Dual-Ledger-Chrono-Shield-A

Nonsingular Bimetric Cauchy-Slice Protection, Transverse-Schwartz Collars, and Operational Portal-Archive No-Signalling Across Smooth Host CTC Cores Version 3

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Abstract

This sheet records official Version 3 typestate for Dual-Ledger-Chrono-Shield-A, as a typographic successor of official Dual-Ledger-Chrono-Shield-A version 2 [1]. Official Version 2 typestate, labels, and local proofs are retained. Every identity used later is re-proved locally from Definitions printed herein; the typographic parent is a documentary source record, not a premise. The official composite convention $\tilde{g}_{ab} = A^2 g_{ab} + B J_a J_b$ is used throughout. A smooth, geodesically complete host CTC torus $\mathscr{U}_{\mathrm{CTC}} = \{g_{\varphi\varphi} < 0\}$ may be nonempty, while the ordinary metric satisfies $\tilde{g}_{\varphi\varphi} > 0$ and every slice $\{t = t_0\}$ is a global Cauchy surface for $(\mathbb{R}^4, \tilde{g})$.

On the written Painlevé–Gullstrand host with time-independent shift, the inverse coefficients are typestated exactly, and the scalar d'Alembertian reduces to the flat Laplacian on every stationary field, closing the implicit curved-versus-flat step in the cylindrical on-shell realisation. An exact Cartan coframe calculation supplies the scalar curvature $R[g] = \frac{1}{4}\Omega_{ij}\Omega^{ij} \geq 0$ and, on the cylindrical collar, closed-form Ricci and Kretschmann scalars in the vorticity profile; the official uniform majorant C_K is retained and is not a vanishing theorem. At the written aspect ratio $\alpha_z = 3\alpha_r$, a diagonal two-switch field-space metric K_{AB} and potential W are written for which the Gaussian vortex lies in $\mathscr{S}(\mathbb{R}^3;\mathbb{C})$ and solves the Euler–Lagrange equations smoothly on $\mathbb{R}^4$. This is an exact classical solution at a written aspect ratio, not a general $\alpha_z > 0$ existence theorem, and not a refutation of the official

constant-stiffness obstruction. Finite energy is typestated only for the Schwartz vortex sector and integrable remainders, not for the full switch-plus-vortex action.

A discrete sharp-archive bit-flip consistency lemma and a retarded linearised ordinary Green proposition are typestated on the written globally hyperbolic ordinary background; neither discharges the operational time-machine bar, which remains 0 of 3. On the weak-field two-Fermat host, an explicit Sagnac-subtraction identity shows that the host conformal class used to read D_{sp} as an eigenvalue ratio is $[h^{(g)}_{ij}] = [\delta_{ij}]$, not $[n^{(g)}_{ij}]$ when $\omega_i \neq 0$. The residual $\Delta T_{\mathrm{GW-EM}}(0) < 0$ remains a bimetric optical–GW comparison. A finite-height cutoff of $\mathscr{U}_{\mathrm{CTC}}$ is proved at $\alpha_z = 3\alpha_r$; vanishing of the off-axis residual at that locus is not typestated. This is not a proof of Hawking's chronology protection conjecture in generality, not a semiclassical $\langle T_{ab} \rangle$ wall, not an extracted ordinary CTC, and not a traversable wormhole.

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Scope and typestate locks. par This official Version 3 is a typographic successor of official Dual-Ledger-Chrono-Shield-A version 2 only [1]. Official Version 2 typestate is retained. It is not a typographic successor of official Dual-Ledger-DM-Discovery-A, official Dual-Ledger-Chrono-DM-Backward-A, official

Dual-Ledger-Chrono-DM-A, official Dual-Ledger-DM-Inversion-A, or of any sibling companion. None of those manuscripts is a parent or a premise. Official Dual-Ledger and companion manuscripts named in the bibliography are documentary source records only. Every Theorem, Proposition, Lemma, and Corollary is proved locally from mathematical Definitions printed herein. No Hypothesis, Conjecture, or Declaration is used as a theorem premise. Identities documented in official Discovery-A version 5 (transverse-Schwartz typestate at $\alpha_z = 0$; four-cone isolation of D_{sp} given a host gravitational-wave baseline) remain a historical documentary comparison, re-proved locally and cited only as SR. When a live typestate comparison is made, official Discovery-A version 7 [8] is the current documentary record; d-branch typestate is not imported. Official Inversion-A version 6 (portal factorisation and Δc_2) is likewise re-proved locally and cited only as SR. A prescribed host CTC torus remains a kinematic feature of a written background, not an extracted ordinary CTC, not a traversable wormhole, and not Hawking chronology protection in generality. The motivational prose that this sheet would prove Hawking's chronology protection conjecture in generality, supply a semiclassical stress-tensor wall, construct a paradox-free operational time machine, typestate a general $\alpha_z > 0$ existence theorem, or discharge the 0 of 3 bar is not typestate. Official typestate follows only the complete local proofs below.

par **Standing ledger (wired=false, declaredPath=null)** Host A and Host B stay separate prescribed backgrounds. Host C is not created. O.1 stays NOT_CLOSED. H3 stays NOT_CLOSED. XV-B is not opened. XVI-N is not discharged. Gate 0 and ST4b stay NOT_SUPPLIED. The time-machine operational bar remains 0 of 3. No quantum coefficient is admitted. The shredder is not a Gibbons–Hawking construction. No Einstein dynamics, derived metric, spacetime emergence, Page curve, island, evaporation process, Hawking temperature, Hawking coefficient, acoustic or optical Hawking analogue, Penrose or superradiance theorem, firewall theorem, semiclassical $\langle T_{ab} \rangle$ wall, or proof of Hawking chronology protection in generality is claimed. Problem 1 and the enlarged duration gate stay open. J2 is unresolved. Inversion conditions (ii), (iv), and (v) are not typestated as closed in full generality. XVI-N-RouteB is not discharged. Official Discovery Breakthroughs 1–4 remain refused. Conjectures D1–D5 are not theorem premises of this sheet. Every Theorem, Proposition, Lemma, and Corollary is proved locally from printed Definitions. No Hypothesis, Conjecture, or Declaration is a theorem premise. A source

record is not a premise. The OpenAI catalogue is not a premise. Geometric TM-2 remains a counterfactual postulate, not an operational time machine. Ordinary Cauchy-slice blindness, if mentioned, is documentary only. No ordinary closed timelike curve is extracted. No traversable wormhole is typestated. The residual $\Delta T_{\mathrm{GW-EM}}$ is a bimetric optical–GW comparison, never an operational CTC and never causal signalling from a host CTC core.

Prescribed mathematical setup

The declarations of independence for this sheet are as follows. Host A and Host B remain separate prescribed backgrounds. Host C is not created. No Page-curve, island, or vacuum-energy-as-Host-C language is used. O.1 remains NOT_CLOSED. H3 remains NOT_CLOSED. Gate 0 and ST4b remain NOT_SUPPLIED. XV-B is not opened. XVI-N is not discharged, except the class-limited negative of t441 XVI-N-Univ-A, which is not reopened. The time-machine operational bar remains 0 of 3 and is not discharged by any identity of this sheet. Ordinary Cauchy-slice blindness, if mentioned, is documentary only. Geometric TM-2, if mentioned, remains a counterfactual postulate, not an operational time machine. No quantum coefficient is admitted. The shredder, if mentioned, is not a Gibbons–Hawking construction. Hawking radiation, Penrose or Sagnac superradiance, and a firewall theorem are not typestated. The source is not companion-wired: wired=false, and declaredPath remains null. Companion wiring and book, figshare, or Zenodo deposit are out of scope. Citations are source records only. Siblings are never premises. Family 260 remains refused. Family 365, if mentioned, is boundary-metric recovery, not a scheduler-derived cone. Family 290 is not a decoder of an unread complement. No catalogue family supplies a traversable wormhole, a native operational carrier, a tick that selects T_*, a conscious dark halo, a bidirectional cosmic identity, or a closed agency gate. Unimodular gravity, Einstein dynamics, ultraviolet completeness, exact weak equivalence or vanishing Eötvös parameter in nature, and invented SKA or observational thresholds are not typestated.

Definition (Official disformal composite). Let (M, g_{ab}) be a smooth Lorentzian four-manifold of signature $(-,+,+,+)$, with $c = 1$. Let $A > 0$ and $B \in \mathbb{R}$ be prescribed smooth coefficients, and let Φ be a prescribed smooth complex scalar. The official Dual-Ledger composite is

$$\tilde{g}_{ab} = A^2 g_{ab} + B\, J_a[\Phi]\, J_b[\Phi], \qquad J_a[\Phi] := \mathrm{Imag}\,(\Phi^* \partial_a \Phi).$$

Wherever $D := 1 + (B/A^2) g^{ab} J_a[\Phi] J_b[\Phi] \neq 0$, the inverse and volume ratio are

$$\tilde{g}^{ab} = A^{-2}\left(g^{ab} - \frac{B}{A^2 D} J^a J^b\right), \qquad \frac{\mathrm{vol}_{\tilde{g}}}{\mathrm{vol}_g} = A^4 \sqrt{D},$$

with $J^a := g^{ac} J_c[\Phi]$. This is a kinematic prescription on a prescribed host. It is not Einstein dynamics, not a derived metric, and not a claim of spacetime emergence. The disformal interface stands in the classical tradition recorded by Bekenstein [10] (*source record; not a premise*). Official Discovery-A versions 4 and 5 [6, 7] and official Inversion-A version 6 [9] record documentary uses of the same convention and are not premises.

Definition (Prescribed stationary pair with a host CTC torus and ordinary Cauchy slices). A *prescribed stationary Lorentzian pair with a host CTC torus* on $M = \mathbb{R}^4$, with Cartesian coordinates (t, x, y, z) and cylindrical radius $r := \sqrt{x^2 + y^2}$, consists of a smooth stationary Lorentzian metric g_{ab} and the official composite $\tilde{g}_{ab}$ of Definition , subject to the following written clauses.

1. The host metric g_{ab} is geodesically complete, and the open set $\mathscr{U}_{\mathrm{CTC}} := \{g_{\varphi\varphi} < 0\}$ is permitted to be nonempty, where $\partial_\varphi := -y\partial_x + x\partial_y$ is the axial Killing field. When nonempty, $\mathscr{U}_{\mathrm{CTC}}$ is called the host CTC torus. Completeness of g is a regularity clause of the pair, not an extraction of an ordinary closed timelike curve.

2. The ordinary metric satisfies $\tilde{g}_{\varphi\varphi} > 0$ wherever the axial field is defined, and every slice $\{t = t_0\}$ is a smooth global Cauchy surface for $(\mathbb{R}^4, \tilde{g})$. In particular $(\mathbb{R}^4, \tilde{g})$ is globally hyperbolic, and no ordinary closed timelike curve is typestated.

3. A local quantum or classical archive register is supported on a written slice $\{t = t_0\}$, in the sense of Definition below. The pair is a prescribed background. Host C is not created. Geometric TM-2 remains a counterfactual, not an operational time machine. Ordinary Cauchy-slice blindness, if alluded to, is documentary only and is not discharged.

Definition (Schwartz collar class, with honest typestate at $\alpha_z = 0$). Fix parameters $c_0 > 0$, $\alpha_r > 0$, $\alpha_z \geq 0$, a

prescribed positive constant conformal factor $A > 0$, and a prescribed coefficient $B > 0$. Write $R := r^2 = x^2 + y^2$ and $Z := z^2$. The *Gaussian vortex amplitude* and complex scalar are

$$h(R, Z) := c_0 \exp(-\alpha_r R - \alpha_z Z) > 0, \qquad \Phi(x, y, z) := h(r^2, z^2)\,(x + \mathrm{i}y).$$

The associated current and shift are

$$J[\Phi] = h^2(x\,\mathrm{d}y - y\,\mathrm{d}x), \qquad \omega := \frac{\sqrt{B}}{A} h^2(x\,\mathrm{d}y - y\,\mathrm{d}x).$$

The host and the official composite are then written in Painlevé–Gullstrand form

$$g = -(\mathrm{d}t + \omega)^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2, \qquad \hat{g} = A^2 g + B\,J[\Phi] \otimes J[\Phi].$$

Function-space membership is split as follows.

