ parameter dependence). *For the compactly supported ω, Φ of Theorem , let $0 \le \varepsilon \le 1$, hold Φ fixed, and set*

$$g_\varepsilon = -(\mathrm{d}t + \varepsilon\omega)^2 + \mathrm{d}\mathbf{x}^2, \qquad B_\varepsilon = \varepsilon^2 B_0.$$

Then $J[\Phi]$ is independent of ε, the identity $B_0 J \otimes J = \omega \otimes \omega$ of Theorem gives $B_\varepsilon J \otimes J = \varepsilon^2 \omega \otimes \omega$, and

$$\tilde{g}_\varepsilon = g_\varepsilon + B_\varepsilon J \otimes J = -\mathrm{d}t^2 - 2\varepsilon\, \mathrm{d}t\, \omega + \mathrm{d}\mathbf{x}^2.$$

The fields Φ and B_ε and the metrics $g_\varepsilon, \tilde{g}_\varepsilon$ are C^∞ in $\varepsilon \in [0,1]$, including at $\varepsilon = 0$. Every $\tilde{g}_\varepsilon$ is globally hyperbolic. Both metrics converge smoothly to Minkowski as $\varepsilon \to 0$. If $\varepsilon \|\omega\|_\infty < 1$, then g_ε is also globally hyperbolic. At $\varepsilon = 1$ the construction coincides with Theorem , so g_1 has an open tube of closed future-directed timelike geodesics on $\mathscr{U}$. For $0 < \varepsilon < 1$, the same angular circles remain g_ε-timelike on the (possibly empty) subset of $\mathscr{U}$ on which $r^2 < \varepsilon^2 h/(1-\varepsilon^2)$; in particular there is a threshold $\varepsilon_* \in (0,1)$, depending only on (r_1, r_2, h), such that the full open tube $\mathscr{U}$ remains loop-traversing for every $\varepsilon \in (\varepsilon_*, 1]$. Closure on all of $\mathscr{U}$ is not asserted for every $\varepsilon > 0$. A linear scaling $B_\varepsilon = \varepsilon B_0$ does not cancel the quadratic term $\varepsilon^2 \omega \otimes \omega$ in g_ε, and is not used.

Proof. Expanding the squared coframe gives $g_\varepsilon = -\mathrm{d}t^2 - 2\varepsilon\, \mathrm{d}t\, \omega - \varepsilon^2 \omega \otimes \omega + \mathrm{d}\mathbf{x}^2$. The disformal term that cancels $-\varepsilon^2 \omega \otimes \omega$ must itself be of order ε^2. Holding Φ fixed keeps J fixed, so that choice is $B_\varepsilon = \varepsilon^2 B_0$, and follows from $B_0 J \otimes J = \omega \otimes \omega$. Each displayed coefficient is a polynomial in ε with smooth spatially compact coefficients, hence C^∞ on $[0,1]$, including at zero. (By contrast, the alternative path $\Phi_\varepsilon = \sqrt{\varepsilon}\, \Phi$ with fixed B_0 produces the same metrics, but Φ_ε fails to be differentiable at $\varepsilon = 0$, and is not used here.) The proof of ordinary Cauchy slices in Theorem applies uniformly to .

For g_ε, the induced slice metric is $\delta - \varepsilon^2 \omega \otimes \omega$. Under the stated strict bound it is uniformly positive definite, and $g_\varepsilon^{tt} = -1 + \varepsilon^2 |\omega|^2 < 0$. The causal coordinate-speed bound is then finite and uniform, so the endpoint argument again proves Cauchy slices.

On $\mathscr{U}$, $\omega = \sqrt{r^2 + h}\, \mathrm{d}\theta$, hence

$$g_\varepsilon(\partial_\theta, \partial_\theta) = r^2 - \varepsilon^2 (r^2 + h) = r^2 (1 - \varepsilon^2) - \varepsilon^2 h.$$

At $\varepsilon = 1$ this equals $-h < 0$ throughout $\mathscr{U}$. For $\varepsilon \in (0,1)$ it is negative precisely when $r^2 < \varepsilon^2 h/(1-\varepsilon^2)$. Since $r \le r_2$ on $\mathscr{U}$, the full tube is retained once $\varepsilon^2 h/(1-\varepsilon^2) > r_2^2$, which determines ε_*.

The canonical vortex stress along this path is not claimed to have quadratic decoupling or to satisfy an Einstein match. The C^{∞} path and the open tube at $\varepsilon = 1$ are obtained together; an open tube for every $\varepsilon \in (0, 1]$ is not obtained, because decoupling the host requires scaling ω down.

For a prescribed FLRW exterior one may similarly replace $d\mathbf{x}^2$ by $a_{\text{base}}(t)^2 d\mathbf{x}^2$. The ordinary metric retains Cauchy slices on the prescribed time interval and tends to that FLRW metric. This substitution does not retain the stationary closed-geodesic proof inside the core.

Architecture boundary. W-CTC proves geometric splitting, including an action-compatible circle-controller test instance. W-FLRW proves positive Einstein residual density and active exchange. They demonstrate nonempty *separate* classes. They do not prove a common global Einstein–controller solution, a cosmological embedding of the CTC core, stability, or access from a shared causal past. Such an amalgamation remains open.

Sagnac fibres and five calibrated rejection contracts

An exact ring observation map

For this subsection only, fix independently calibrated ring coordinates with angular period 2π, stationary detector proper time t, and conformal factor $A = 1$. Consider

$$g_{\text{ring}} = -dt^2 - 2a\, dt\, d\theta + c\, d\theta^2, \qquad \tilde{g}_{\text{ring}} = -dt^2 - 2a\, dt\, d\theta + H\, d\theta^2,$$

where

$$a > 0, \qquad c > -a^2, \qquad b \geq 0, \qquad H = c + b > 0.$$

The cylindrical CTC case has $c = -h < 0$. Guided ring arrivals are null routes on the stipulated ring interface; a guide and its forces are part of that interface. They are not automatically free bulk null geodesics in an arbitrary rotating host.

Theorem (Ordinary Sagnac reconstruction and chronology fibre). *Let $D_o = \sqrt{a^2 + H}$. The two ordinary future return times are*

$$t_+^{(o)} = 2\pi(D_o - a), \qquad t_-^{(o)} = 2\pi(D_o + a).$$

They reconstruct

$$a = \frac{t_-^{(o)} - t_+^{(o)}}{4\pi}, \qquad H = \frac{t_+^{(o)} t_-^{(o)}}{4\pi^2},$$

but not c or b separately. At every such (a, H)*, the fibre*

$$c = H - b, \qquad 0 \le b < H + a^2$$

contains both $c > 0$ *and* $c < 0$ *completions. No ordinary-only decision rule uniformly identifies their chronology sign; for equal-prior paired completions its worst-case error is at least* 1/2.

Proof. Null slopes satisfy $(\mathrm{d}t/\mathrm{d}\theta)^2 + 2a(\mathrm{d}t/\mathrm{d}\theta) - H = 0$. The future branches give . Difference and product yield . The stated fibre follows from $c > -a^2$. Choosing $b < H$ gives $c > 0$; choosing $H < b < H + a^2$ gives $c < 0$. Both choices have identical ordinary data. If a randomized rule outputs "CTC" with probability p on those data, its two errors are p and $1 - p$; one is at least 1/2.

The non-identification statement is about this specified observation map, not every possible ordinary experiment.

Theorem (One host-null datum and the one-sided CTC window). *Put* $d_g = \sqrt{a^2 + c} > 0$*. The future host-null ring returns are*

$$t_+^{(g)} = 2\pi(d_g - a), \qquad t_-^{(g)} = 2\pi(d_g + a).$$

For $c < 0$*,*

$$t_+^{(g)} < 0, \qquad 2\pi a < t_-^{(g)} < 4\pi a.$$

Thus a future co-rotating return occurs at negative coordinate elapsed time; it is not absent merely because no positive-time arrival exists.

Given ordinary times and one independently identified datum $\tau_g = t_-^{(g)}$*, one reconstructs*

$$d_g = \frac{\tau_g}{2\pi} - a, \qquad c = d_g^2 - a^2, \qquad b = H - c.$$

The forward Jacobian satisfies

$$\det \frac{\partial(t_+^{(o)}, t_-^{(o)}, t_-^{(g)})}{\partial(a, c, b)} = \frac{4\pi^3}{D_o d_g} \neq 0.$$

Proof. The host null slopes are $-a \pm d_g$. With increasing θ, the + root has $\dot{t} + a\dot{\theta} = d_g\dot{\theta} > 0$ and is future. With decreasing θ, the other root is future. Integration gives . For $c < 0$, $0 < d_g < a$, proving . Solving the counter-rotating equation gives .

The change $(a, c, b) \mapsto (a, H, d_g)$ has determinant $-1/(2d_g)$. The change $(a, H, d_g) \mapsto (t_+^{(o)}, t_-^{(o)}, t_-^{(g)})$ has determinant $-8\pi^3/D_o$. Their product is .

A named host-null channel is an additional physical interface, not an ordinary photon secretly relabelled as a host probe.

Corollary (Factorisation remains unidentified). *If $b = B\kappa^2$, ordinary plus host-null times identify b but leave the continuum*

$$(B, \kappa)\text{longmapsto}\,(B/\lambda^2, \lambda\kappa), \qquad \lambda > 0$$

unidentified. An independent winding datum $\kappa > 0$ gives a full four-variable local inverse, with determinant

$$\det \frac{\partial(t_+^{(o)}, t_-^{(o)}, t_-^{(g)}, \kappa)}{\partial(a, c, B, \kappa)} = \frac{4\pi^3\kappa^2}{D_o d_g} \neq 0.$$

Proof. The scaling preserves $B\kappa^2$. The last determinant multiplies by the Jacobian κ^2 of $b = B\kappa^2$ at fixed κ.

Proposition (Winding kinetic-scalar calibration of the four-variable inverse). *On the printed ring , let $q = \kappa\,\mathrm{d}\theta$ and write*

$$\rho_q := \frac{1}{2} f_q^2 g^{ab} q_a q_b = \frac{1}{2} f_q^2 \frac{\kappa^2}{a^2 + c} = \frac{1}{2} f_q^2 \frac{\kappa^2}{d_g^2}$$

for the printed kinetic scalar of the circle-valued controller, at fixed stiffness $f_q > 0$. *This is not an ordinary photon observable, and it is not typestated as a particular observer's energy density beyond the printed contraction. On the domain* $D_o > 0$, $d_g > 0$, $\kappa > 0$, $a > 0$, $c > -a^2$, $b = B\kappa^2 \geq 0$, *and* $H = c + b > 0$,

$$\det \frac{\partial(t_+^{(o)}, t_-^{(o)}, t_-^{(g)}, \rho_q)}{\partial(a, c, b, \kappa)} = \frac{4\pi^3 f_q^2 \kappa}{D_o d_g^3} \neq 0,$$

while in the factorisation coordinates (a, c, B, κ),

$$\det \frac{\partial(t_+^{(o)}, t_-^{(o)}, t_-^{(g)}, \rho_q)}{\partial(a, c, B, \kappa)} = \frac{4\pi^3 f_q^2 \kappa^3}{D_o d_g^3} \neq 0.$$

Thus, with f_q *held fixed, the four-tuple* $(t_+^{(o)}, t_-^{(o)}, t_-^{(g)}, \rho_q)$ *is a local inverse for* (a, c, B, κ). *Ordinary-only data remain fibre-blind by Theorem . The neighbouring winding-inclusive inverse of official Dual-Ledger-Chrono-DM-A version 4 is a source record only [5]; it is neither imported as a premise nor discharged or replaced here.*

Proof. The (t, θ) block of has determinant $-(a^2 + c)$, hence $g^{\theta\theta} = 1/(a^2 + c)$ and $g^{ab} q_a q_b = \kappa^2/(a^2 + c)$, which is the displayed formula for ρ_q. The three times depend on (a, c, b) but not on κ, while ρ_q depends on (a, c, κ) but not on b. The Jacobian matrix in coordinates (a, c, b, κ) is therefore block triangular, with upper 3×3 block and lower-right entry $\partial\rho_q/\partial\kappa = f_q^2 \kappa / d_g^2$. The product is . The change of parameters $b = B\kappa^2$ at fixed (a, c, κ) has Jacobian factor κ^2, and follows.

These results locally establish the calibration distinction documented in the chronology companions [5, 6].

Coverage-qualified rejection, not fitting

Definition (Rejection contract). Before examining the test data, specify:

1. a parameter/nuisance domain $\mathscr{K}$;

2. one same-model forward map F_i for each named channel;

3. independent calibration regions and a deterministic model discrepancy set $\mathscr{E}_i$;

4. a simultaneous data confidence region $\mathscr{C}_i$. The channel acceptance set is

$$\mathcal{A}_i = \{\theta \in \mathcal{K} : \theta \text{satisfies the calibrations and} F_i(\theta) \in \mathcal{C}_i + \mathcal{E}_i\}.$$

Reject the declared instance or class exactly when $\mathcal{A}_i = \varnothing$. For a joint test use one shared parameter vector: $\mathcal{A}_{\text{joint}} = \bigcap_i \mathcal{A}_i$. A nonempty set means *not rejected*, not proved, identified, or empirically fitted.

Lemma (Coverage control). *If the true parameter belongs to the declared model and the calibration, data, and discrepancy regions jointly cover their true values with probability at least* $1 - \delta$*, then rejection of that true parameter by the stated contract has probability at most* δ*.*

Proof. On the simultaneous coverage event, the true parameter itself belongs to the acceptance set, so that set is nonempty. Rejection can occur only outside the coverage event.

If a required channel or calibration is unavailable, the status is NOT_TESTED. Missing data do not constitute a binary rejection result.

The five contracts

Declaration (C1-Chrono-Fibre). Test the ring subclass $a > 0$, $-a^2 < c < 0$, $b \geq 0$, $H > 0$. Ordinary data first constrain a, H through .

• With ordinary data alone, a chronology identification is refused whenever both sign-completion sets remain nonempty. This is non-identification, not rejection of the model.

• With a calibrated counter-rotating host datum, reject when no common a, H, τ_g satisfies

$$2\pi a < \tau_g < 4\pi a, \qquad c = (\tau_g/2\pi - a)^2 - a^2, \qquad b = H - c \geq 0.$$

• A correctly classified future co-rotating host return whose calibrated elapsed-time interval lies strictly above zero rejects this $c < 0$ ring instance. Finite uncertainty is handled by acceptance-set emptiness, not by replacing the strict window with an unsupported universal error constant. No physical host-null ring apparatus is supplied here.

Declaration (C2-Temp-FreeFall). Anchor the host frame by an independent gravitational standard. Test the clock and momentum relations –, together with dark geodesic transport. An active instance must specify an

independently calibrated nonzero portal history, not merely a freely adjustable coupling that can vanish after the data are seen.

Reject when no same-model history, including instrumental and astrophysical nuisance allowances, reproduces those relations. If a predeclared active modulation is excluded while a dark non-geodesic modification is required, the stipulated instance is rejected. Agreement with zero modulation does not reject the decoupled or screened class.

A direct dark-clock observation is not assumed to exist. Any inference of dark transport from dynamics requires its own mass, collision, and gravitational calibration.

Declaration (C3-Merger-Centroid). Declare a merger forward model linking host gravity, ordinary lensing on $\hat{g}$, gas, stellar matter, controller stress, and the dark residual. Specify before testing a resolved positive residual mass $M_{\mathrm{d}} \geq M_{\min} > 0$ and an offset $d_{\mathrm{gas,d}} \geq d_{\min} > 0$, with explicit projection and centroid allowances. Reject if the simultaneous confidence region is disjoint from these positive-offset predictions.

A residual merely "consistent with zero" is not automatically a rejection: a confidence interval may include both zero and the required positive values. Electromagnetic silence alone neither predicts a merger centroid nor bounds dark self-interactions. Bullet-Cluster evidence motivates this comparison [18]; it is not a CTC observation.

Declaration (C4-Decoupling). Audit the exact zero-controller action limit and a named family approaching it. Reject a claimed controlled extension if its limiting metric or equations remain chronology violating in the intended ordinary domain, or if a declared finite-window observable has a nonzero limiting deviation from the fixed GR+ baseline.

W-FLRW passes the printed background audit by Theorem . The fixed-cylinder $B \to 0$ proposal fails the globally hyperbolic audit by Proposition . The clamped relative-contrast identity and the named quasistatic unclamped remainder of Proposition may be included in the ε_4 budget; they are not a growth or Planck fit. Growth, acoustic peaks, recombination, damping, and light-deflection comparisons require their stated forward systems and calibrations. The background theorem alone is not a Planck fit [19].

Declaration (C5-MultiMessenger). Specify the full photon and tensor-wave paths, local propagation laws, source emission lag, and detector timing. Use the measured multi-messenger constraint, commonly summarised for GW170817 as an order-10^{-15} relative-speed bound under the stated source assumptions [17]. Reject an unscreened instance when its calibrated directional or integrated-delay prediction is incompatible with the simultaneous observational region.

A fully conformal vacuum path has identical metric null cones by Proposition ; an intersecting spacelike vortex has a direction-dependent penalty. Neither statement supplies an emission model or proves that actual observed paths avoid the vortex support.

Optional equatorial first-integral supplement

Contract and Theorems and remain the official guided-ring typestate. They are not withdrawn. The following optional equatorial scattering supplement, labelled informally C1'-FreeLensing, records first integrals, a capture band, and pointwise coordinate-time signs on the official unit-lapse collar, together with off-equatorial first integrals on a z-independent collar. It is not a replacement contract, not a 2D sky-plane shadow theorem, and not a typestate of a negative Shapiro jump.

Lemma (Equatorial first integrals on the official collar). *On the stationary unit-lapse collar*

$$g = -\mathrm{d}t^2 - 2a(r,z)\,\mathrm{d}t\,\mathrm{d}\varphi + c(r,z)\,\mathrm{d}\varphi^2 + \mathrm{d}r^2 + \mathrm{d}z^2,$$

with $F := a^2 + c > 0$, let γ be an affine g-null geodesic confined to the equatorial plane $z = 0$, and write $E := -g(\dot\gamma, \partial_t)$ and $L := g(\dot\gamma, \partial_\varphi)$ for the conserved Killing energy and angular momentum. Then

$$\dot t = \frac{Ec - aL}{F}, \qquad \dot\varphi = \frac{L + aE}{F}, \qquad \dot r^2 = E^2 - \frac{(L + aE)^2}{F}.$$

If the same collar is Minkowskian at spatial infinity, with $a \to 0$ and $F \sim r^2$, the impact parameter $b_{\mathrm{imp}} := L/E$ (for $E \neq 0$) coincides with the usual asymptotic intercept. Along a future-directed ray with $E > 0$, the radial inequality $\dot r^2 \geq 0$ is $|b_{\mathrm{imp}} + a| \leq \sqrt{F}$.

Proof. The two Killing identities are $\dot{t} + a\dot{\varphi} = E$ and $-a\dot{t} + c\dot{\varphi} = L$. Solving the linear system gives the first two displays, since the determinant is $c + a^2 = F$. The unit-lapse null condition expands as

$$-(\dot{t} + a\dot{\varphi})^2 + F\dot{\varphi}^2 + \dot{r}^2 = 0,$$

which is the third display. The asymptotic identification of b_{imp} is the Minkowski reduction $\dot{r}^2/E^2 = 1 - b_{\text{imp}}^2/r^2$.