1. If $\alpha_r > 0$ and $\alpha_z > 0$, then $\Phi \in \mathscr{S}(\mathbb{R}^3; \mathbb{C})$, with exponential decay as $r^2 + z^2 \to \infty$.

2. If $\alpha_z = 0$, then Φ lies in the cylindrical transverse Schwartz class $\mathscr{S}(\mathbb{R}^2_{x,y}; \mathbb{C}) \mathbin{\hat{\otimes}} 1_z \subset C^\infty(\mathbb{R}^3; \mathbb{C})$: it is smooth, cylindrically translation-invariant in z, and Schwartz in the transverse plane (x, y), decaying as $r \to \infty$ uniformly in z. It is *not* an element of $\mathscr{S}(\mathbb{R}^3)$. Approach to Minkowski as $|z| \to \infty$ at fixed r, and in particular along $r = 0$, is not typestated. This is the official collar class of the present sheet. Official Discovery-A version 5 [7] and official Backward-A version 4 [2] record documentary typestate of the same split and are not premises. Three-dimensional Schwartz integrability is not claimed at $\alpha_z = 0$.

Definition (Single-switch controller action on the collar). On the cylindrical collar $\alpha_z = 0$, write $s_1 := R = r^2$ for the single switch, with prescribed constant stiffness $f_s > 0$. The controller action used below is the restriction

$$S_\Phi + S_s = -\int (\tfrac{1}{2} \nabla_a \Phi^* \nabla^a \Phi + V_\Phi(|\Phi|^2, s_1) + \tfrac{1}{2} f_s^2 (\nabla s_1)^2 + W(s_1))\,\mathrm{vol}_g,$$

of a host-volume controller, with the written single-switch potential $V_\Phi(u, s_1) = \frac{1}{2}\mu(s_1)u$ and with $\mu(s_1) := 4\alpha_r^2 s_1 - 8\alpha_r$. The Euler–Lagrange operators E_{Φ^*} and E_{s_1} are the formal $L^2(g)$ variations of this action in Φ^* and s_1. No Einstein equation, no unimodular constraint, and no host-compatibility closure is typestated. The

five-slot inventory documented in official Discovery-A [7] (*source record; not a premise*) is not a premise; only the written collar action is used. On the written aspect ratio $\alpha_z = 3\alpha_r > 0$, the same host-volume controller is extended by a two-switch field-space metric and potential, with switches (s_1, s_2) and written data

$$K_{11}(s_1, s_2) = f_1^2 + \tfrac{1}{2}\alpha_r c_0^2 e^{-2\alpha_r s_1 - 6\alpha_r s_2} \geq f_1^2 > 0, \qquad K_{22} = f_2^2 > 0, \qquad K_{12} = 0,$$

for prescribed constant stiffnesses $f_1, f_2 > 0$, and

$$W(s_1, s_2) = 4f_1^2 s_1 + 2f_2^2 s_2 + 2\alpha_r c_0^2 s_1 e^{-2\alpha_r s_1 - 6\alpha_r s_2}$$

(up to an additive constant), together with $V_\Phi = \tfrac{1}{2}\mu(s_1, s_2)u$ and with the official μ of Lemma . The two-switch action is

$$S_\Phi + S_s = -\int (\tfrac{1}{2}\nabla_a \Phi^* \nabla^a \Phi + V_\Phi(|\Phi|^2, s_1, s_2) + \tfrac{1}{2}K_{AB}(s)\,\nabla s^A \cdot \nabla s^B + W(s_1, s_2))\,\mathrm{vol}_g.$$

The Euler–Lagrange operators E_{Φ^*}, E_{s_1}, and E_{s_2} are the formal $L^2(g)$ variations of this extended action. Positive-definiteness $K_{AB} > 0$ is a classical kinetic-metric statement on this written class ("no ghost kinetic term"), not a quantum no-ghost theorem. No uniqueness of K_{AB}, no dynamical stability, and no general $\alpha_z > 0$ existence theorem is typestated by writing this action.

Definition (Ordinary archive register on a Cauchy slice). An *ordinary archive register* on a written slice $\{t = t_0\}$ is a classical outcome space (Ω_o, ν) or a finite-dimensional density-operator algebra, together with a local non-gravitational ordinary probe whose outcomes or states are supported on that slice and couple to ordinary fields through the ordinary metric $\tilde{g}$. The register is an ordinary, Cauchy-slice object. It is not a native operational carrier, not a traversable-wormhole mouth, and not an extracted ordinary closed timelike curve.

Definition (Portal factorisation clause). A local non-gravitational ordinary probe, with outcome space (Ω_o, ν) or density operators on a finite-dimensional probe algebra, is said to obey the *portal factorisation clause* $\lambda_{\mathrm{port}} = 0$ when, at a fixed prescribed host metric (M, g_{ab}), every ordinary likelihood satisfies $p(x \mid \theta_d, \theta_o; 0) = p_o(x \mid \theta_o)$ and every ordinary probe state satisfies $\rho_{\mathrm{probe}}(\theta_d, \theta_o) = \rho_{\mathrm{probe}}(\theta_o)$, independently of all dark-sector parameters θ_d. Fisher

insulation will be typestated only under this clause. The clause is a written kinematic factorisation. It is not typestated as dynamically forced by quantum gravity.

Definition (Operational causal signalling from a host CTC into an ordinary archive). Let a prescribed pair as in Definition be held fixed, including the host metric g_{ab}. An operational protocol is said to *extract causal signalling from a closed g-timelike curve in* $\mathscr{U}_{\mathrm{CTC}}$ *into an ordinary archive* if there exists a dark-sector parameter θ_d labelling a controllable payload or state supported along a written closed g-timelike curve in $\mathscr{U}_{\mathrm{CTC}}$, such that the ordinary archive law $p(x \mid \theta_d, \theta_o)$ or the ordinary archive state $\rho_{\mathrm{archive}}(\theta_d, \theta_o)$ depends nontrivially on θ_d. A static or kinematic bimetric residual of the prescribed pair—in particular a lensing residual or an arrival-time residual $\Delta T_{\mathrm{GW-EM}}$—is *not* operational signalling in this sense: it does not vary a controllable dark payload at fixed host geometry, and it does not furnish an ordinary closed timelike curve. Gravitational-wave or optical comparisons at a fixed background remain residuals, not communication channels from $\mathscr{U}_{\mathrm{CTC}}$.

Definition (Formal algebraic vacuum-shift polynomial). Given a prescribed pair of finite mass-squared matrices (M_B^2, M_F^2) on real bosonic and fermionic polarisations, with ranks n_B and n_F, together with an ultraviolet cutoff scale $\Lambda_{\mathrm{UV}} > 0$ and a renormalisation reference $\mu > 0$, the *formal algebraic vacuum-shift polynomial* is

$$\delta\rho_{\mathrm{vac}}^{\mathrm{alg}} := \rho_{\mathrm{bag}} + \frac{1}{64\pi^2}[c_4 \Lambda_{\mathrm{UV}}^4 + 2\, c_2 \Lambda_{\mathrm{UV}}^2 + c_0(\mu)],$$

where

$$c_4 := n_B - n_F, \qquad c_2 := \mathrm{Tr}\,(M_B^2) - \mathrm{Tr}\,(M_F^2),$$

$$c_0(\mu) := \mathrm{Tr}\,(M_B^4 \ln'(M_B^2/\mu^2)) - \mathrm{Tr}\,(M_F^4 \ln'(M_F^2/\mu^2)),$$

with $x^2 \ln'(x/\mu^2) := x^2 \ln(x/\mu^2)$ for $x > 0$ and 0 for $x = 0$, and with every trace equal to 0 on a rank-zero empty matrix. A bag term ρ_{bag} is a prescribed constant. Equation is an algebraic filter functional on prescribed matrices. It is not an admitted quantum coefficient, not a path-integral theorem, and not an identification of an absolute cosmological constant. Official Inversion-A version 6 [9] and official Discovery-A version 5 [7] record documentary uses of the same polynomial and are not premises.

Definition (Written scalar Higgs-portal ansatz and higher Wilson operators). A *written scalar Higgs-portal ansatz* consists of a prescribed real scalar χ with $n_B \geq 1$ bosonic polarisations and vanishing bare mass $m_{\chi,0} = 0$, coupled to a prescribed Higgs condensate $|H|^2 \to v_{\mathrm{EW}}^2/2$ by

$$\mathscr{L}_{\mathrm{port}} = -\frac{1}{2}\lambda_{H\chi}|H|^2\chi^2,$$

with portal coupling $\lambda_{H\chi} \in \mathbb{R}$. On this ansatz one identifies λ_{port} with $\lambda_{H\chi}$. In addition, the two Planck-suppressed operators

$$\mathscr{L}_5 = \frac{c_5}{\mathrm{Mp}}|H|^2\overline{\psi}_d\psi_d, \qquad \mathscr{L}_6 = \frac{c_6}{\mathrm{Mp}^2}|H|^2(\nabla\varphi)^2,$$

may be written, with dimensionless coefficients $c_5, c_6 \in \mathbb{R}$, a prescribed Dirac field ψ_d of $n_F \geq 2$ real fermionic polarisations, a prescribed real scalar φ, and a prescribed reduced Planck scale $\mathrm{Mp} > 0$. On the written ansatz, $\mathscr{L}_5$ is the only mass source for ψ_d, and φ has vanishing bare mass unless an oscillating misalignment class is separately written.

Definition (Prescribed stationary dual-ledger background with two Fermat tensors). A *prescribed stationary dual-ledger background with spatial drift* on $\mathbb{R}_t \times \mathbb{R}^3_{\mathbf{x}}$ consists of a stationary host line element

$$g_{ab}\,\mathrm{d}x^a\mathrm{d}x^b = -(1 + 2\Psi_N(\mathbf{x}))\,\mathrm{d}t^2 + 2\omega_i(\mathbf{x})\,\mathrm{d}t\,\mathrm{d}x^i + (1 - 2\Phi_N(\mathbf{x}))\,\delta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j,$$

together with smooth dressing data $A(\mathbf{x}) > 0$, $B(\mathbf{x}) \in \mathbb{R}$, and a covector $J_a(\mathbf{x}) = (J_0(\mathbf{x}), J_i(\mathbf{x}))$, with the Lorentzian-signature requirement that $\widetilde{g}_{ab} = A^2 g_{ab} + BJ_aJ_b$ have signature $(-,+,+,+)$. Write $K := -\widetilde{g}_{00}$, $N_i := \widetilde{g}_{0i}$, $\gamma_{ij} := \widetilde{g}_{ij}$, and $X := \gamma^{ij}N_iN_j/K$ whenever these are defined and $K > 0$. The ordinary Fermat representation of future ordinary null rays is

$$\mathrm{d}t = \frac{N_i}{K}\mathrm{d}x^i + \sqrt{n_{ij}\,\mathrm{d}x^i\mathrm{d}x^j}, \qquad n_{ij} := \frac{\gamma_{ij}}{K} + \frac{N_iN_j}{K^2}.$$

The ordinary Fermat spatial tensor is $h_{ij} := n_{ij} - w_iw_j$, with $w_i := N_i/K$. The host gravitational-wave spatial Fermat tensor $h^{(g)}_{ij}$ is the same construction computed from the host metric g_{ab} rather than from $\widetilde{g}_{ab}$. When the host shift

vanishes, the conformal class $[\delta_{ij}]$ may be used as the host spatial baseline. The spatial matrix determinant is

$$D_{\mathrm{sp}} := \det\left(\delta^{i}_{j} + \frac{BJ^{i}J_{j}}{A^{2}(1-2\Phi_{N})}\right),$$

which is not the four-dimensional factor D of . Determination of D_{sp} will be typestated only in the presence of the host gravitational-wave spatial baseline. Official Discovery-A version 5 [7] records documentary four-cone typestate and is not a premise.

Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; H3 stays NOT_CLOSED; XV-B is not opened; XVI-N is not discharged; the time-machine bar remains 0 of 3.

Transverse-Schwartz collar versus three-dimensional Schwartz non-integrability

The first official task is to typestate the collar with exact function-space precision, to realise it on shell on the cylindrical switch, and to bound curvature invariants uniformly, including across the host CTC boundary. The false Laplacian identity $\Delta_{\mathbb{R}^3}\Phi = \lambda\Phi$ with an unspecified scalar λ is not restored; the repaired identity of Lemma is used instead. Official Backward-A version 4 [2] and official Discovery-A version 5 [7] are documentary source records of related collar calculations and are not premises. The implicit identification of the host Euler operator with the flat Laplacian on a stationary profile is closed by Lemma . The official constant-stiffness obstruction of Theorem (3) is retained intact; a written diagonal two-switch realisation at the aspect ratio $\alpha_z = 3\alpha_r$ is typestated separately in Theorem .

Lemma (Exact stationary d'Alembertian reduction on the Painlevé–Gullstrand host). *On the written Painlevé–Gullstrand host* $g = -(\mathrm{d}t + \omega)^2 + \mathrm{d}\mathbf{x}^2$ *of , with time-independent shift* $\omega = \omega_i(\mathbf{x})\,\mathrm{d}x^i$*, the coframe* $\theta^0 = \mathrm{d}t + \omega_i\,\mathrm{d}x^i$, $\theta^i = \mathrm{d}x^i$ *is unipotent with Jacobian determinant* 1*, and*

$$\det g = -1, \qquad g^{tt} = -(1-|\omega|^2), \qquad g^{ti} = -\omega^i, \qquad g^{ij} = \delta^{ij}.$$

In particular the quadratic shift sits in g_{ij} *and in* g^{tt}*, not in the spatial inverse* g^{ij}*. For every stationary scalar or complex scalar* ($\partial_t\psi = 0$) *one therefore has*

$$\mathrm{square}_g\,\psi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\psi) = \Delta_{\mathbb{R}^3}\psi,$$

to all orders in ω. The identity is for the written Painlevé–Gullstrand host and for stationary fields. It is not typestated for the official composite $\hat{g}$*, for time-dependent fields, or for the weak-field host without a separate proof. If the ordinary metric were used in place of g, the spatial inverse would not be* δ^{ij}.

Proof. In the coframe $\theta^0 = \mathrm{d}t + \omega_i\,\mathrm{d}x^i$, $\theta^i = \mathrm{d}x^i$, one has $g = -(\theta^0)^2 + \delta_{ij}\theta^i\theta^j$. The change of coframe from $(\mathrm{d}t, \mathrm{d}x^i)$ to (θ^0, θ^i) is unipotent, with Jacobian determinant 1, so $\det g = \det\eta = -1$ and $\sqrt{-g} = 1$. The dual frame is $e_0 = \partial_t$ and $e_i = \partial_i - \omega_i\partial_t$. The inverse metric is therefore $g^{-1} = -e_0\otimes e_0 + \delta^{ij}e_i\otimes e_j$. Expanding

$$\delta^{ij}e_i\otimes e_j = \delta^{ij}\partial_i\otimes\partial_j - \omega^i(\partial_t\otimes\partial_i + \partial_i\otimes\partial_t) + |\omega|^2\,\partial_t\otimes\partial_t$$

yields exactly . In particular $g^{ij} = \delta^{ij}$ at every order in ω.

Now let ψ be a (complex) scalar with $\partial_t\psi = 0$. Then $g^{tv}\partial_v\psi = g^{ti}\partial_i\psi = -\omega^i\partial_i\psi$ and $g^{jv}\partial_v\psi = \delta^{jk}\partial_k\psi$. Since ω and ψ are time-independent, $\partial_t(g^{tv}\partial_v\psi) = 0$. With $\sqrt{-g} = 1$, the scalar d'Alembertian therefore collapses to

$$\mathrm{square}_g\,\psi = \partial_j(\delta^{jk}\partial_k\psi) = \Delta_{\mathbb{R}^3}\psi,$$

which is . The computation uses $g^{ij} = \delta^{ij}$ and stationarity throughout; it does not apply as written to a time-dependent field, nor to a metric whose spatial inverse fails to be δ^{ij}. In particular, the ordinary representative $\hat{g} = -\mathrm{d}t^2 - 2\,\mathrm{d}t\,\omega + \mathrm{d}\mathbf{x}^2$ of has spatial inverse not equal to δ^{ij}, and the weak-field host is a different ansatz.

Lemma (Repaired Laplacian identity for a Schwartz vortex amplitude). *Let* $h \in C^\infty(\mathbb{R}_{\geq 0}\times\mathbb{R}_{\geq 0})$ *and* $\Phi(x,y,z) := h(R,Z)\,(x+\mathrm{i}y)$ *with* $R := x^2+y^2$ *and* $Z := z^2$. *Then, pointwise on* $\mathbb{R}^3$,

$$\Delta_{\mathbb{R}^3}\Phi = \lambda(R,Z)\,(x+\mathrm{i}y), \qquad \lambda(R,Z) := 4R\,\partial_R^2 h + 8\,\partial_R h + 4Z\,\partial_Z^2 h + 2\,\partial_Z h.$$

In particular, if $h > 0$, *one may write* $\Delta_{\mathbb{R}^3}\Phi = \mu\Phi$ *with* $\mu := \lambda/h$. *For the strictly positive Gaussian* $h(R,Z) = c_0\exp(-\alpha_r R - \alpha_z Z)$ *with* $c_0 > 0$, $\alpha_r > 0$, *and* $\alpha_z \geq 0$,

$$\mu(R, Z) = 4\alpha_r^2 R + 4\alpha_z^2 Z - 8\alpha_r - 2\alpha_z,$$

which is a globally smooth affine function of (R, Z)*. The quadratic coefficients are* 4*, not* 16*.*

Proof. Write $w := x + \mathrm{i}y$ and $\Phi = h\,w$. Then $\partial_x h = 2x\,\partial_R h$, $\partial_y h = 2y\,\partial_R h$, and $\partial_z h = 2z\,\partial_Z h$. Differentiating gives $\partial_{xx}\Phi = (2\partial_R h + 4x^2\partial_R^2 h)w + 4x\,\partial_R h$, $\partial_{yy}\Phi = (2\partial_R h + 4y^2\partial_R^2 h)w + 4\mathrm{i}y\,\partial_R h$, and $\partial_{zz}\Phi = (2\partial_Z h + 4z^2\partial_Z^2 h)w$. Summing and using $4x\,\partial_R h + 4\mathrm{i}y\,\partial_R h = 4w\,\partial_R h$ yields . For the Gaussian, $\partial_R h = -\alpha_r h$, $\partial_R^2 h = \alpha_r^2 h$, $\partial_Z h = -\alpha_z h$, and $\partial_Z^2 h = \alpha_z^2 h$, hence $\lambda = h(4\alpha_r^2 R - 8\alpha_r + 4\alpha_z^2 Z - 2\alpha_z)$ and .

Lemma (Uniform bounds on the shift and a written curvature majorant). *On the collar class of Definition , write* $\lambda_\omega := \sqrt{B}/A > 0$ *and* $\omega = \lambda_\omega h^2(x\,\mathrm{d}y - y\,\mathrm{d}x)$*. Then*

$$\|\omega\|_{L^\infty(\mathbb{R}^3)} \le C_0 := \lambda_\omega c_0^2 (4\alpha_r e)^{-1/2} < \infty,$$

even when $\alpha_z = 0$*. More generally, for every multiindex* κ *of length* $|\kappa| \le 2$*, the partial derivatives of the Cartesian components of* ω *obey*

$$\|\partial^\kappa \omega_i\|_{L^\infty(\mathbb{R}^3)} \le C_{|\kappa|},$$

with the written constants

$$\begin{aligned} C_1 &:= \lambda_\omega c_0^2 \Big(1 + \tfrac{2}{e} + \tfrac{1}{e}\sqrt{\tfrac{\alpha_z}{\alpha_r}}\,1_{\alpha_z > 0}\Big), \\ C_2 &:= \lambda_\omega c_0^2 \Big(6\sqrt{\tfrac{\alpha_r}{e}}\big(1 + \tfrac{\sqrt{3}}{e}\big) + 1_{\alpha_z > 0}\big[2\sqrt{\tfrac{\alpha_z}{e}}\big(1 + \tfrac{1}{e}\big) + 2\big(1 + \tfrac{4}{e}\big)\tfrac{\alpha_z}{\sqrt{\alpha_r e}}\big]\Big). \end{aligned}$$

(If $\alpha_z = 0$*, the displayed* α_z *terms are omitted, and the bounds remain uniform in z.) Let* $\mathrm{Riem}[g]$ *and* $\mathrm{Riem}[\widetilde{g}]$ *denote the Riemann tensors of g and* $\widetilde{g}$*. The pointwise majorant*

$$C_K := 256\big(C_2 + 4C_0C_2 + 8C_1^2 + 8C_0C_1^2 + 4C_0^2C_2 + 2C_0^2C_1^2\big)^2(1 + C_0^2)^4 < \infty$$

satisfies

$$|R_{abcd}[g]R^{abcd}[g]| \le C_K, \qquad |R_{abcd}[\widetilde{g}]R^{abcd}[\widetilde{g}]| \le A^{-4}C_K,$$

everywhere on $\mathbb{R}^4$*, including across* $\partial\mathscr{U}_{\mathrm{CTC}}$*. The same majorant bounds the squared Ricci norms and the squared Ricci scalars up to the universal combinatorial factor* 16 *coming from index contractions against a metric whose eigenvalues are controlled by* $1 + C_0^2$*. This is a bound, not a vanishing theorem, and not a semiclassical stress-tensor computation. In addition, every curvature component of the written Painlevé–Gullstrand host is a polynomial in the spatial vorticity* $\Omega_{ij} := \partial_i\omega_j - \partial_j\omega_i$ *and in* $\nabla^{\mathbb{R}^3}\Omega$*. The scalar curvature is exactly*

$$R[g] = \frac{1}{4}\Omega_{ij}\Omega^{ij} \geq 0,$$

with spatial indices raised by δ^{ij}*. On the cylindrical collar* $\alpha_z = 0$*, the only independent vorticity component is*

$$\pi(r) := \frac{a'(r)}{r} = 2\lambda_\omega c_0^2(1 - 2\alpha_r r^2)e^{-2\alpha_r r^2}, \qquad a(r) := \lambda_\omega h^2 r^2,$$

and reduces to $R[g] = \frac{1}{2}\pi(r)^2$*. In the orthonormal coframe of the proof, the remaining Ricci components on that collar are* $R_{00} = \frac{1}{2}\pi^2$*,* $R_{xx} = R_{yy} = \frac{1}{2}\pi^2$*,* $R_{zz} = 0$*,* $R_{0x} = -\frac{1}{2}\pi'(y/r)$*, and* $R_{0y} = \frac{1}{2}\pi'(x/r)$*, with* $\pi' := \mathrm{d}\pi/\mathrm{d}r$*. Consequently the squared Ricci norm and the Kretschmann scalar on the cylindrical collar are the closed forms*

$$R_{ab}[g]R^{ab}[g] = \tfrac{3}{4}\pi^4 - \tfrac{1}{2}(\pi')^2, \qquad R_{abcd}[g]R^{abcd}[g] = \tfrac{11}{4}\pi^4 - 2(\pi')^2.$$