Proposition (Retrograde capture band on the official tube). *In the interior tube of Theorem , one has* $a = \sqrt{r^2 + h}$ *and* $F = r^2$ *for some* $h > 0$*, hence* $a > r > 0$*. A future-directed equatorial g-null ray can meet that tube only if*

$$b_{\text{imp}} \in [-a - r, -a + r] \subset (-\infty, 0)$$

at each such radius. In particular, with the convention that $b_{\text{imp}} > 0$ *is co-rotating (prograde) relative to the positive drag* $a > 0$*, no prograde equatorial ray enters the tube. The penetrating band is strictly retrograde. The same interval written as* $[-\mathscr{A} - r, -\mathscr{A} + r]$ *with* $\mathscr{A} = a$ *is this band, and is not a prograde window.*

Proof. On the tube, $|b_{\text{imp}} + a| \le r$. Since $a > r$, the interval $[-a - r, -a + r]$ lies entirely on the negative half-line. A positive impact parameter cannot satisfy the inequality.

Proposition (Pointwise coordinate-time signs on the official tube). *In the interior tube of Theorem , one has* $a = \sqrt{r^2 + h}$ *and* $F = r^2$*, with* $h > 0$ *and* $a > r > 0$*. At a radius on that tube write* $\xi := b_{\text{imp}} + a(r)$*, so that* $\xi \in [-r, r]$ *if and only if the impact parameter lies in the capture band . Then, for* $E \neq 0$*,*

$$\frac{\dot{t}}{E} = 1 - \frac{a(r)\xi}{r^2}, \qquad \frac{\dot{\varphi}}{E} = \frac{\xi}{r^2}.$$

For $E > 0$*, the interval* $\xi \in (r^2/a(r), r]$ *is nonempty, because* $a > r$ *forces* $r^2/a < r$*, and on that interval* $\dot{t} < 0$ *holds strictly while* $\dot{\varphi} > 0$*. These signs are pointwise identities at radii on the tube. They do not by themselves typestate a global sign of* $\dot{t}$ *along an entire geodesic, because* $\xi = b_{\text{imp}} + a(r)$ *varies with r whenever a is not constant on the relevant radial interval. The penetrating band remains strictly retrograde, and prograde rays with* $b_{\text{imp}} > 0$ *still do*

not enter the tube.

Proof. On the tube, $c = r^2 - a^2 = -h$, so the first display of becomes $\dot{t}/E = (c - ab_{\mathrm{imp}})/r^2$. Substituting $b_{\mathrm{imp}} = \xi - a$ yields $c - ab_{\mathrm{imp}} = c - a\xi + a^2 = F - a\xi = r^2 - a\xi$, which is . The second identity is $(b_{\mathrm{imp}} + a)/r^2 = \xi/r^2$. Since $a > r > 0$, one has $0 < r^2/a < r$, so the interval $(r^2/a, r]$ is a nonempty sub-band of $\xi \in [-r, r]$. On it, $a\xi/r^2 > 1$, hence $\dot{t}/E < 0$, while $\xi > 0$ forces $\dot{\varphi}/E > 0$. For $E > 0$ these are the stated signs. Because $a = \sqrt{r^2 + h}$ depends on r on the official tube, ξ is not constant along a radial excursion, and no global sign of $\dot{t}$ is claimed.

Coordinate-time accumulation along an equatorial ray is $\mathrm{d}t/\mathrm{d}r = \dot{t}/\dot{r}$, with the quotient determined by . A finite renormalised coordinate-time comparison $\Delta T^{(g)}_{\mathrm{ren}}(b_{\mathrm{imp}}) - \Delta T^{(\mathrm{o})}_{\mathrm{ren}}(b_{\mathrm{imp}})$, defined by a cutoff about an outermost turning radius, is not evaluated here: the official metric is explicit on the tube and is Minkowski outside the compact support of w, but is not an explicit elementary profile in the transition region, and no closed-form antiderivative is invented. In particular, no sign $\Delta T^{(g)}_{\mathrm{ren}} < 0$ on the penetrating band, and no negative Shapiro jump, is typestated. The optional supplement remains an auxiliary record alongside Contract , not a replacement of Theorems and , and not a two-dimensional sky-plane shadow.

Lemma (Off-equatorial first integrals on a z-independent collar). *On the z-independent unit-lapse collar of Theorem (2), with $g = -(\mathrm{d}t + \omega)^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\varphi^2 + \mathrm{d}z^2$ and $\omega = a(r)\,\mathrm{d}\varphi$, the vector ∂_z is Killing, and $p_z := g(\dot{\gamma}, \partial_z)$ is conserved along any affine geodesic γ. In these coordinates $p_z = \dot{z}$. For a g-null geodesic, writing $E := -g(\dot{\gamma}, \partial_t)$ and $L := g(\dot{\gamma}, \partial_\varphi)$ as in Lemma , and $F := r^2$, one has*

$$\dot{r}^2 = (E^2 - p_z^2) - \frac{(L + a(r)E)^2}{F(r)}.$$

In particular, $\dot{r}^2 \geq 0$ if and only if $|L + aE| \leq \sqrt{F}\,\sqrt{E^2 - p_z^2}$, provided $|p_z| \leq |E|$. If the collar contains an official-tube radius with $a = \sqrt{r^2 + h}$ and $h > 0$, a future-directed ray with $E > 0$ can meet that radius only if

$$b_{\mathrm{imp}} \in [-a - r\mu, -a + r\mu] \subset (-\infty, 0), \qquad \mu := \sqrt{1 - p_z^2/E^2},$$

so the penetrating band remains strictly retrograde. Observer screen coordinates $(\alpha_{\mathrm{scr}}, \beta_{\mathrm{scr}})$*, a closed-form sky-plane capture-strip inequality, a two-dimensional crescent or toroidal shadow, and a negative Shapiro jump, are not typestated.*

Proof. z-independence of the metric coefficients makes ∂_z Killing, so $p_z = g(\gamma, \partial_z)$ is conserved. With $g_{zz} = 1$ and no mixed z terms, this momentum is $\dot{z}$. The two Killing identities for E and L are unchanged from Lemma , because the (t, φ) block does not involve z. The unit-lapse null condition expands as

$$-(\dot{t} + a\dot{\varphi})^2 + F\dot{\varphi}^2 + \dot{r}^2 + \dot{z}^2 = 0,$$

and substituting $\dot{t} + a\dot{\varphi} = E$, $\dot{\varphi} = (L + aE)/F$, and $\dot{z} = p_z$ yields . The displayed inequality is the condition $\dot{r}^2 \geq 0$. At an official-tube radius, $|b_{\mathrm{imp}} + a| \leq r\mu$ with $\mu = \sqrt{1 - p_z^2/E^2} \leq 1$. Since $a > r \geq r\mu$, the interval $[-a - r\mu, -a + r\mu]$ lies entirely on the negative half-line. No screen-coordinate identification or integrated arrival comparison is performed.

Additional compatibility obligations

toprule Constraint Obligation retained in this paper

midrule MICROSCOPE and lunar laser ranging Universal ordinary metric coupling controls test-body universality, but controller forces, self-gravity, and environmental dependence must be compared with the calibrated experiments [14, 15]. No numerical acceptance is claimed.

Cassini and Solar-System optics The ordinary metric, source-generated host geometry, and ranging clocks must satisfy the measured light-deflection and Shapiro-delay constraints [16]. Conformal null equality does not by itself constrain the sourced host metric.

CMB and linear growth Residual portals must be propagated through a complete perturbation and recombination system. Fixed-window continuity does not establish a sound-horizon or damping likelihood.

Laboratory chronology The ordinary metric has Cauchy slices in the witnesses. Full-system protection with

accessible host-cone channels is not established; Proposition remains applicable.

bottomrule

No joint observational acceptance set has been evaluated or proved nonempty. A data-free synthetic feasibility construction would not discharge any of C1–C5.

Release theorem, debugging audit, and documentary hygiene

Theorem (Conditional backward-inference release). *For the classes and assumptions printed in this paper:*

1. *the geometric backward postulate is realised by complete prescribed cylindrical and compactly supported smooth chronology-split witnesses;*

2. *the action ledger has exact ordinary/controller exchange, independently conserved dark stress, and five-slot switch-shell closure;*

3. *off-shell electromagnetic and controller silence follows from robust direct insulation, not from CTC capability alone;*

4. *dark-slot uniqueness is relative to a supplied additive inventory and does not select a microscopic species;*

5. *the separate FLRW comparison has positive dark residual density, active exchange, and a quadratic approach to its specified background;*

6. *fixed-host controller removal has a closed-null chronology threshold without the asserted spacetime-signature degeneracy; correlated host removal avoids this obstruction;*

7. *ordinary ring data are chronology-fibre blind, one calibrated host-null return reconstructs a, c, b, and an independent winding kinetic-scalar datum at fixed stiffness supplies a four-variable local inverse of the $B\kappa^2$ factorisation;*

8. *a controller cone inequality, with a pinching hypothesis, replaces qualitative controller silence, while $A \equiv 1$ remains a frame gauge;*

9. *on-shell vortex bookkeeping is admitted under the repaired Laplacian identity, with Schrödinger coefficient μ, under the printed two-switch closedness condition when μ is smooth, or on a z-independent radial collar under smoothness of μ, and not for a withdrawn single-invariant pullback; the closedness condition is universal in form for every smooth* $V_{\Phi}(u, s_1, s_2)$ *and is not automatic; a compactly supported two-switch realisation with locally bounded μ is obstructed, and the clean Schwartz realisation is the z-independent Gaussian collar, not a compact two-switch example;*

10. *a global radial symmetron profile exists by shooting, with the Yukawa comparison as a lower bound on the exterior deviation, and with the nonlinear two-sided bracket in the bare parameters; the linearised matching identity is an upper bound, not an equality;*