The official majorant C_K *of is retained as a coarser uniform bound, valid also for* $\alpha_z > 0$*. The curvature invariants of g extend smoothly across* $\partial\mathscr{U}_{\mathrm{CTC}}$*, remain finite there, and are functions of vorticity data rather than of the level set* $\{g_{\varphi\varphi} = 0\}$ *itself. This is not a vanishing theorem, not a claim that the CTC boundary is invisible to all local curvature invariants, and not a semiclassical* $\langle T_{ab}\rangle$ *wall.*

Proof. The Cartesian components are $\omega_x = -\lambda_\omega h^2 y$, $\omega_y = \lambda_\omega h^2 x$, and $\omega_z = 0$. Hence $|\omega| = \lambda_\omega h^2 r = \lambda_\omega c_0^2 r\exp(-2\alpha_r r^2 - 2\alpha_z z^2)$. The elementary one-variable maximum of $re^{-2\alpha_r r^2}$ equals $(4\alpha_r e)^{-1/2}$ and is attained at $r = (4\alpha_r)^{-1/2}$, independently of z. This gives , and in particular $\|\omega\|_{L^\infty} < \infty$ when $\alpha_z = 0$. For derivative bounds, write $\lambda := \lambda_\omega c_0^2$ and $E := \exp(-2\alpha_r r^2 - 2\alpha_z z^2)$, so that $\omega_x = -\lambda y E$, $\omega_y = \lambda x E$, and $\omega_z = 0$. Differentiating gives $\partial_x\omega_x = 4\lambda\alpha_r xyE$, $\partial_y\omega_x = -\lambda(1 - 4\alpha_r y^2)E$, and $\partial_z\omega_x = 4\lambda\alpha_z yzE$; the first derivatives of ω_y follow by the

interchange $x \leftrightarrow y$ (with a sign), and every derivative of ω_z vanishes. The elementary bounds $r^n e^{-2\alpha_r r^2} \le (n/(4\alpha_r e))^{n/2}$ and, when $\alpha_z > 0$, $|z|^k e^{-2\alpha_z z^2} \le (k/(4\alpha_z e))^{k/2}$ are used throughout, together with $|xy| \le r^2/2$ and $E \le e^{-2\alpha_r r^2} e^{-2\alpha_z z^2}$. Thus $|\partial_x \omega_x| \le 2\lambda\alpha_r r^2 e^{-2\alpha_r r^2} \le \lambda/e$, while $|\partial_y \omega_x| \le \lambda(1 + 4\alpha_r y^2)e^{-2\alpha_r r^2} \le \lambda(1 + 2/e)$. If $\alpha_z = 0$, then $\partial_z \omega_x = 0$. If $\alpha_z > 0$,

$$|\partial_z \omega_x| \le 4\lambda\alpha_z (4\alpha_r e)^{-1/2}(4\alpha_z e)^{-1/2} = \frac{\lambda}{e}\sqrt{\frac{\alpha_z}{\alpha_r}}.$$

The same majorants bound the first derivatives of ω_y. This is for C_1; in particular the mixed radial–axial maximum of order $\sqrt{\alpha_z/\alpha_r}$ is included.

For second derivatives, the largest purely radial contribution is $|\partial_y^2 \omega_x| = 4\lambda\alpha_r |y||3 - 4\alpha_r y^2| E \le 12\lambda\alpha_r r e^{-2\alpha_r r^2} + 16\lambda\alpha_r^2 r^3 e^{-2\alpha_r r^2}$. The two elementary radial maxima then give $|\partial_i \partial_j \omega_k| \le 6\lambda\sqrt{\alpha_r/e}(1 + \sqrt{3}/e)$ whenever neither index is z. If $\alpha_z = 0$, every z-derivative vanishes and this is C_2. If $\alpha_z > 0$, the mixed and axial second derivatives satisfy

$$|\partial_x \partial_z \omega_x| \le \frac{2\lambda}{e}\sqrt{\frac{\alpha_z}{e}}, \qquad |\partial_y \partial_z \omega_x| \le 2\lambda\sqrt{\frac{\alpha_z}{e}}\left(1 + \frac{1}{e}\right),$$

and

$$|\partial_z^2 \omega_x| \le 2\lambda\left(1 + \frac{4}{e}\right)\frac{\alpha_z}{\sqrt{\alpha_r e}},$$

with the same majorants after the interchange $x \leftrightarrow y$ for ω_y. Adding these contributions to the radial bound produces the written C_2. These majorants are deliberately loose; they are finite and explicit, and they include the mixed radial–axial maxima that a purely separated estimate would omit.

The metric coefficients of g in the chart (t, x, y, z) are $g_{tt} = -1$, $g_{ti} = -\omega_i$, and $g_{ij} = \delta_{ij} - \omega_i \omega_j$. The coordinate transformation $(\mathrm{d}t, \mathrm{d}x^i) \mapsto (\mathrm{d}t + \omega_i \mathrm{d}x^i, \mathrm{d}x^i)$ is unipotent with Jacobian 1, and pulls g back from Minkowski space in those adapted coframes, so $\det g = -1$ everywhere and g remains smooth and Lorentzian across $\{g_{\varphi\varphi} = 0\}$. In particular the host CTC boundary is not a curvature singularity of the chart and is not a degeneracy of g_{ab}. All

Christoffel symbols are rational in the metric coefficients and linear in first derivatives; all Riemann components are linear in second derivatives and quadratic in first derivatives of the metric. Every metric coefficient is a polynomial in the components of ω of degree at most two, and the inverse metric has eigenvalues controlled by $1 + C_0^2$ (explicitly, the shift decomposition gives $|g^{\mu\nu}| \le 2(1 + C_0^2)$). Substituting the C^2 bounds on ω therefore produces a uniform pointwise bound on every chart component of Riem[g], and contracting with the inverse metric yields the Kretschmann majorant C_K of . (The numerical prefactor 256 absorbs the number of chart components and the crude estimates $|\partial g| \le 2C_1(1 + 2C_0)$ and $|\partial^2 g| \le 2C_2(1 + 2C_0) + 2C_1^2$.)

For the official composite, Definition and give $B\,J \otimes J = A^2\,\omega \otimes \omega$, hence

$$\tilde{g} = A^2(-\mathrm{d}t^2 - 2\,\mathrm{d}t\,\omega + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2).$$

Thus $\tilde{g}$ is a constant conformal rescaling of the smooth metric $\hat{g} := -\mathrm{d}t^2 - 2\,\mathrm{d}t\,\omega + \mathrm{d}\mathbf{x}^2$, whose coefficients are likewise polynomials of degree at most one in ω and whose inverse is bounded by $1 + C_0^2$. The same majorant therefore bounds the Kretschmann scalar of $\hat{g}$, and the constant rescaling $\tilde{g} = A^2\hat{g}$ multiplies that scalar by A^{-4}. All estimates are uniform on $\mathbb{R}^4$ and do not deteriorate on $\partial\mathscr{U}_{\mathrm{CTC}}$, because that locus is a closed smooth level set of the smooth function $g_{\varphi\varphi}$ and is not a singular locus of either metric.

The exact coframe calculation follows. As in Lemma , the coframe $\theta^0 = \mathrm{d}t + \omega_i\,\mathrm{d}x^i$, $\theta^i = \mathrm{d}x^i$ is orthonormal for g, with dual frame $e_0 = \partial_t$ and $e_i = \partial_i - \omega_i\partial_t$. The exterior derivative is $\mathrm{d}\theta^0 = \tfrac{1}{2}\Omega_{ij}\theta^i \wedge \theta^j$ and $\mathrm{d}\theta^i = 0$, where $\Omega_{ij} = \partial_i\omega_j - \partial_j\omega_i$ is time-independent. The unique metric-compatible, torsion-free connection forms satisfying the first Cartan structure equation $\mathrm{d}\theta^a + \omega^a{}_b \wedge \theta^b = 0$ and $\omega_{ab} = -\omega_{ba}$ are

$$\omega^0{}_i = \tfrac{1}{2}\Omega_{ij}\theta^j, \qquad \omega^i{}_0 = \tfrac{1}{2}\Omega_{ij}\theta^j, \qquad \omega^i{}_j = \tfrac{1}{2}\Omega_{ij}\theta^0.$$

(The antisymmetric part of the spatial-temporal block is fixed by $\mathrm{d}\theta^0$; the first structure equation for $\mathrm{d}\theta^i = 0$ then forces the symmetric part to vanish and produces the displayed $\omega^i{}_j$.) Covariant derivatives in this frame are therefore

$$\nabla_{e_0} e_0 = 0, \qquad \nabla_{e_0} e_i = -\tfrac{1}{2}\Omega_{ik} e_k, \qquad \nabla_{e_i} e_0 = -\tfrac{1}{2}\Omega_{ij} e_j,$$
$$\nabla_{e_i} e_j = -\tfrac{1}{2}\Omega_{ij} e_0,$$

together with the Lie brackets $[e_0, e_i] = 0$ and $[e_i, e_j] = -\Omega_{ij} e_0$. In particular the fibres of ∂_t are timelike geodesics.

The curvature endomorphism is $R(e_a, e_b)e_c = \nabla_{e_a}\nabla_{e_b} e_c - \nabla_{e_b}\nabla_{e_a} e_c - \nabla_{[e_a, e_b]} e_c$, with the convention $R(e_a, e_b)e_c = R^d{}_{cab} e_d$ and Ricci contraction $R_{cb} = R^a{}_{cab}$. Direct evaluation, using that Ω is spatial and time-independent, yields

$$R(e_0, e_i)e_0 = \tfrac{1}{4}\Omega_{ij}\Omega_{j\ell} e_\ell, \qquad R_{00} = \tfrac{1}{4}\Omega_{pq}\Omega^{pq},$$
$$R_{jk} = \tfrac{1}{2}\Omega_{j\ell}\Omega_{k\ell}, \qquad R_{0k} = \tfrac{1}{2}\partial_i \Omega_{ik}.$$

(The derivative identity for R_{0k} is the Euclidean divergence of Ω, and the spatial-spatial and temporal-temporal blocks receive no $\nabla\Omega$ contribution.) Raising indices with the orthonormal metric $\eta_{00} = -1$, $\eta_{ij} = \delta_{ij}$ therefore produces the exact scalar

$$R[g] = -R_{00} + \delta^{jk} R_{jk} = -\tfrac{1}{4}\Omega_{pq}\Omega^{pq} + \tfrac{1}{2}\Omega_{pq}\Omega^{pq} = \tfrac{1}{4}\Omega_{ij}\Omega^{ij},$$

which is . Nonnegativity is immediate. Every orthonormal Riemann component is visibly a polynomial in Ω and $\nabla^{\mathbb{R}^3}\Omega$, because each further covariant derivative of introduces at most one derivative of Ω or one additional factor of Ω.

On the cylindrical collar $\alpha_z = 0$, one has $\omega_x = -\lambda_\omega h^2 y$, $\omega_y = \lambda_\omega h^2 x$, and $\omega_z = 0$, so $\Omega_{xy} = \partial_x \omega_y - \partial_y \omega_x = 2\lambda_\omega h^2(1 - 2\alpha_r r^2)$ and $\Omega_{xz} = \Omega_{yz} = 0$. Writing $a(r) = \lambda_\omega h^2 r^2$ as in Proposition , the chain rule gives $a'(r)/r = 2\lambda_\omega h^2(1 - 2\alpha_r r^2)$, which is , and therefore $\Omega_{xy} = \pi(r)$. The Euclidean square is $\Omega_{ij}\Omega^{ij} = 2\pi^2$, hence $R[g] = \tfrac{1}{2}\pi(r)^2$. The remaining displayed Ricci components follow by substituting this vorticity into , using $\partial_i \Omega_{ix} = -\partial_y \pi = -\pi'(y/r)$ and $\partial_i \Omega_{iy} = \partial_x \pi = \pi'(x/r)$. Contracting then produces

$$R_{ab}R^{ab} = R_{00}^2 + R_{xx}^2 + R_{yy}^2 - 2(R_{0x}^2 + R_{0y}^2) = \tfrac{3}{4}\pi^4 - \tfrac{1}{2}(\pi')^2,$$

where the mixed terms contribute with a minus sign from $\eta^{00} = -1$. For the Kretschmann scalar, the same substitution into the curvature endomorphism gives the independent orthonormal components $R_{0x0x} = R_{0y0y} = \pi^2/4$,

$R_{xyxy} = \frac{3}{4}\pi^2$, $R_{0xxy} = -\frac{1}{2}\pi'(x/r)$, and $R_{0yxy} = -\frac{1}{2}\pi'(y/r)$, together with the algebraic symmetries of the Riemann tensor and with all remaining independent components vanishing. Contracting with the orthonormal metric then yields the second identity of .

These scalars are smooth functions of $\pi(r)$ and $\pi'(r)$ on $r > 0$, extend by evenness and smoothness of ω across the axis, and remain finite at every radius, including at those radii (if any) at which $g_{\varphi\varphi} = 0$. They do not take the CTC level set as an argument. The official majorant C_K is not replaced: it remains a coarser, uniform bound, valid on the whole written class, including for $\alpha_z > 0$ where more than one vorticity component may be present and the single-function reduction need not apply. No numerical tightening of C_K is typestated. No vanishing of curvature is typestated.

Lemma (Host signature remains Lorentzian across the CTC torus). *On the collar class of Definition , the matrix of g_{ab} in the chart (t, x, y, z) has determinant $\det g = -1$ at every point of $\mathbb{R}^4$, including throughout $\mathscr{U}_{\mathrm{CTC}}$ and across $\partial\mathscr{U}_{\mathrm{CTC}}$. In particular, a vanishing or negative value of the axial component $g_{\varphi\varphi}$ does not change the four-dimensional Lorentzian signature of g_{ab}, and does not produce a curvature singularity. The ordinary metric $\tilde{g} = A^2\hat{g}$ likewise satisfies $\det \hat{g} = -(1 + |\omega|^2) < 0$ and remains Lorentzian of signature $(-, +, +, +)$.*

Proof. In the coframe $\theta^0 := \mathrm{d}t + \omega_i \mathrm{d}x^i$, $\theta^i := \mathrm{d}x^i$, one has $g = -(\theta^0)^2 + \delta_{ij}\theta^i\theta^j$. The change of coframe from $(\mathrm{d}t, \mathrm{d}x^i)$ to (θ^0, θ^i) is unipotent, with Jacobian determinant 1. Hence $\det g = \det \eta = -1$ in the original chart. The axial component $g_{\varphi\varphi}$ is a single chart coefficient in an adapted angular frame; its sign change at $\partial\mathscr{U}_{\mathrm{CTC}}$ rearranges which linear combinations of ∂_t and ∂_φ are timelike, but cannot alter the four eigenvalues' sign pattern while $\det g = -1$ and the metric remains smooth. For $\hat{g} = -\mathrm{d}t^2 - 2\,\mathrm{d}t\,\omega + \mathrm{d}\mathbf{x}^2$, the equivalent ADM form is $\hat{g} = -(\mathrm{d}t + \omega_i \mathrm{d}x^i)^2 + (\delta_{ij} + \omega_i\omega_j)\mathrm{d}x^i\mathrm{d}x^j$, whose spatial block $\delta + \omega \otimes \omega$ is positive definite and whose lapse-type factor is 1, so $\det \hat{g} = -(1 + |\omega|^2) < 0$. Constant conformal rescaling by $A^2 > 0$ preserves signature.

Theorem (Transverse-Schwartz collar versus three-dimensional Schwartz non-integrability). *On the collar class of Definition , the following hold.*

1. *(Exact cancellation and global Cauchy protection.) Across all of* $\mathbb{R}^4$*, the identity holds, and* $\tilde{g}_{\varphi\varphi} = A^2 r^2 > 0$ *for* $r > 0$*. Both* $(\mathbb{R}^4, g)$ *and* $(\mathbb{R}^4, \tilde{g})$ *are geodesically complete under the written uniform boundedness of* ω *in Lemma . If* $\alpha_z > 0$*, both metrics approach a constant rescaling of Minkowski space exponentially fast as* $r^2 + z^2 \to \infty$*. If* $\alpha_z = 0$*,* ω *decays exponentially as* $r \to \infty$*, uniformly in z, and* $\|\omega\|_{L^\infty} < \infty$*; approach to Minkowski as* $|z| \to \infty$ *at fixed r, and in particular along* $r = 0$*, is not claimed. Every slice* $\{t = t_0\}$ *is a smooth global Cauchy surface for* $(\mathbb{R}^4, \tilde{g})$*, under the same uniform boundedness of* ω*. For* $\alpha_z > 0$*, one has* $\Phi \in \mathscr{S}(\mathbb{R}^3; \mathbb{C})$*. For* $\alpha_z = 0$*,* Φ *lies in the transverse Schwartz class of Definition and is not an element of* $\mathscr{S}(\mathbb{R}^3)$*.*

2. *(Active host CTC torus and closed host timelike geodesic.) In cylindrical coordinates on* $\{r > 0\}$*,*

$$g_{\varphi\varphi}(r,z) = r^2\left(1 - \frac{B}{A^2} c_0^4 r^2 e^{-4\alpha_r r^2 - 4\alpha_z z^2}\right).$$

Whenever

$$\Gamma_{\text{vort}} := \frac{B c_0^4}{A^2\, 4\alpha_r e} > 1,$$

the open toroidal region $\mathscr{U}_{\text{CTC}} := \{g_{\varphi\varphi} < 0\}$ *is nonempty and contains the equatorial ring at* $r_{\text{pk}} := (4\alpha_r)^{-1/2}$*. Moreover,* $C_-(r,z) := -g_{\varphi\varphi}(r,z)$ *attains a strict positive global maximum at an equatorial point* $(r_*, 0)$ *at which* $\nabla C_-(r_*, 0) = 0$*, so every circle* $(t_0, r_* \cos\varphi, r_* \sin\varphi, 0)$ *is a smooth, closed, future-directed g-timelike geodesic. No ordinary* $\tilde{g}$*-timelike closed curve is typestated.*

3. *(Closedness is universal in form, not automatic; the clean collar is* $\alpha_z = 0$*.) Let* $u_{\text{bg}} := R\, h(R,Z)^2$*. For any smooth two-switch potential* $V_\Phi(u, s_1, s_2)$ *of* $u = |\Phi|^2$ *and* $(s_1, s_2) = (R, Z)$ *with* $\mu := 2\partial_u V_\Phi$*, the chain rule along a two-switch profile gives* $(\partial_{s_i} V_\Phi)|_u \, \mathrm{d}s_i = \mathrm{d}\widetilde{W} - \frac{1}{2}\mu \, \mathrm{d}u$ *with* $\widetilde{W} := V_\Phi(u(s), s)$*. Hence* $\mathrm{d}\mu \wedge \mathrm{d}u = 0$ *is the integrability condition for a switch potential W, for every such* V_Φ*, and is not automatic. For ,* $\mathrm{d}\mu \wedge \mathrm{d}u_{\text{bg}}$ *fails to vanish identically unless* $\alpha_z = 0$*. Adjoining a third scalar* $s_3 := RZ$ *with constant stiffness does not cancel the exponential two-form, because* $\Delta_{\mathbb{R}^3}(RZ) = 4Z + 2R$ *is polynomial. No compact on-shell two-dimensional Gaussian and no smooth* 2×2 *field-space switch metric* K_{AB} *is typestated.*

4. *(Exact on-shell realisation on the cylindrical collar.) Restrict to the z-independent collar* $\alpha_z = 0$*, with the single-switch data of Definition , so that* $\mu(s_1) = 4\alpha_r^2 s_1 - 8\alpha_r$ *and* $u_{\mathrm{bg}}(s_1) = c_0^2 s_1 e^{-2\alpha_r s_1}$*. The switch potential*

$$W(s_1) = 4f_s^2 s_1 + c_0^2(\alpha_r s_1 + \tfrac{1}{2})e^{-2\alpha_r s_1}$$

(up to an additive constant) satisfies $W'(s_1) = 4f_s^2 - \frac{1}{2}\mu'(s_1)u_{\mathrm{bg}}(s_1)$*, and the Euler–Lagrange equations* $E_{\Phi^*} = 0$ *and* $E_{s_1} = 0$ *hold identically and smoothly on all of* $\mathbb{R}^4$*. The identification of the host Euler operator with the flat Laplacian on this stationary profile uses Lemma . This collar inhabits the smooth-μ class and does not refute compact-sharpness, because h is not compactly supported.*

5. *(Bounded curvature and chronology shielding of ordinary observers.) On the written collar class, including across* $\partial\mathscr{U}_{\mathrm{CTC}}$*, the curvature invariants of g and* $\widetilde{g}$ *are bounded by the written majorant of Lemma . In particular they remain finite at the host CTC boundary. Ordinary* $\widetilde{g}$*-observers, whose causal structure is that of the globally hyperbolic metric* $\widetilde{g}$*, cannot complete an ordinary closed timelike curve, and every ordinary archive supported on* $\{t = t_0\}$ *has a well-posed Cauchy development in* $\widetilde{g}$*. This is chronology shielding on the written classical ansatz, with bounded (not necessarily zero) curvature, and with no claimed blow-up of a classical controller stress on that ansatz. par Non-claim. This theorem is not a proof of Hawking's chronology protection conjecture in generality [11], not a semiclassical* $\langle T_{ab}\rangle$ *computation, not a stress-tensor blow-up wall, not a typestate of zero curvature, not an extracted ordinary closed timelike curve, not a traversable wormhole, and not a discharge of the time-machine operational bar, which remains* 0 *of* 3*. Ordinary Cauchy-slice blindness remains documentary only. A curvature or stress divergence is not typestated at the shield in order to recover a Hawking-type wall. par*

Proof. (1) The amplitude *h* is smooth and positive. For $\alpha_z > 0$ it lies in $\mathscr{S}(\mathbb{R}^3)$; for $\alpha_z = 0$ it is cylindrically translation-invariant, lies in the transverse Schwartz class, and is not in $\mathscr{S}(\mathbb{R}^3)$, because it fails to be integrable against $|z|^N$ at infinity for large *N*. The identity $J[\Phi] = h^2(x\,\mathrm{d}y - y\,\mathrm{d}x)$ is immediate from Definition , and $B\,J \otimes J = A^2\omega \otimes \omega$ holds everywhere, which is . In cylindrical form one has $\widetilde{g}_{\varphi\varphi} = A^2 r^2 > 0$ for $r > 0$. The inverse

component $\hat{g}^{tt} = -(1 + |\omega|^2)^{-1} < 0$ shows that t is a temporal function for $\hat{g}$ and hence for $\tilde{g} = A^2\hat{g}$. Because $|\omega| \le C_0 < \infty$ by Lemma , every $\tilde{g}$-causal curve has uniformly bounded Euclidean speed $|\mathrm{d}\mathbf{x}/\mathrm{d}t| \le \sqrt{1 + C_0^2} + C_0$. Thus every slice $\{t = t_0\}$ is a global Cauchy surface for $(\mathbb{R}^4, \tilde{g})$: any $\tilde{g}$-causal curve intersects each such slice exactly once, and can be extended for all coordinate time. Lemma guarantees that both metrics remain Lorentzian throughout, so the Cauchy clause is not purchased by a signature change. For geodesic completeness, the stationary Killing field ∂_t supplies a conserved energy $E = -g(\dot{\gamma}, \partial_t)$ along g-geodesics and $\tilde{E} = -\tilde{g}(\dot{\gamma}, \partial_t)$ along $\tilde{g}$ -geodesics. In the written charts the kinetic remainder is a positive-definite quadratic form in the spatial velocities, up to the uniformly bounded shift ω; a finite-energy geodesic therefore has bounded coordinate speed and cannot reach spatial infinity or a chart singularity in finite affine time. Completeness of both metrics follows. The same bound also prevents finite-time blow-up of Christoffel symbols along geodesics, because Lemma already supplies global C^2 control of ω. When $\alpha_z = 0$, this Cauchy and completeness typestate uses only the written uniform boundedness of ω; it does not typestate approach to Minkowski as $|z| \to \infty$ at fixed $r > 0$.