11. *C1–C5 are declared rejection contracts, not performed observational tests, an optional equatorial first-integral supplement, including the pointwise signs of Proposition , does not replace C1 or the guided-ring theorems, a renormalised coordinate-time comparison and a negative Shapiro jump are not typestated, and Track II programmes of Appendix remain unfinished or refused except for the local fragments actually proved there;*

12. *uniqueness of* $J[\Phi]$ *is typestated only within the zero-distortion class (U1)–(U2);*

13. *Standard-Model silence of listed direct couplings is typestated only under the optional postulates (BP2')–(BP3'), and not as a consequence of geometric TM-2 alone; the principal-symbol half of (BP2'), for the operators of Lemma , is the characteristic variety of* $\hat{\tilde{g}}$*;*

14. *the clamped relative-growth identity is retained, and the unclamped Helmholtz elimination is typestated only under the two-step remainder hypotheses of Proposition (2). None of these assertions certifies observed dark matter as a CTC carrier, constructs a laboratory time machine, or supplies a common global Einstein–matter realisation of both witnesses. The time-machine bar remains* 0 *of* 3*. Host C is not created.*

Proof. Item 1 is Theorems and . Item 2 is Theorem and Proposition . Items 3–4 are Propositions and , Theorem , and Theorem . Item 5 is Theorems and , with Proposition . Item 6 is Proposition and Theorem . Item 7 is Theorems and , with Corollary and Proposition . Item 8 is Theorem . Item 9 is Theorems and , together with

Proposition . Item 10 is Lemma . Item 11 is Definitions , Declarations –, Lemma , Propositions and , Lemma , and Appendix . Item 12 is Theorem and Proposition . Item 13 is Assumption , Lemma , Theorem , and Proposition . Item 14 is Proposition . The proofs prescribe metrics, use no operational return protocol, and do not identify the separate witnesses.

Debugging audit

toprule Proposed assertion Corrected release

midrule endhead "A time machine exists now" is a premise, while Bar 3 stays open. The premise is counterfactual geometric TM-2. Operational existence is neither assumed as evidence nor certified.

CTCs or a Cauchy horizon imply free closed dark geodesics. Closed timelike geodesics are separately required and explicitly proved in the cylinder and toroid. A horizon alone is insufficient.

CTC capability forces off-shell electromagnetic silence. False without extra assumptions. Robust direct insulation is printed separately and implies silence by a local variational proof.

Maxwell-on-host necessarily has a closed photon geodesic. Not proved and not used. The exclusion concerns the failure of global hyperbolicity of a universally adopted ordinary metric.

The unique carrier is a unique dark-matter species. Only the inventory-relative maximal aggregate is unique. Microscopic decompositions remain unidentified.

A real scalar or winding controller is the unique minimal mechanism. The Rolle obstruction excludes one exact real-gradient correction from uniformly excluding the same loop. It does not classify every mechanism. The complex current can be smooth and nonclosed.

$B\kappa^2 = h$ causes Lorentzian signature degeneracy. The spacetime remains Lorentzian. The angular circle and induced slice become null, and global hyperbolicity fails.

Vortex-free propagation automatically satisfies GW170817. Exact conformal vacuum cone equality is local and path-qualified. Emission, path coverage, and timing calibration remain necessary.

$J(k) = 0$ means zero angular momentum. Not in a metric with time–angle cross terms. The condition is covector orthogonality, not a conserved-momentum identification.

A smooth compact vortex is automatically an action solution. A general compact profile remains kinematic.

On-shell bookkeeping holds for a two-switch potential under closedness, or on a z-independent radial collar, not for a single-invariant σ pullback.

Positive conserved dust on the flat CTC host is an Einstein match. The null-contraction obstruction leaves a nonzero residual. The positive Einstein-residual witness is the separate FLRW model.

The Gödel winding stress is cancelled by changing dust and Λ. Its spatial anisotropy survives in the printed source class. Additional anisotropic sources would be a new model.

A nonempty joint acceptance set is a rejection condition. Rejection requires emptiness of a calibrated same-model acceptance set. Nonempty does not mean identified or verified.

One host-null datum still leaves host chronology unidentified. In the calibrated ring class it reconstructs c and b. Only the factorisation of b remains uncalibrated.

A residual consistent with zero rejects positive dark matter. Only incompatibility with a predeclared resolved positive prediction rejects the named instance.

Ordinary Cauchy slices imply no backward signal to apparatus. They protect ordinary trajectories. Accessible host-cone channels prevent an automatic full-system temporal-function theorem.

Quadratic background convergence proves CMB and growth acceptance. It proves finite-interval background convergence and conditional continuity statements, not a complete perturbation calculation or likelihood analysis.

Any controller coupling flips the azimuthal cone and forces controller silence on the loop sector. False on a collar with $c < 0$. The proved inequality is $B_I[J_\varphi]^2 < A_I^2|c|$. Forcing $B_I = 0$ requires a pinching hypothesis. $A \equiv 1$ is a frame gauge.

Linear-in-ε scaling of B with a correlated host shift $\varepsilon\omega$ remains an exact open-tube cancellation and is C^∞. The

cancelling disformal term is of order ε^2. Holding Φ fixed and taking $B_\varepsilon = \varepsilon^2 B_0$ is C^∞ and cancels exactly. The linear scaling is the wrong power. An open tube for every $\varepsilon \in (0, 1]$ is not obtained.

A Yukawa tail bounds the symmetron deviation from above, and $\varphi(R)/\varphi_\infty$ is a closed nonlinear form in bare parameters. Yukawa is a lower bound on $\varphi_\infty - \varphi$; the massless envelope is the generally valid upper bound. A proved two-sided bracket in the bare parameters is typestated; the linearised matching identity is an upper bound, not an equality.

The four-variable Jacobian in (a, c, B, κ) equals $4\pi^3 f_q^2 \kappa/(D_o d_g^3)$. That expression is the Jacobian in coordinates (a, c, b, κ). In factorisation coordinates (a, c, B, κ) an extra factor κ^2 appears.

A closed Y_k unclamped growth multiplier is an exact identity of Contract C4. Only a Helmholtz elimination with a two-step remainder is typestated, under stated slow-forcing and homogeneous-wave hypotheses. Homogeneous free waves are not silently dropped. The pasted prefactor is not used.

$\Delta_{\mathbb{R}^3}\Phi = \lambda\Phi$ for $\Phi = h(r^2, z^2)(x + \mathrm{i}y)$, with λ the differential polynomial in h, so a compact two-switch example is only a matter of checking closedness. The identity is $\Delta_{\mathbb{R}^3}\Phi = \lambda(x + \mathrm{i}y)$. The Schrödinger coefficient is $\mu = \lambda/h$, which need not extend boundedly across $\{h = 0\}$. With a locally bounded extension, unique continuation obstructs compact support. No compact example is exhibited.

$J[\Phi]$ is the unique smooth tensor shield on $\mathbb{R}^4$, and a monopole defect is visible even where $g = \eta$. Uniqueness is typestated only in the class (U1)–(U2). A monopole calculation is not written. Quantitative Fourier bounds from the Backward-B draft are not typestated.

Full Standard-Model darkness follows from backward chronology protection, and $B_{\mathrm{dm}} = 0$ holds without pinching. Silence of listed couplings is conditioned on (BP2')–(BP3'). Underdetermination is retained. Pinching remains the official horizon-approaching hypothesis, and $A \equiv 1$ is a frame gauge.

Prograde rays with $b_{\mathrm{imp}} > 0$ enter the official CTC tube and produce a negative Shapiro jump, replacing Contract C1. The official capture band is retrograde, $b_{\mathrm{imp}} \in [-a - r, -a + r] \subset (-\infty, 0)$. Pointwise $\dot{t} < 0$ holds on a nonempty

retrograde sub-band at tube radii, not necessarily along an entire geodesic. No Shapiro jump is typestated, and the renormalised integral is uncomputed. C1 and the guided-ring theorems are not replaced.

The truncated Backward-C Einstein-tensor formulae, a zero-ADM no-go, a curved-core NEC cure, and a Chronology-Core Jeans theorem, are released. The first Cartan equation repairs the spatial connection sign. Einstein-tensor components are not typestated by quotation. $M_{\mathrm{ADM}} = 0$ for compact deviations is a tautology, not a no-go. The named C theorems and Breakthroughs 1–5 remain refused.

Closedness $\mathrm{d}\mu \wedge \mathrm{d}u = 0$ holds automatically for every smooth potential, and Schwartz or dead-core profiles bypass compact sharpness. The chain-rule identity shows that the *form* of closedness is universal, not that closedness holds automatically. Schwartz Gaussians inhabit the smooth-μ class and do not refute compact sharpness; two-switch closedness fails unless $\beta = 0$. Dead-core potentials are the blow-up branch, not a bypass. No compact example is exhibited.

The truncated Backward-C Einstein-tensor formulae, a Global Curved-Core NEC Dichotomy, a Painlevé–Gullstrand amalgamation discharging Obligation 1, a cratered Vlasov torus, a CTC-background small-gain theorem, or a 3D sky-plane capture strip, are released. Static-isotropic and unit-lapse reduction identities for a future Einstein-tensor typestate are recorded; general $G_{\hat{a}\hat{b}}$ and a global NEC dichotomy remain refused. A kinematic W-CTC/W-FLRW interpolation with a Minkowski buffer is typestated and does not discharge Obligation 1. The Vlasov torus, a small-gain theorem on a CTC background, and screen-coordinate capture strips remain refused. A finite-slab ordinary-metric contraction is a restricted claim only. None of Breakthroughs 1–5 closes the Section 7.3 obligations.

Rank-k exact gradients, Standard-Model silence, free-geodesic negative Shapiro delays, curved-core NEC cures, Swiss-cheese unification, Planck-suppressed leakage bounds, or a Moser-trapped warm Vlasov tube, are released flagship theorems. These programmes are recorded in Appendix as unfinished or refused, except the class-limited, conditional, and local fragments actually proved in the main text or appendix.

bottomrule

Version hygiene

This official version 5 is a typographic successor of official version 4 (request t514u). Official version 4 typestate, labels, and local proofs are retained. The authorised Version 5 work is typestate hygiene, ordinary quotation marks, a Scholar-parseable bibliography without an invented DOI, and the documentary negative-control table of Section . Siblings remain source records at their official currently delivered versions; unwritten t563u successors are not cited. Card 11 remains 0 of 3.