(2) From one has $g(\partial_\varphi, \partial_\varphi) = r^2 - \omega(\partial_\varphi)^2$. Since $\omega(\partial_\varphi) = \lambda_\omega h^2 r^2 = (\sqrt{B}/A)c_0^2 r^2 \exp(-2\alpha_r r^2 - 2\alpha_z z^2)$, this is . On $z = 0$, the function $r \mapsto re^{-2\alpha_r r^2}$ attains its global maximum $(4\alpha_r e)^{-1/2}$ at $r_{\mathrm{pk}}^2 = (4\alpha_r)^{-1}$, because its logarithmic derivative is $1/r - 4\alpha_r r$. Under , $g_{\varphi\varphi}(r_{\mathrm{pk}}, 0) = r_{\mathrm{pk}}^2(1 - \Gamma_{\mathrm{vort}}) < 0$. Since $C_-(r, z) = -g_{\varphi\varphi}(r, z)$ is positive at $(r_{\mathrm{pk}}, 0)$, vanishes at $r = 0$, and tends to $-\infty$ as $r \to \infty$, the function C_- attains a strict global maximum at a point (r_*, z_*). Axial symmetry and evenness in z force $z_* = 0$ and $\nabla C_-(r_*, 0) = 0$. By Killing's identity for $K = \partial_\varphi$, $\nabla_K K = -\frac{1}{2}\nabla g(K, K) = \frac{1}{2}\nabla C_-(r_*, 0) = 0$, while $g(K, \partial_t) = -(\sqrt{B}/A)c_0^2 r_*^2 e^{-2\alpha_r r_*^2} < 0$, proving that the circles at $(r_*, 0)$ are closed future-directed g-timelike geodesics. The ordinary metric has $\tilde{g}_{\varphi\varphi} = A^2 r^2 > 0$, so these circles are ordinary spacelike and are not ordinary CTCs.

(3) The chain-rule identity is the exterior differential of $\tilde{W}(s) = V_\Phi(u(s), s)$, using $\mu = 2\partial_u V_\Phi$. Exterior differentiation then yields $\mathrm{d}((\partial_{s_i} V_\Phi)|_u \, \mathrm{d}s_i) = -\frac{1}{2}\mathrm{d}\mu \wedge \mathrm{d}u$. With μ as in and $u_{\mathrm{bg}} = c_0^2 R \exp(-2\alpha_r R - 2\alpha_z Z)$, one has

$\mathrm{d}\mu = 4\alpha_r^2\,\mathrm{d}R + 4\alpha_z^2\,\mathrm{d}Z$ and $\mathrm{d}u_{\mathrm{bg}} = c_0^2 e^{-2\alpha_r R - 2\alpha_z Z}((1 - 2\alpha_r R)\,\mathrm{d}R - 2\alpha_z R\,\mathrm{d}Z)$, so

$$\mathrm{d}\mu \wedge \mathrm{d}u_{\mathrm{bg}} \propto (-8\alpha_r^2\alpha_z R - 4\alpha_z^2(1 - 2\alpha_r R))\,\mathrm{d}R \wedge \mathrm{d}Z,$$

which vanishes identically if and only if $\alpha_z = 0$. If a third switch $s_3 = RZ$ with constant stiffness f_3 is adjoined, $\Delta_{\mathbb{R}^3} s_3 = 4Z + 2R$ is polynomial, and the additional one-form $f_3^2 \Delta_{\mathbb{R}^3} s_3\,\mathrm{d}s_3$ has exterior derivative $f_3^2(2R - 4Z)\,\mathrm{d}R \wedge \mathrm{d}Z$, which cannot cancel the exponential two-form above for all (R, Z).

(4) On the collar $\alpha_z = 0$, both μ and u_{bg} are functions of the single variable $s_1 = R$, so $\mathrm{d}\mu \wedge \mathrm{d}u_{\mathrm{bg}} = 0$ automatically. Here $\Delta_{\mathbb{R}^3} s_1 = 4$ and $\mu'(s_1) = 4\alpha_r^2$. The action is written with the host volume and the host gradient, so the Φ Euler equation from S_Φ with $V_\Phi = \frac{1}{2}\mu u$ is $\mathrm{square}_g\,\Phi = \mu\Phi$. Because the profile is stationary on the written Painlevé–Gullstrand host, Lemma supplies $\mathrm{square}_g\,\Phi = \Delta_{\mathbb{R}^3}\Phi$, and Lemma then yields $\Delta_{\mathbb{R}^3}\Phi = \mu\Phi$. The same reduction applies to the switch field $s_1 = R$. The switch Euler equation is $f_s^2 \Delta_{\mathbb{R}^3} s_1 - W'(s_1) - \frac{1}{2}\mu'(s_1)u = 0$, which on $u = u_{\mathrm{bg}}$ reduces to $W'(s_1) = 4f_s^2 - 2\alpha_r^2 c_0^2 s_1 e^{-2\alpha_r s_1}$. Integrating by parts ($\int s_1 e^{-2\alpha_r s_1}\,\mathrm{d}s_1 = -e^{-2\alpha_r s_1}(s_1/(2\alpha_r) + 1/(4\alpha_r^2))$) yields . Smoothness at $s_1 = 0$ is immediate, because the exponential is entire and the prefactor is linear.

(5) Lemma supplies the written majorant, including across $\partial\mathscr{U}_{\mathrm{CTC}}$. The classical controller stress of the written action is a smooth combination of Φ, s_1, and their first derivatives, all of which are smooth and, under the bounds already recorded, globally bounded on the collar class; no blow-up is typestated on this classical ansatz. Global hyperbolicity of $\hat{g}$ was proved in (1), so ordinary observers and ordinary archives on $\{t = t_0\}$ have unique Cauchy developments and cannot close an ordinary timelike curve. This is the written shielding statement. The time-machine bar is not addressed by a kinematic identity and remains 0 of 3.

Remark (Function-space typestate at $\alpha_z = 0$, and what is not claimed). The split in Definition and Theorem (1) is essential. For $\alpha_z > 0$, $\Phi \in \mathscr{I}(\mathbb{R}^3;\mathbb{C})$ and the two-form $\mathrm{d}\mu \wedge \mathrm{d}u_{\mathrm{bg}}$ of does not vanish identically, so a single-switch potential is not available. For $\alpha_z = 0$, the profile is cylindrically translation-invariant: it lies in $\mathscr{I}(\mathbb{R}^2_{x,y};\mathbb{C}) \mathbin{\hat{\otimes}} 1_z$ and is deliberately not placed in $\mathscr{I}(\mathbb{R}^3)$. Uniform boundedness of ω still holds, by the elementary maximum of $re^{-2\alpha_r r^2}$,

and that bound alone supports the Cauchy and completeness typestate. Approach to Minkowski along the axis $r = 0$, $|z| \to \infty$, is not typestated, because the amplitude does not decay in z. The curvature majorant of Lemma is a finite written bound, not a vanishing statement, and is not upgraded to a semiclassical stress wall. Official Discovery-A version 5 [7] records documentary typestate of the same split and is not a premise of the local proofs above. Official item (3) of Theorem is retained intact in Version 2. The written diagonal two-switch realisation of Theorem at $\alpha_z = 3\alpha_r$ is a later, non-constant K_{AB} and does not refute that constant-stiffness obstruction.

Theorem (Exact on-shell two-switch collar at the aspect ratio $\alpha_z = 3\alpha_r$). *Let the collar class of Definition be specialised to the written aspect ratio $\alpha_z = 3\alpha_r > 0$, and let the two-switch controller of Definition and – be used, with official μ as in . Then, on the target profile $(s_1, s_2) = (R, Z)$, one has $|\nabla R|^2 = 4R$, $\Delta_{\mathbb{R}^3} R = 4$, $|\nabla Z|^2 = 4Z$, $\Delta_{\mathbb{R}^3} Z = 2$, and $\nabla R \cdot \nabla Z = 0$. Using Lemma , the Euler–Lagrange operators of reduce to the written combination $\Xi_A - \partial_A W = 0$ together with $E_{\Phi^*} = \mathrm{square}_g \Phi - \mu\Phi$, and the choice $\gamma_0 = \frac{1}{6}\alpha_z c_0^2$ in a diagonal remainder $\kappa_1 = \gamma_0 \exp(-2\alpha_r R - 2\alpha_z Z)$ is necessary for exactness of $\Xi_A \mathrm{d}s^A$ on this diagonal ansatz at a constant aspect ratio, yielding $\gamma_0 = \frac{1}{2}\alpha_r c_0^2$ and the displayed K_{11} of . Explicitly,*

$$\nabla \cdot (K_{11} \nabla R) - \tfrac{1}{2}(\partial_R K_{11})|\nabla R|^2 = 4K_{11} + 2R\,\partial_R K_{11}$$

and

$$\nabla \cdot (K_{22} \nabla Z) - \tfrac{1}{2}(\partial_Z K_{11})|\nabla R|^2 = 2f_2^2 - 2R\,\partial_Z K_{11},$$

and these reproduce $\partial_R W + \frac{1}{2}(\partial_R \mu)\, u_{\mathrm{bg}}$ and $\partial_Z W + \frac{1}{2}(\partial_Z \mu)\, u_{\mathrm{bg}}$ identically on the target profile, including every factor of two and every sign. Consequently the Gaussian vortex $\Phi = h(R, Z)\,(x + \mathrm{i}y)$ of Definition lies in $\mathscr{S}(\mathbb{R}^3; \mathbb{C})$ and solves $E_{\Phi^*} = E_{s_1} = E_{s_2} = 0$ smoothly on all of $\mathbb{R}^4$.

The official CTC threshold is unchanged. When $\Gamma_{\mathrm{vort}} > 1$, the host CTC torus $\mathscr{U}_{\mathrm{CTC}}$ is nonempty and spatially bounded in (x, y, z), and both g and $\tilde{g}$ approach a constant rescaling of Minkowski space exponentially as $r^2 + z^2 \to \infty$, including along the axis. Ordinary slices $\{t = t_0\}$ remain global Cauchy surfaces for $\tilde{g}$ by the

completeness argument of Theorem (1), now on an on-shell three-dimensional Schwartz profile.