The current typestate pins for documentary neighbours, used only as source records, are as follows. Earlier briefs' outdated official version labels are not used as the current typestate.

toprule Record Official pin Documentary status

midrule Chrono-DM-A Version 4, t506u Source record, not a premise. Official titles are copied from the parent copy.

Chrono-DM-B Version 4, t509u Source record, not a premise. Official residuals on unshifted families are not withdrawn by this sheet.

Chrono-DM-C Version 3, t508u Source record, not a premise. The withdrawn single-invariant on-shell pullback of official C version 2 is not reused.

Temporal-A Version 5, t504u Source record, not a premise.

Mobility-A Version 5, t495u Source record, not a premise.

Big06-A Version 2 Used only as a documentary source record, where referred to.

Chrono-DM-D Not cited This sheet does not cite or depend on official Chrono-DM-D.

bottomrule

No companion browse URL, invented DOI, figshare, Zenodo, or Scholar landing page is used. Journal DOIs already present in the typographic parent are retained. The source is not companion-wired: wired=false;

declaredPath remains null.

Remaining obligations and potential significance

The mathematically released results are conditional comparisons. Their principal potential value is methodological: they isolate what a sector-exclusive chronology hypothesis actually requires, show which attractive backward deductions are not logically forced, and identify the extra channel that converts one concrete non-identification problem into an inverse problem.

The following substantial obligations remain:

1. a single global action-compatible Einstein–matter model joining the CTC and cosmological witnesses, with every controller and residual source accounted for;

2. stability, formation or access from a specified common causal past, and a physically admissible dark transport continuation on the non-globally-hyperbolic host;

3. a complete signal-causality analysis including gravity, controller propagation, and apparatus coupling;

4. a global environmental screening solution compatible with laboratory, Solar-System, cluster, and cosmological data;

5. complete perturbation, recombination, growth, and merger forward models, followed by calibrated empirical tests;

6. quantum-state, renormalisation, and chronology-horizon analysis in the actual coupled theory. The Track II programmes recorded in Appendix are instances of these obligations, or proposed strengthenings of them; they are not discharged by being listed. Breakthroughs 1–5 of the t514u brief (and of the t513u brief) remain refused as closures of these obligations. A kinematic Painlevé–Gullstrand interpolation joining W-CTC and W-FLRW on one manifold is typestated in the appendix and does not discharge Obligation 1. A global NEC dichotomy, a cratered 3D Vlasov torus, a small-gain theorem on a CTC background, and a celestial-screen capture strip remain untypestated as such. Local fragments actually proved (reduction checks, the Painlevé–Gullstrand coordinate

identity, off-equatorial first integrals, and a restricted ordinary-metric contraction) do not close the six obligations.

The last obligation cannot be replaced by an iε slogan or by importing a preferred continuation from a different manuscript. Results such as the compactly generated Cauchy-horizon obstruction in quantum field theory [11] motivate caution, but do not constitute a quantum analysis of the present two-metric class.

Final release boundary par The title phrase "dark matter is the only matter that can time-travel" is not established as a statement about nature. Its admissible release is: *under a prescribed geometric chronology split and an additional direct-insulation hypothesis, one may define a maximal silent host-transport matter slot, while retaining explicit non-identification and observational rejection contracts.*

Time-machine bar: 0 of 3. Host C: not created. O.1: NOT_CLOSED. H3: NOT_CLOSED. Gate 0 and ST4b: NOT_SUPPLIED. No derived metric, spacetime emergence, empirical fit, microscopic species selection, quantum coefficient admission, machine-checked proof, or executed compilation is asserted. Companion manuscripts remain documentary records, never theorem premises. The source is not companion-wired: wired=false; declaredPath remains null.

appendix

Track II programmes: typestate and explicit refusals

The items below were proposed as flagship strengthenings in the enhancement briefs attached to requests t511u, t513u, and t514u, and in the two working supplements (a Backward-B draft and a truncated Backward-C draft) used as source records for official version 3. Mid-argument "check", "wait", "let us state", or "CHECK THAT INEQUALITY!" language in those drafts is scratch, not typestate, and is not carried into this source. Each programme is recorded as unfinished or refused, except where a complete local proof is written in the main text or in this appendix (Theorems and , Proposition , Lemma , Propositions and , Lemma , Proposition , Lemma , the repaired Theorem , Theorem , Lemma , Proposition , and Proposition). No official Backward-B or Backward-C companion package is created. None of the refused programmes, and none of these local fragments, is Bar 1, Bar

2, or Bar 3; prescribed hosts do not discharge Bar 3; and TM-1 typestate remains undischarged.

Rank-k exact real gradients and minimal distortion. Proposition already records that a Rolle obstruction for one exact real gradient does not classify every mechanism. A rank-two sum of exact gradients can remain non-vanishing along a circle. Theorem now typestates uniqueness of $J[\Phi]$ only within the zero-radial, zero-vertical, and zero time-time class (U1)–(U2). Proposition records a qualitative radial-distortion obstruction for exact real-gradient shields whose weights remain positive on a full disk. Quantitative Fourier constants, Sobolev cutoffs, and a monopole "even where $g = \eta$" without a complete linearised gravity or optics calculation, remain refused. Official winding controllers and disformal stresses outside the class (U1)–(U2) are not eliminated.

Non-Abelian Standard-Model silence and portal exclusion. Lemma typestates the characteristic variety of Lorenz-gauge Yang–Mills and of a tetrad-coupled Dirac operator when those operators are written with kinetic metric $\tilde{g}$. That is the principal-symbol half of optional (BP2'), conditional on those operators being the ones assigned. It does not derive (BP2') or (BP3') from Assumption . High-frequency characteristics following g rather than $\tilde{g}$, if that were the printed kinetic metric, can conflict with a printed ordinary-protection postulate for those fields; that is not a deduction of QCD, weak, Higgs, or neutrino silence, and it does not withdraw Proposition . Official Chrono-DM-A already typestates, as a source record, that electromagnetic silence is not an absolute firewall [5]. Theorem is conditioned explicitly on the optional postulates (BP2')–(BP3') of Assumption . "Full Standard-Model darkness purely from backward chronology protection" is refused. The Backward-B pinching claim $B_{\mathrm{dm}} = 0$ without the horizon-approaching hypothesis of Theorem (3) is refused; $A \equiv 1$ remains a Contract C2 frame gauge.

Unconstrained free-geodesic lensing and a negative Shapiro jump. Lemma and Proposition record equatorial first integrals and a strictly retrograde capture band $b_{\mathrm{imp}} \in [-a - r, -a + r] \subset (-\infty, 0)$ on the official tube. Proposition records the pointwise signs $\dot{t} < 0$ and $\dot{\varphi} > 0$ on the nonempty sub-band $\xi \in (r^2/a, r]$ at radii on the tube; no global sign along an entire geodesic is typestated. A draft label identifying $b_{\mathrm{imp}} > 0$ as the penetrating, prograde window is false on this collar and is not used. The slogans $\Delta t_{\mathrm{grav}}(b_{\mathrm{imp}}) < 0$ for g-null rays, a strictly positive

ordinary delay at the same impact parameter, and a 2D sky-plane crescent or toroidal shadow, are not typestated, and the renormalised coordinate-time comparison remains uncomputed. Contract is not replaced by an astrophysical observable. Guided-ring typestate and ordinary fibre-blindness (Theorems and) are not withdrawn. Lemma typestates off-equatorial first integrals and an inclined retrograde capture condition on a z-independent collar; observer screen coordinates and a sky-plane capture strip remain a programme.

Cartan connection of a curved-core stationary coframe. Let a stationary curved-core host be written

$$g = -e^{2\Phi(\mathbf{x})}(\mathrm{d}t + \omega_i(\mathbf{x})\,\mathrm{d}x^i)^2 + e^{2\Sigma(\mathbf{x})}\delta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j,$$

with $\Phi, \Sigma \in C^\infty(\mathbb{R}^3)$ and a smooth spatial covector ω_i. The orthonormal coframe

$$\Theta^{\hat{0}} := e^{\Phi}(\mathrm{d}t + \omega_i\,\mathrm{d}x^i), \qquad \Theta^{\hat{i}} := e^{\Sigma}\,\mathrm{d}x^i$$

is Lorentzian, with dual frame $e_{\hat{0}} = e^{-\Phi}\partial_t$ and $e_{\hat{i}} = e^{-\Sigma}(\partial_i - \omega_i\partial_t)$. Writing $\Phi_i := \partial_i\Phi$, $\Sigma_i := \partial_i\Sigma$, and $\mathscr{F}_{ij} := e^{\Phi-2\Sigma}(\partial_i\omega_j - \partial_j\omega_i)$, the first structure equations are

$$\mathrm{d}\Theta^{\hat{0}} = e^{-\Sigma}\Phi_i\,\Theta^{\hat{i}} \wedge \Theta^{\hat{0}} + \tfrac{1}{2}\mathscr{F}_{ij}\,\Theta^{\hat{i}} \wedge \Theta^{\hat{j}},$$

$$\mathrm{d}\Theta^{\hat{i}} = e^{-\Sigma}\Sigma_j\,\Theta^{\hat{j}} \wedge \Theta^{\hat{i}}.$$