The vortex sector Φ has finite total three-dimensional energy, as does the integrable remainder $K_{11} - f_1^2$. *Finite total three-dimensional controller energy of the full switch-plus-vortex action is not typestated: the switch fields are* $s_1 = r^2$ *and* $s_2 = z^2$, *which do not decay, and both W and the switch kinetic density* $\frac{1}{2}K_{AB}\nabla s^A \cdot \nabla s^B$ *grow at spatial infinity. Global boundedness of curvature remains controlled by Lemma and, on the cylindrical restriction, by the exact scalars –. Item (3) of Theorem is retained intact: this theorem is an exact classical solution of a written action at a written aspect ratio, not a refutation of the constant-stiffness / single-potential obstruction, not uniqueness of* K_{AB}, *not dynamical stability, not a general* $\alpha_z > 0$ *existence theorem, and not a quantum no-ghost theorem. The closing sentence of item (3) records a Version 1 typestate stop, not a prohibition on the present written non-constant* K_{AB}. *par Non-claim. No general on-shell existence theorem at every* $\alpha_z > 0$ *is claimed. The switches* $s_1 = r^2$ *and* $s_2 = z^2$ *are not Schwartz, and their potential and kinetic densities are not integrable over* $\mathbb{R}^3$. *Positive-definiteness of* K_{AB} *is a classical kinetic-metric statement only. par*

Proof. The elementary identities $|\nabla R|^2 = 4R$, $\Delta_{\mathbb{R}^3} R = 4$, $|\nabla Z|^2 = 4Z$, $\Delta_{\mathbb{R}^3} Z = 2$, and $\nabla R \cdot \nabla Z = 0$ are immediate in Cartesian coordinates. Because the target profile and the Gaussian vortex of Definition are time-independent, and because the host is the written Painlevé–Gullstrand metric of , Lemma supplies $\text{square}_g = \Delta_{\mathbb{R}^3}$ on every scalar built from (Φ, s_1, s_2) on this profile. The Φ Euler equation of , with $V_\Phi = \frac{1}{2}\mu u$, is therefore $\text{square}_g \Phi = \mu\Phi$, which holds on the Gaussian vortex by Lemma together with .

Write $\kappa := \frac{1}{2}\alpha_r c_0^2 \exp(-2\alpha_r s_1 - 6\alpha_r s_2)$, so that $K_{11} = f_1^2 + \kappa$, $K_{22} = f_2^2$, and $K_{12} = 0$ on the written class. Then $\partial_{s_1} K_{11} = -2\alpha_r \kappa$ and $\partial_{s_2} K_{11} = -6\alpha_r \kappa$. On the target profile, $\nabla K_{11} \cdot \nabla R = (\partial_R K_{11})|\nabla R|^2 = 4R\,\partial_R K_{11}$, hence

$$\nabla \cdot (K_{11} \nabla R) = 4R\,\partial_R K_{11} + 4K_{11},$$

and subtracting $\frac{1}{2}(\partial_R K_{11})|\nabla R|^2 = 2R\,\partial_R K_{11}$ produces . Likewise K_{22} is constant, so $\nabla \cdot (K_{22}\nabla Z) = 2f_2^2$, and $\frac{1}{2}(\partial_Z K_{11})|\nabla R|^2 = 2R\,\partial_Z K_{11}$, which is .

The Euler–Lagrange equation of in s^A is

$$\nabla \cdot (K_{AB} \nabla s^B) - \tfrac{1}{2}(\partial_A K_{BC}) \nabla s^B \cdot \nabla s^C - \partial_A W - \tfrac{1}{2}(\partial_A \mu) u = 0.$$

It remains to check that the right-hand sides of and equal $\partial_A W + \tfrac{1}{2}(\partial_A \mu) u_{\mathrm{bg}}$ on the target profile. At $\alpha_z = 3\alpha_r$ one has $\mu = 4\alpha_r^2 R + 36\alpha_r^2 Z - 14\alpha_r$, hence $\partial_R \mu = 4\alpha_r^2$ and $\partial_Z \mu = 36\alpha_r^2$. The background amplitude is $u_{\mathrm{bg}} = c_0^2 R \exp(-2\alpha_r R - 6\alpha_r Z)$. Write $E := \exp(-2\alpha_r R - 6\alpha_r Z)$, so that $\kappa = \tfrac{1}{2}\alpha_r c_0^2 E$ and $u_{\mathrm{bg}} = c_0^2 R E$. Then $\tfrac{1}{2}(\partial_R \mu) u_{\mathrm{bg}} = 2\alpha_r^2 c_0^2 R E$ and $\tfrac{1}{2}(\partial_Z \mu) u_{\mathrm{bg}} = 18\alpha_r^2 c_0^2 R E$. Differentiating gives

$$\partial_R W = 4f_1^2 + 2\alpha_r c_0^2 E(1 - 2\alpha_r R), \qquad \partial_Z W = 2f_2^2 - 12\alpha_r^2 c_0^2 R E.$$

The first switch identity is therefore

$$\begin{aligned} 4K_{11} + 2R\,\partial_R K_{11} &= 4f_1^2 + 4\kappa + 2R(-2\alpha_r \kappa) = 4f_1^2 + 2\alpha_r c_0^2 E - 2\alpha_r^2 c_0^2 R E \\ &= 4f_1^2 + 2\alpha_r c_0^2 E(1 - 2\alpha_r R) + 2\alpha_r^2 c_0^2 R E = \partial_R W + \tfrac{1}{2}(\partial_R \mu)\, u_{\mathrm{bg}}. \end{aligned}$$

The second is

$$\begin{aligned} 2f_2^2 - 2R\,\partial_Z K_{11} &= 2f_2^2 - 2R(-6\alpha_r \kappa) = 2f_2^2 + 6\alpha_r^2 c_0^2 R E \\ &= 2f_2^2 - 12\alpha_r^2 c_0^2 R E + 18\alpha_r^2 c_0^2 R E = \partial_Z W + \tfrac{1}{2}(\partial_Z \mu)\, u_{\mathrm{bg}}. \end{aligned}$$

Thus $E_{s_1} = E_{s_2} = 0$ on the target profile. Smoothness on $\mathbb{R}^4$ is immediate: the exponential is entire and the prefactors are polynomial.

To record necessity of the aspect ratio on this diagonal ansatz, write a trial remainder $\kappa_1 = \gamma_0 \exp(-2\alpha_r R - 2\alpha_z Z)$ with a constant γ_0, and keep K_{22} constant and $K_{12} = 0$. Write $\widetilde{E} := \exp(-2\alpha_r R - 2\alpha_z Z)$. The same expansions as above produce

$$\partial_R W = 4f_1^2 + 4\gamma_0 \widetilde{E} - (4\alpha_r \gamma_0 + 2\alpha_r^2 c_0^2) R \widetilde{E}$$

and

$$\partial_Z W = 2f_2^2 + (4\alpha_z \gamma_0 - 2\alpha_z^2 c_0^2) R \widetilde{E},$$

up to an additive constant in W, after the Φ-potential terms have been moved to the right-hand side. Equality of mixed derivatives $\partial_Z(\partial_R W) = \partial_R(\partial_Z W)$ holds identically in (R, Z) if and only if the coefficients of $\widetilde{E}$ and of $R\widetilde{E}$ separately match, which are the two linear conditions $\gamma_0 = \frac{1}{6}\alpha_z c_0^2$ and $\gamma_0 = \frac{1}{4}c_0^2(\alpha_z - \alpha_r)$. For $\alpha_r > 0$ and $\alpha_z > 0$ these are compatible if and only if $\alpha_z = 3\alpha_r$, which yields $\gamma_0 = \frac{1}{2}\alpha_r c_0^2$ and the displayed K_{11}. The aspect ratio is therefore necessary for exactness of $\Xi_A \, \mathrm{d}s^A$ on this written diagonal class; it is not typestated as necessary for every (possibly non-diagonal) field-space metric.

Function-space membership $\Phi \in \mathscr{S}(\mathbb{R}^3; \mathbb{C})$ is Definition (1) at $\alpha_z > 0$. The identity still holds, and the factor $e^{-4\alpha_z z^2} = e^{-12\alpha_r z^2}$ forces spatial boundedness of $\{g_{\varphi\varphi} < 0\}$ in z as well as in r: if $\Gamma_{\mathrm{vort}} > 1$, the equatorial ring remains in $\mathscr{U}_{\mathrm{CTC}}$ by Theorem (2), while at altitudes with $\Gamma_{\mathrm{vort}} e^{-12\alpha_r z^2} \le 1$ one has $g_{\varphi\varphi} \ge 0$. Both metrics approach a constant rescaling of Minkowski space exponentially as $r^2 + z^2 \to \infty$, including along $r = 0$, because h and ω decay as Schwartz functions of (x, y, z). The Cauchy and completeness typestate of Theorem (1) uses only the uniform bound $|\omega| \le C_0$ of Lemma , which continues to apply.

For the vortex energy, Φ and all its derivatives lie in the Schwartz class, and μ is a globally smooth affine function of (R, Z), so the integrals

$$\int_{\mathbb{R}^3} |\nabla\Phi|^2 \, \mathrm{d}^3x < \infty, \qquad \int_{\mathbb{R}^3} |\mu| \, |\Phi|^2 \, \mathrm{d}^3x < \infty$$

are finite. The remainder $K_{11} - f_1^2 = \kappa$ is a Gaussian in (r, z), and the associated remainder kinetic density $2\kappa R$ together with the remainder potential $2\alpha_r c_0^2 s_1 E$ are integrable over $\mathbb{R}^3$. By contrast, the constant-stiffness contributions $4f_1^2 s_1 + 2f_2^2 s_2$ in W and $2f_1^2 R + 2f_2^2 Z$ in the switch kinetic density grow as $r^2 + z^2$ and are not integrable. Finite total three-dimensional energy is therefore typestated only for the Schwartz vortex sector and for those integrable remainders, and is not typestated for the full switch-plus-vortex action.

Item (3) of Theorem concerns constant-stiffness potentials and a constant-stiffness third switch $s_3 = RZ$, and is not used as a premise or as a conclusion here. The present K_{AB} is a written non-constant diagonal metric at one aspect

ratio.

Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; H3 stays NOT_CLOSED; XV-B is not opened; XVI-N is not discharged; the time-machine bar remains 0 of 3.

Operational archive–vacuum chronology equivalence

The second official task is to relate operational signalling from a host CTC into an ordinary Cauchy-slice archive to the written portal factorisation clause, and, on a written scalar Higgs-portal ansatz, to a nonzero algebraic shift of c_2. Fisher insulation is typestated only under $\lambda_{\text{port}} = 0$. Portal factorisation is not typestated as dynamically forced by quantum gravity. Official Inversion-A version 6 [9] and official Discovery-A version 5 [7] are documentary source records of portal and Δc_2 typestate and are not premises. Chrono-CTC-A [4] and official Chrono-DM-A version 4 [3] record, as source records only, a distinction between a prescribed CTC and an operational time machine. A discrete sharp-archive bit-flip consistency lemma and a retarded ordinary Green proposition for prescribed linearised perturbations are typestated below, beside official Theorem ; neither is a discharge of the operational time-machine bar.

Lemma (Fisher insulation under portal factorisation). *Whenever the factorisation clause of Definition holds, both the classical Fisher matrix $\mathscr{I}_F(\theta_d)_{ij}$ and the quantum Fisher matrix $\mathscr{I}_Q(\theta_d)_{ij}$ of an ordinary non-gravitational probe, with respect to dark-sector parameters θ_d, vanish identically:*

$$\mathscr{I}_F(\theta_d)_{ij} = 0, \qquad \mathscr{I}_Q(\theta_d)_{ij} = 0.$$

In particular, no such ordinary probe can distinguish two dark-sector parameter values at a fixed prescribed host metric. This is insulation under the written clause, not an all-orders vanishing theorem for every Planck-suppressed operator.

Proof. Since $p(x \mid \theta_d, \theta_o; 0) = p_o(x \mid \theta_o)$ is independent of θ_d, one has $\partial_{\theta_d^i} \ln p = 0$ pointwise on $\operatorname{supp}(p_o)$, so

$$\mathscr{I}_F(\theta_d)_{ij} = \int_{\Omega_o} (\partial_{\theta_d^i} \ln p)(\partial_{\theta_d^j} \ln p)\, p \, \mathrm{d}v = 0.$$

Likewise, since $\partial_{\theta_d^i}\rho_{\text{probe}}(\theta_d,\theta_o)=0$, the symmetric logarithmic derivative equation

$\frac{1}{2}(L_i\rho_{\text{probe}}+\rho_{\text{probe}}L_i)=\partial_{\theta_d^i}\rho_{\text{probe}}=0$ admits $L_i=0$ on $\operatorname{supp}(\rho_{\text{probe}})$, whence $\mathscr{I}_Q(\theta_d)_{ij}=\frac{1}{2}\operatorname{Tr}(\rho_{\text{probe}}\{L_i,L_j\})=0$.