These two displays are independently recomputed from and are typestated. The unique metric-compatible, torsion-free connection compatible with is

$$\omega^{\hat{0}}{}_{\hat{i}} = e^{-\Sigma}\Phi_i\,\Theta^{\hat{0}} + \tfrac{1}{2}\mathscr{F}_{ij}\,\Theta^{\hat{j}},$$

$$\omega^{\hat{i}}{}_{\hat{j}} = \tfrac{1}{2}\mathscr{F}_{ij}\,\Theta^{\hat{0}} + e^{-\Sigma}(\Sigma_j\Theta^{\hat{i}} - \Sigma_i\Theta^{\hat{j}}).$$

(The spatial Σ terms are opposite to a displayed formula in the truncated Backward-C draft: the first Cartan equation $\mathrm{d}\Theta^{\hat{i}} = -\omega^{\hat{i}}{}_{\hat{0}} \wedge \Theta^{\hat{0}} - \omega^{\hat{i}}{}_{\hat{j}} \wedge \Theta^{\hat{j}}$ forces , and the draft's last step identifies $\Theta^{\hat{i}} \wedge \Theta^{\hat{j}}$ with $\Theta^{\hat{j}} \wedge \Theta^{\hat{i}}$.) The orthonormal Einstein-tensor components displayed in that draft, including the isotropic-gauge formula carrying an

exclamation mark, were computed from the unrepaired connection and are not typestated. No component of $G^{\wedge\wedge}_{ab}$ is admitted by quotation. The following static reduction identity is recorded as a check that any future typestate of $G^{\wedge\wedge}_{ab}$ must satisfy. It is not a typestate of the general curved-core Einstein tensor. In the static case $\omega_i = 0$, the induced spatial metric is $\gamma_{ij} = e^{2\Sigma}\delta_{ij}$ and the second fundamental form vanishes. The three-dimensional conformal formula ${}^{(3)}R = -2e^{-2\Sigma}(2\Delta\Sigma + |\nabla\Sigma|^2)$, together with the Hamiltonian constraint, yields the orthonormal component

$$G^{\wedge\wedge}_{00} = -e^{-2\Sigma}(2\Delta\Sigma + |\nabla\Sigma|^2),$$

and $G^{\wedge\wedge}_{0i} = 0$. This vanishes for the isotropic Schwarzschild conformal factor $\Sigma = 2\log(1 + m/(2\rho))$ outside the origin, because $\psi = 1 + m/(2\rho)$ is harmonic and $2\Delta\Sigma + |\nabla\Sigma|^2 = 0$. In the unit-lapse frame-dragging case $\Phi = \Sigma = 0$, the metric is $g = -(\mathrm{d}t + \omega_i\,\mathrm{d}x^i)^2 + \mathrm{d}\mathbf{x}^2$. This is not the ADM dataset $N = 1$, $\gamma_{ij} = \delta_{ij}$, $\beta_i = \omega_i$: the induced metric on a constant-t slice is $\gamma_{ij} = \delta_{ij} - \omega_i\omega_j$, the shift is $\beta_i = -\omega_i$, and the lapse satisfies $N^2 = 1/(1 - |\omega|^2)$, which is available only where $|\omega| < 1$ (so that the slice is spacelike). In particular ${}^{(3)}R$ need not vanish, and K_{ij} is not $-\frac{1}{2}(\partial_i\omega_j + \partial_j\omega_i)$ computed with a unit lapse on a flat slice. Those identifications are not typestated, and no orthonormal component of $G^{\wedge\wedge}_{ab}$ is typestated in this reduction. The static check stands. Neither the static reduction nor this withdrawn unit-lapse identification typestates a general formula for $G^{\wedge\wedge}_{ab}$, and neither is the truncated draft's exclamation-mark isotropic display.

Zero ADM mass as a surface-integral tautology, not a no-go. On an asymptotically Euclidean spatial slice that coincides with Euclidean three-space outside a compact set, the ADM mass is a limit of surface integrals of $\partial_j h_{ij} - \partial_i h_{jj}$ over large spheres. If the deviation h_{ij} is compactly supported, every sufficiently large sphere sees vanishing integrand, and $M_{\mathrm{ADM}} = 0$. The same conclusion holds if the deviation decays faster than any inverse power, or more generally whenever the surface integrand is $o(r^{-2})$. This identity is a surface-integral tautology. It is not, by itself, an obstruction to a chronology-violating core, and it does not force a null-energy violation. A no-go that combines $M_{\mathrm{ADM}} = 0$, a compact CTC torus, and an energy condition is not proved here. The truncated abstract claims labelled thm : zero-adm-nogo and thm : curved-nec-cure are refused.

Curved-core or Weyl–Lewis–Papapetrou exteriors as a NEC cure. Proposition records a residual obstruction on the flat cylindrical host. Official Chrono-DM-C typestate, as a source record, and the unit-lapse compact-support class of Theorem , leave open a boundary-shell null-energy violation for compact bumps on flat spatial slices. A complete NEC-satisfying asymptotically flat core, or an exact global no-go, is not proved here. The static reduction identity recorded after is a check, not that proof. A "Global Curved-Core NEC Dichotomy" or "Global Tipler–Hawking versus Weyl–Lewis–Papapetrou dichotomy theorem" is not typestated. In particular, it is not claimed that an $M_{\mathrm{ADM}} > 0$ dipolar Kerr or halo tail eliminates an outer curl-shell NEC violation, and it is not claimed that $\pi_1(\mathbb{R}^4) = 0$ forces an inner-hole topological obstruction, unless both sides are actually proved, which they are not. Testing $G_{ab}k^a k^b$ on a general ansatz, or a transition-zone identity, is not a global verdict. A convected Helmholtz resolvent is not that proof, and is not a repair of the unit-lapse obstruction. Vanishing ADM mass for compact deviations remains the surface-integral tautology already recorded. This is Breakthrough 1 of the t514u brief, and it remains refused as a global theorem.

McVittie, Swiss-cheese, or core-in-FLRW unification. This is Breakthrough 2 of the t514u brief. After Theorem it is already recorded that replacing $\mathrm{d}\mathbf{x}^2$ by $a(t)^2\mathrm{d}\mathbf{x}^2$ does not retain the stationary closed-geodesic proof. The following coordinate identity and kinematic interpolation place W-CTC and W-FLRW on one manifold without discharging Obligation 1.

Lemma (Painlevé–Gullstrand form of flat FLRW). *Let* $g_{\mathrm{FLRW}} = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\mathbf{x}^2$ *be a flat Friedmann–Lemaître–Robertson–Walker metric with* $a(t) > 0$ *and* $H := \dot{a}/a$. *In Eulerian coordinates* $\mathbf{X} := a(t)\mathbf{x}$, *one has the identity*

$$g_{\mathrm{FLRW}} = -\mathrm{d}t^2 + |\mathrm{d}\mathbf{X} - H(t)\mathbf{X}\,\mathrm{d}t|^2 = -(1 - H^2|\mathbf{X}|^2)\mathrm{d}t^2 - 2H\mathbf{X}\cdot\mathrm{d}\mathbf{X}\,\mathrm{d}t + \mathrm{d}\mathbf{X}^2.$$

In this presentation the lapse-shift data are $N = 1$, $\gamma_{ij} = \delta_{ij}$, *and* $\beta = -H\mathbf{X}$, *so* $g^{tt}_{\mathrm{FLRW}} = -1$ *and t is temporal everywhere, including outside the Hubble radius.*

Proof. From $\mathbf{x} = \mathbf{X}/a$ one has $\mathrm{d}\mathbf{x} = a^{-1}\mathrm{d}\mathbf{X} - H\mathbf{x}\,\mathrm{d}t$, hence $a^2\mathrm{d}\mathbf{x}^2 = |\mathrm{d}\mathbf{X} - H\mathbf{X}\,\mathrm{d}t|^2$. Expanding the square yields the

second display. The ADM inverse for $N = 1$ and flat γ is $g^{tt} = -1$, independently of the size of $|\mathbf{X}|$.

Proposition (Kinematic W-CTC/W-FLRW interpolation with a Minkowski buffer). *Let $I \subset \mathbb{R}_t$ be a time interval on which the W-FLRW witness of Theorem exists, with $H(t) > 0$. Fix radii $0 < R_{\mathrm{core}} < R_{\mathrm{ext}} < R_{\mathrm{match}}$, and assume $R_{\mathrm{match}} < \inf_I H(t)^{-1}$ if a sub-Hubble shell is desired (this bound is not required for the Cauchy property of $\tilde{g}$ below). Place the compact spatial support of the toroidal witness of Theorem inside $\{|\mathbf{X}| \le R_{\mathrm{core}}\}$. Let $\chi \in C^{\infty}([0,\infty);[0,1])$ satisfy $\chi \equiv 1$ on $[0, R_{\mathrm{ext}}]$ and $\chi \equiv 0$ on $[R_{\mathrm{match}}, \infty)$. On $M = I \times \mathbb{R}^3_{\mathbf{X}}$ define a host metric g by*

$$g = \begin{cases} g_{\mathrm{CTC}} & \text{on} |\mathbf{X}| \le R_{\mathrm{ext}}, \\ -\mathrm{d}t^2 + |\mathrm{d}\mathbf{X} - (1-\chi)H(t)\mathbf{X}\,\mathrm{d}t|^2 & \text{on} |\mathbf{X}| \ge R_{\mathrm{ext}}, \end{cases}$$

where g_{CTC} is the official toroidal host of . Let the ordinary metric, vortex, and disformal data of Theorem be used unchanged on $|\mathbf{X}| \le R_{\mathrm{ext}}$, and let the ordinary metric, conformal factor, and matter data of Theorem be used unchanged on $|\mathbf{X}| \ge R_{\mathrm{match}}$, with the homogeneous scalar $\varphi(t)$ interpolated in the shell by a smooth cutoff in $|\mathbf{X}|$ from a constant value with $A = 1$ at R_{ext} to the W-FLRW value at R_{match}. Then:

1. *g and $\tilde{g}$ are smooth and Lorentzian on M;*

2. *on a neighbourhood of the core, $|\mathbf{X}| \le R_{\mathrm{ext}}$, the data $(g, \tilde{g}, \Phi, B_0 J \otimes J)$ are identically those of the stationary witness of Theorem , including its closed dark geodesics;*

3. *on $|\mathbf{X}| \ge R_{\mathrm{match}}$, the data are identically those of Theorem , including a positive dark residual density;*

4. *every constant-t slice is a Cauchy surface for $\tilde{g}$ on M;*

5. *writing $T_{\mathrm{res}} := M_{\mathrm{p}}^2(G[g] + \Lambda g) - T_{\mathrm{o}} - T_{\mathrm{r}} - T_{\varphi} - T_{\mathrm{d}}$ for the printed stresses of the interpolated fields, the shell residual $T_{\mathrm{res}}^{\mathrm{shell}} := T_{\mathrm{res}}$ is supported in the closed shell $\overline{\{R_{\mathrm{ext}} \le |\mathbf{X}| \le R_{\mathrm{match}}\}}$, up to the already recorded core residual of the official W-CTC witness, and is computed from the ADM data of the interpolated metric: on the shell one has $N = 1$, $\gamma_{ij} = \delta_{ij}$, and $\beta = f(t,R)\mathbf{X}$ with $f = (1-\chi)H$ and $R = |\mathbf{X}|$, whence $K_{ij} = -f\delta_{ij} - (f_R/R)X_iX_j$, and the Einstein tensor is the standard ADM functional of $(K_{ij}, \partial_t K_{ij}, \partial_R f, H, \dot{H})$, together with the stresses of the radially interpolated*

scalar. The construction is kinematic: the interpolated fields are prescribed, and $T^{\text{shell}}_{\text{res}}$ *is defined as the Einstein discrepancy, not obtained as the on-shell stress of a printed action with controlled energy conditions. It is not an Einstein–Straus mass match (the compact core has vanishing ADM mass by the tautology already recorded). Completing the square inside the Hubble radius does not discharge Obligation 1 of Section . The interpolation remains a prescribed background, not a single global action-compatible Einstein–matter model.*

Proof. On $|\mathbf{X}| \le R_{\text{core}}$ the metric is the official toroidal host, already smooth and Lorentzian. In the buffer $R_{\text{core}} \le |\mathbf{X}| \le R_{\text{ext}}$ one has $\omega = 0$ and $\chi = 1$, so both the official exterior and the interpolated formula reduce to Minkowski, and the gluing is identically Minkowski on an open set, hence smooth. On $|\mathbf{X}| \ge R_{\text{ext}}$ the display $-\mathrm{d}t^2 + |\mathrm{d}\mathbf{X} - (1-\chi)H\mathbf{X}\,\mathrm{d}t|^2$ is of ADM form with $N = 1$ and flat γ, therefore Lorentzian, and at $|\mathbf{X}| \ge R_{\text{match}}$ it coincides with . The ordinary metric matches similarly: $\tilde{g}$ is the official ordinary toroid inside the core, Minkowski in the buffer, and $A^2 g_{\text{FLRW}}$ outside R_{match}, with A interpolated smoothly through the shell. This is (1)–(3). Because the interior data are literally those of Theorem on an open neighbourhood of the tube, the closed dark geodesics and the identification $B_0 J \otimes J = \omega \otimes \omega$ persist.

For (4), t is temporal for $\tilde{g}$ in each region: inside the core, official $\tilde{g}^{tt} = -1/(1 + |\omega|^2) < 0$; in the buffer and exterior, $\tilde{g}^{tt} = A^{-2} g^{tt} = -A^{-2} < 0$. Along a future $\tilde{g}$-causal curve, t is therefore strictly increasing, so each slice is crossed at most once. In the core, the official spatial-speed bound of Theorem applies. In the interpolated region, $|d\mathbf{X}/dt - \beta|^2 \le 1$, hence $|d\mathbf{X}/dt| \le 1 + |\beta|$. For $|\mathbf{X}| \le R_{\text{match}}$ one has $|\beta| \le H R_{\text{match}}$, a bound depending only on t; for $|\mathbf{X}| \ge R_{\text{match}}$ the inequality $\mathrm{d}R/\mathrm{d}t \le 1 + HR$ integrates to the standard comoving bound $\mathrm{d}(R/a)/\mathrm{d}t \le 1/a$. In all cases a finite terminal t-range yields a finite spatial path in complete Euclidean space, hence an endpoint and a continuation. Every inextendible $\tilde{g}$-causal curve therefore meets every constant-t slice exactly once.

For (5), the Einstein tensor of the official core is already accounted for as a residual in the W-CTC typestate, and the exterior residual is the official positive dark density of Theorem . On the shell, the only non-FLRW, non-Minkowski data are the interpolated shift and the radially interpolated scalar, both supported in

$\{R_{\mathrm{ext}} \le R \le R_{\mathrm{match}}\}$. The second fundamental form of the flat slices is $K_{ij} = -\partial_{(i}\beta_{j)}$. With $\beta = f\mathbf{X}$ and $f = f(t,R)$, one has $\partial_i\beta_j = f\delta_{ij} + (f_R/R)X_iX_j$, which is the displayed K_{ij}. The ADM Einstein tensor is then a local functional of K_{ij} and of the acceleration and spatial derivatives of the shift, together with the stress of $\varphi(t,R)$. The construction specifies the fields rather than solving an action, so the residual is kinematic.

Quantitative chronology-leakage bounds. The formula $\mathcal{O}(G^2E_{\mathrm{lab}}M_{\mathrm{core}}/R_{\mathrm{lab}}^4)$ is not derived and is not pasted as typestate. Official Chrono-DM-A records, as a source record, a formal first-order increment under a declared cover assumption, not this bound [5]. TM-1 typestate remains exact (undischarged). Proposition continues to block an automatic full-system no-signalling theorem. The host g of the official CTC core is not globally hyperbolic. A time-periodic compactification of $P_g = -\mathrm{Box}_g + m_{\mathrm{d}}^2$ is not automatically well-posed, and a non-resonance hypothesis is not automatic. Unique fixed-point solvability on a CTC background is not typestated. The following restricted claim is on the ordinary metric only.

Proposition (Restricted finite-slab small-gain on the ordinary metric). *Let $(M,\tilde{g})$ be the ordinary metric of Theorem on $\mathbb{R}_t \times \mathbb{R}^3$, which is globally hyperbolic, with Cauchy slices $\{t = \mathrm{const}\}$. Let $P := -\mathrm{Box}_{\tilde{g}} + m_{\mathrm{d}}^2$ with $m_{\mathrm{d}} > 0$. Fix smooth cutoffs χ_{lab} and χ_{core} with $\mathrm{supp}\,\chi_{\mathrm{lab}} \subset \{r \ge R_{\mathrm{lab}}\}$ and $\mathrm{supp}\,\chi_{\mathrm{core}} \subset \{r \le R_{\mathrm{ext}}\}$, and write $D := R_{\mathrm{lab}} - R_{\mathrm{ext}} > 0$. Let V_{core} be a smooth, real, compactly supported potential, and let $\mathcal{N}_{\mathrm{lab}}$ be a (possibly nonlocal in space, but pointwise in time) Lipschitz map of Sobolev spaces $H^s \to H^{s-1}$ for some $s > 2$, with $\mathcal{N}_{\mathrm{lab}}(0) = 0$. On a finite slab $[0,T] \times \mathbb{R}^3$, consider the retarded problem*

$$Pu = \kappa_{\mathrm{lab}}\chi_{\mathrm{lab}}\mathcal{N}_{\mathrm{lab}}[u] + \kappa_{\mathrm{core}}\chi_{\mathrm{core}}V_{\mathrm{core}}u + f_{\mathrm{in}},$$

with vanishing data at $t = 0$ and with $f_{\mathrm{in}} \in L^1([0,T];H^{s-1}(\mathbb{R}^3))$. Let $E^s(T) := C([0,T];H^s) \cap C^1([0,T];H^{s-1})$. There is a constant $C_*(T) < \infty$, depending on T, m_{d}, s, and the stationary geometry of $\tilde{g}$, such that the retarded solution of $Pu = F$ satisfies $\|u\|_{E^s(T)} \le C_*(T)\|F\|_{L^1([0,T];H^{s-1})}$. Consequently, if

$$L := C_*(T)(|\kappa_{\mathrm{lab}}|\mathrm{Lip}\,(\mathcal{N}_{\mathrm{lab}}) + |\kappa_{\mathrm{core}}|\|\chi_{\mathrm{core}}V_{\mathrm{core}}\|_{C^{s-1}}) < 1,$$

the retarded problem has a unique solution in $E^s(T)$*. Writing* u_{dec} *for the decoupled retarded solution with the same* f_{in} *and with either* $\kappa_{\mathrm{core}} = 0$ *or* $\chi_{\mathrm{core}} = 0$*, and writing* u_{lab} *and* u_{core} *for the solutions with only the laboratory or only the core coupling, the interaction remainder* $r := u - u_{\mathrm{dec}} - u^{\mathrm{lin}}_{\mathrm{core}}$ *(the part of u sourced by the composition of the two couplings) satisfies* $\|r\|_{E^s(T)} \le C|\kappa_{\mathrm{lab}}\kappa_{\mathrm{core}}|$ *for sufficiently small couplings. If T is smaller than the* $\tilde{g}$*-travel time across the gap D, the two supports are causally disjoint on the slab and the interaction remainder vanishes identically. This proposition does not treat the host operator* Box_g*, does not typestate well-posedness of a time-periodic problem on a CTC background, does not invent a closed contraction constant in* $(\kappa_{\mathrm{lab}}, \kappa_{\mathrm{core}}, \mu_{\mathrm{gap}}, D)$ *beyond the displayed L and the causal-gap vanishing, and does not withdraw Proposition . It does not close Obligation 3 of Section .*

Proof. The ordinary metric of Theorem is a smooth, stationary, compactly supported perturbation of Minkowski, and t is a Cauchy temporal function. The massive wave operator P is therefore a normally hyperbolic operator on a globally hyperbolic spacetime, and the Cauchy problem on $[0, T]$ is well-posed in the Sobolev energy spaces $E^s(T)$. Because the coefficients are stationary and smooth, and agree with Minkowski outside a compact set, one may commute spatial derivatives with P and apply the standard energy identity for the stress of a massive Klein–Gordon field. Integrating in time yields the estimate with a finite $C_*(T)$; on this stationary background $C_*(T)$ grows at most linearly in T. The map

$$\mathcal{K}[u] := P^{-1}_{\mathrm{ret}}(\kappa_{\mathrm{lab}}\chi_{\mathrm{lab}}\mathcal{N}_{\mathrm{lab}}[u] + \kappa_{\mathrm{core}}\chi_{\mathrm{core}}V_{\mathrm{core}}u + f_{\mathrm{in}})$$

is then a contraction of $E^s(T)$ of Lipschitz constant L whenever $L < 1$, hence has a unique fixed point. The decoupled and single-coupling solutions exist by the same argument. Subtracting the integral equations, the source of the interaction remainder is a bilinear (or Lipschitz-bilinear) expression in the two couplings, and is therefore $O(|\kappa_{\mathrm{lab}}\kappa_{\mathrm{core}}|)$ for small couplings, with the energy estimate upgrading this to the stated $E^s(T)$ bound. Finite propagation speed for P at the ordinary light speed of $\tilde{g}$, together with the spatial separation D, implies that the two cutoffs have disjoint influence domains on any slab shorter than the travel time across the gap.

This is a restricted ordinary-metric claim, not Breakthrough 4 of the t514u brief as stated. The latter remains refused on a CTC background.

Chronology-core Jeans theorems and Moser-trapped warm tubes. Linear Jacobi typestate must follow the signs recorded in official Chrono-DM-C version 3 as a source record: at a strict local maximum of $C_{-} = -g_{\theta\theta}$, the vertical mode is hyperbolic; a cratered local minimum of C_{-} is a different printed class [7]. On the official unit-lapse tube, with $a = \sqrt{r^2 + h}$ and $F = r^2$, the effective potential of timelike g-geodesics with fixed Killing constants is $V_{\text{eff}} = (L + aE)^2/F - E^2 + \varepsilon$. At the official circular orbit one finds $V_{rr} > 0$, but if h has a strict local maximum in z at the equatorial plane, as any nonnegative compactly supported height function on a bounded tube must have at its peak, then $V_{zz} < 0$. Thus the official compact unit-lapse tube does not furnish a positive-definite Hessian of V_{eff} at an interior maximum of h. A lapse well or a non-official (a, c) profile that would produce $\nabla^2_{r,z} V_{\text{eff}}(r_*, 0) \operatorname{succ} 0$ inside $\{c < 0\}$, together with a smooth compactly supported Vlasov density on trapped shells, is not written. The truncated Backward-C abstract's claims labelled prop : boltzmann-blowup and thm : chrono-jeans are absent from the truncated file and are not typestated. Joint (E, L_z) decay criteria, Moser twist, an energy–Casimir Lyapunov functional, a perpetually trapped positive-measure six-dimensional Vlasov tube, and dynamical Einstein back-reaction, are not typestated. Official C propositions on Jacobi and cratered rings remain documentary only. This is Breakthrough 3 of the t514u brief, and it remains refused.

Non-equatorial 3D gravito-optical shadow. Lemma typestates conservation of p_z, the independently checked three-dimensional null radial equation , and the inclined retrograde capture condition on a z-independent collar. Observer screen coordinates $(\alpha_{\text{scr}}, \beta_{\text{scr}})$, a closed-form sky-plane capture-strip inequality, and a complete two-dimensional shadow theorem with a crescent or toroidal time-advance boundary, are not proved. No negative Shapiro jump is typestated. This is Breakthrough 5 of the t514u brief as a screen-and-shadow programme, and that programme remains refused.

Architecture reminder. A prescribed chronology-split witness, an on-shell test potential, a cone-threshold lemma, a Sagnac Jacobian, a symmetron sandwich or bare-parameter bracket, a class-limited vortex

representation, a conditional insulation postulate, an equatorial first-integral band, a kinematic W-CTC/W-FLRW interpolation, a restricted ordinary-metric contraction, or any comparison in this appendix is not Bar 1, not Bar 2, and not Bar 3. Host C is not created. The working supplements used as source records are not official companions.

Documentary negative-control catalogue

The following one-page table isolates unverified external catalogue entries and refused slogans as documentary negative controls only [20]. They are not theorem premises, not scheduler-derived cones, not a time machine, not a Page curve, not a free clock, not worldline return, and not a decoder of an unread complement. They do not move the ordinary operational time-machine bar, which remains 0 of 3, and they do not typestate uniqueness of dark matter or a chronology engine in nature. Clock Track-I identities are not imported from this catalogue, and no Host C, quantum coefficient, island, bounce, or de Sitter state is typestated.

References

[1] Cheung, N. (2026). The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement. *Frontiers in Physics*, **14**, 1954943. DOI: 10.3389/fphy.2026.1954943. *Source record; not a premise.*

[2] Cheung, N. (2026). *Big06-A*, official version 2. Enter Calculus Companion, official version 2. *Source record; not a premise.* Documentary source record, including the discussed electromagnetic Euler-current typestate.

[3] Cheung, N. (2026). *Dual-Ledger-Mobility-A: Mobility asymmetry between an ordinary slot and a current-free dark slot on a prescribed host: a conditional fluid-plus-controller class, its conservation ledger, and declared inverse problems*. Enter Calculus Companion, official version 5 (request t495u). *Source record; not a premise.*

[4] Cheung, N. (2026). *Dual-Ledger-Temporal-A: Sector-Dependent Clock Rates and Transport on a Prescribed Lorentzian Host. Conservation, No-Freezing Results, Controlled Cosmological Witnesses, and Observational Rejection Contracts*. Enter Calculus Companion, official version 5 (request t504u). *Source record; not a premise.*

[5] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-A: Winding, Chronology Asymmetry, and Operational*

Insulation on Prescribed Lorentzian Hosts. Exact Geometric Witnesses, Corrected No-Paradox Results, and Complete-Record Rejection Contracts. Enter Calculus Companion, official version 4 (request t506u). *Source record; not a premise.*

[6] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-B: Class-Limited Einstein Matches, a Closed Causal-Geodesic Obstruction, On-Shell Ordinary Shields, a Prescribed Misner Overlay, and One-Sided Sagnac Identifiability on Prescribed Hosts*. Enter Calculus Companion, official version 4 (request t509u). *Source record; not a premise.*

[7] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-C: Continuum Dark-Field Self-Consistency, Detector Seam Rules, and a Prescribed Toroidal Chronology Split on* $\mathbb{R}^4$. Enter Calculus Companion, official version 3 (request t508u). *Source record; not a premise.* The official version 2 single-invariant on-shell pullback is withdrawn as not coordinate-consistent; the constructions of the present sheet are proved from definitions printed here.

[8] Bekenstein, J. D. (1993). The relation between physical and gravitational geometry. *Physical Review D*, **48**, 3641–3647. DOI: 10.1103/PhysRevD.48.3641.

[9] Gödel, K. (1949). An example of a new type of cosmological solutions of Einstein's field equations of gravitation. *Reviews of Modern Physics*, **21**, 447–450. DOI: 10.1103/RevModPhys.21.447.

[10] Hawking, S. W. (1992). Chronology protection conjecture. *Physical Review D*, **46**, 603–611. DOI: 10.1103/PhysRevD.46.603.

[11] Kay, B. S., Radzikowski, M. J., and Wald, R. M. (1997). Quantum field theory on spacetimes with a compactly generated Cauchy horizon. *Communications in Mathematical Physics*, **183**, 533–556. DOI: 10.1007/s002200050236.

[12] Aronszajn, N. (1957). A unique continuation theorem for solutions of elliptic partial differential equations or inequalities of second order. *Journal de Mathématiques Pures et Appliquées*, **36**, 235–249.

[13] Hinterbichler, K., and Khoury, J. (2010). Screening long-range forces through local symmetry restoration. *Physical Review Letters*, **104**, 231301. DOI: 10.1103/PhysRevLett.104.231301.

[14] Touboul, P., et al. (2022). MICROSCOPE mission: Final results of the test of the equivalence principle. *Physical Review Letters*, **129**, 121102. DOI: 10.1103/PhysRevLett.129.121102.

[15] Williams, J. G., Turyshev, S. G., and Boggs, D. H. (2004). Progress in lunar laser ranging tests of relativistic gravity. *Physical Review Letters*, **93**, 261101. DOI: 10.1103/PhysRevLett.93.261101.

[16] Bertotti, B., Iess, L., and Tortora, P. (2003). A test of general relativity using radio links with the Cassini spacecraft. *Nature*, **425**, 374–376. DOI: 10.1038/nature01997.

[17] Abbott, B. P., et al. (2017). Gravitational waves and gamma-rays from a binary neutron star merger: GW170817 and GRB 170817A. *The Astrophysical Journal Letters*, **848**, L13. DOI: 10.3847/2041-8213/aa920c.

[18] Clowe, D., Brada v c, M., Gonzalez, A. H., Markevitch, M., Randall, S. W., Jones, C., and Zaritsky, D. (2006). A direct empirical proof of the existence of dark matter. *The Astrophysical Journal Letters*, **648**, L109–L113. DOI: 10.1086/508162.

[19] Planck Collaboration, Aghanim, N., et al. (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, **641**, A6. DOI: 10.1051/0004-6361/201833910.

[20] OpenAI mathematics catalogue entries Oai365-A (Family 365), Oai290-A (Family 290), Oai265-A/B, Oai276-A, Oai212-A, and refused Family 260. Documentary negative controls only, isolated to Section ; never premises. Family 260 remains refused.