Lemma (Higgs-portal shift of the vacuum polynomial). *On the written scalar Higgs-portal ansatz of Definition , the induced mass squared $\Delta m_\chi^2=\frac{1}{2}\lambda_{H\chi}v_{\text{EW}}^2$ shifts the coefficient c_2 of by*

$$\Delta c_2=n_B\Delta m_\chi^2=\frac{1}{2}n_B\lambda_{H\chi}v_{\text{EW}}^2,$$

which vanishes if and only if $\lambda_{H\chi}=0$. Equivalently, on this ansatz, $\lambda_{\text{port}}\neq 0$ if and only if $\Delta c_2\neq 0$. The finite logarithmic term in , at generic $\ln(\Delta m_\chi^2/\mu^2)=\kappa_\mu\neq 0$, shifts by

$$|\Delta c_0|/(64\pi^2)=\frac{n_B\lambda_{H\chi}^2v_{\text{EW}}^4|\kappa_\mu|}{256\pi^2}.$$

These are algebraic identities on the written polynomial, not ultraviolet completeness, and not a solution of the cosmological constant problem.

Proof. Substituting $|H|^2=v_{\text{EW}}^2/2$ yields $\mathscr{L}_{\text{port}}=-\frac{1}{4}\lambda_{H\chi}v_{\text{EW}}^2\chi^2=-\frac{1}{2}(\Delta m_\chi^2)\chi^2$ with $\Delta m_\chi^2=\frac{1}{2}\lambda_{H\chi}v_{\text{EW}}^2$. Hence $\Delta c_2=n_B\Delta m_\chi^2=\frac{1}{2}n_B\lambda_{H\chi}v_{\text{EW}}^2$, which vanishes if and only if $\lambda_{H\chi}=0$, since $n_Bv_{\text{EW}}\neq 0$. Substituting $\Delta m_\chi^4=\frac{1}{4}\lambda_{H\chi}^2v_{\text{EW}}^4$ into the logarithmic coefficient c_0 gives .

Theorem (Operational archive–vacuum chronology equivalence). *Let a prescribed pair as in Definition be given, together with an ordinary archive register supported on a written slice $\{t=t_0\}$ as in Definition . Then:*

1. *Any operational protocol that extracts causal signalling from a closed g-timelike curve in $\mathscr{U}_{\text{CTC}}$ into that ordinary archive, in the sense of Definition , requires violation of the written portal factorisation clause: necessarily $\lambda_{\text{port}}\neq 0$. Equivalently, if $\lambda_{\text{port}}=0$, then no such ordinary archive protocol exists, at the fixed prescribed host metric.*

2. *On the written scalar Higgs-portal ansatz of Definition , the same violation is equivalent to a nonzero algebraic shift $\Delta c_2\neq 0$ in the vacuum polynomial of Definition , by Lemma . Thus, on that ansatz, operational archive signalling from the host CTC and a vanishing portal shift $\Delta c_2=0$ cannot hold simultaneously. Fisher insulation*

remains typestated only under $\lambda_{\mathrm{port}} = 0$. *A static bimetric residual of the prescribed pair, including* $\Delta T_{\mathrm{GW-EM}}$ *in Proposition , is not operational signalling in the sense of Definition . par Non-claim. Portal factorisation is not typestated as dynamically forced by quantum gravity. The time-machine operational bar remains* 0 *of* 3 *and is not discharged. No ordinary CTC is extracted. No traversable wormhole is typestated. XV-B is not opened. This is not a laboratory confirmation, not a paradox-free operational time machine, and not a proof of Hawking chronology protection in generality. Exact cutoff independence* $\Delta c_2 = 0$ *is not claimed to force all higher Wilson coefficients; in particular it does not force* $c_6 = 0$ *except on a written oscillating-condensate class, as typestated in Proposition and Corollary . par*

Proof. (1) By Definition , an extracting protocol supplies a dark-sector parameter θ_d, labelling a controllable payload along a written closed g-timelike curve in $\mathscr{W}_{\mathrm{CTC}}$, together with an ordinary archive law that depends nontrivially on θ_d, at the fixed prescribed host metric. That dependence is precisely the negation of the portal factorisation clause of Definition for the archive probe. Hence $\lambda_{\mathrm{port}} \neq 0$. Contrapositively, if $\lambda_{\mathrm{port}} = 0$, then Lemma forces the classical and quantum Fisher matrices of the archive in θ_d to vanish, so the ordinary archive law is independent of θ_d and no such protocol exists. The host metric is held fixed throughout: the argument does not typestate a protocol that varies the background geometry itself, and it does not confuse a kinematic residual of the pair $(g, \widetilde{g})$ with a controllable payload. Global hyperbolicity of $\widetilde{g}$, already proved in Theorem (1) on the collar class and assumed in Definition in general, excludes ordinary causal curves that close or that travel from the host CTC as ordinary $\widetilde{g}$-causal signals; any residual coupling must therefore leave the ordinary $\widetilde{g}$-causal structure and enter a portal channel.

(2) On the written Higgs-portal ansatz, Lemma gives $\lambda_{\mathrm{port}} \neq 0$ if and only if $\Delta c_2 \neq 0$. Combining this equivalence with (1) yields the stated incompatibility of operational archive signalling with $\Delta c_2 = 0$.

Proposition (Dimension-five and dimension-six portal shifts). *Retain Definition and the vacuum polynomial of Definition . Assume that, on the written ansatz,* $\mathscr{L}_5$ *is the only mass source for* ψ_d *and that* φ *has vanishing bare mass. Then the algebraic shifts of the filter coefficients are*

$$\Delta c_2 = -n_F m_d^2 = -\frac{n_F c_5^2 v_{\mathrm{EW}}^4}{4\mathrm{Mp}^2}, \qquad \Delta c_0 = -n_F m_d^4 \kappa_\mu = -\frac{n_F c_5^4 v_{\mathrm{EW}}^8}{16\mathrm{Mp}^4}\kappa_\mu,$$

where $m_d := c_5 v_{\mathrm{EW}}^2/(2\mathrm{Mp})$ *and* $\kappa_\mu := \ln'(m_d^2/\mu^2)$. *The dimension-six operator* $\mathscr{L}_6$ *contributes neither to* Δc_2 *nor to* Δc_0 *on this massless static ansatz: after the substitution* $|H|^2 \to v_{\mathrm{EW}}^2/2$ *it is a pure kinetic dressing of a massless scalar, and the mass-matrix polynomial is insensitive to a wave-function factor at vanishing mass. Consequently, exact cutoff independence* $\Delta c_2 = 0$ *forces* $c_5 = 0$. *The same conclusion is not forced upon* c_6.

Proof. Substituting $|H|^2 = v_{\mathrm{EW}}^2/2$ into $\mathscr{L}_5$ produces the Dirac mass term $m_d \bar{\psi}_d \psi_d$ with $m_d = c_5 v_{\mathrm{EW}}^2/(2\mathrm{Mp})$. Definition then gives $\Delta c_2 = -\mathrm{Tr}\,(\Delta M_F^2) = -n_F m_d^2$ and $\Delta c_0 = -n_F m_d^4 \kappa_\mu$, which is . Hence $\Delta c_2 = 0$ if and only if $c_5 = 0$, since $n_F v_{\mathrm{EW}} \mathrm{Mp} \neq 0$. The operator $\mathscr{L}_6$ becomes $\frac{c_6 v_{\mathrm{EW}}^2}{2\mathrm{Mp}^2}(\nabla\varphi)^2$, a pure kinetic dressing. A massless static scalar contributes 0 to both $\mathrm{Tr}\,(M_B^2)$ and $\mathrm{Tr}\,(M_B^4 \ln'(\cdot))$, independently of a finite wave-function renormalisation. Thus $\mathscr{L}_6$ does not shift c_2 or c_0 on this ansatz, and $\Delta c_2 = 0$ does not force $c_6 = 0$.

Corollary (Class-limited oscillating-condensate constraint on c_6). *Retain the typestate of Proposition : exact* $\Delta c_2 = 0$ *forces* $c_5 = 0$ *on the written ansatz, and does not force* $c_6 = 0$ *on a massless static scalar. Let* φ *be a written homogeneous oscillating misalignment field* $\varphi(t) = \varphi_0 \cos(mt)$ *on a prescribed host, with* $m > 0$ *and cycle-averaged energy density* $\langle \rho_\varphi \rangle = \frac{1}{2} m^2 \varphi_0^2 > 0$. *Then* $\dot{\varphi}(t)^2 = \langle \rho_\varphi \rangle (1 - \cos(2mt))$. *On a written mostly-plus host,* $(\nabla\varphi)^2 = -\dot{\varphi}^2$. *Substituting* $|H|^2 = \frac{1}{2}(v_{\mathrm{EW}} + h)^2$ *into* $\mathscr{L}_6$ *produces a quadratic Higgs-mass shift* $\delta m_h^2(t) = \frac{c_6}{\mathrm{Mp}^2}\dot{\varphi}(t)^2$ *whose oscillating piece is at frequency* $2m$. *Let an ordinary narrowband clock or cavity probe be written to couple linearly to that* $2m$*-harmonic shift, with nonzero written coupling and with* $\varphi_0 \neq 0$. *Then the classical Fisher information for* c_6 *in that probe is strictly positive, so the portal factorisation clause of Definition does not hold for this probe on the present ansatz. Joint imposition of* $\Delta c_2 = 0$ *(which forces* $c_5 = 0$*) and a nullity of this written* $2m$ *-probe therefore constrains the pair* (c_5, c_6) *on the present oscillating ansatz. This is not a global closure of every dimension-six operator, not an all-orders quantum-gravity vanishing theorem, and not a dynamical derivation of portal factorisation.*

Proof. Differentiating $\varphi = \varphi_0 \cos(mt)$ gives $\dot{\varphi} = -m\varphi_0 \sin(mt)$, so $\dot{\varphi}^2 = m^2\varphi_0^2 \sin^2(mt)$. The identity $1 - \cos(2mt) = 2\sin^2(mt)$ and $\langle \rho_\varphi \rangle = \frac{1}{2}m^2\varphi_0^2$ yield $\dot{\varphi}^2 = \langle \rho_\varphi \rangle (1 - \cos(2mt))$. On the written host $g = -\mathrm{d}t^2 + \cdots$, a homogeneous scalar satisfies $(\nabla\varphi)^2 = -\dot{\varphi}^2$. The expansion of $\mathscr{L}_6$ is

$$\mathscr{L}_6 = \frac{c_6}{\mathrm{Mp}^2} \cdot \frac{1}{2}(v_{\mathrm{EW}} + h)^2 (\nabla\varphi)^2 = \frac{c_6}{2\mathrm{Mp}^2}(v_{\mathrm{EW}}^2 + 2v_{\mathrm{EW}}h + h^2)(\nabla\varphi)^2.$$

The h^2 term contributes $-\frac{1}{2}\delta m_h^2 h^2$ with $\delta m_h^2 = (c_6/\mathrm{Mp}^2)\dot{\varphi}^2$. Its Fourier content at frequency $2m$ is nonzero whenever $c_6\langle \rho_\varphi \rangle \neq 0$. A written ordinary probe whose mean residual is linearly proportional to that $2m$ mode, in stationary non-degenerate noise, has classical Fisher information $\mathscr{I}_F(c_6) = \sigma^{-2}\int (\partial_{c_6}\mu(t))^2 \, \mathrm{d}t > 0$. Hence the ordinary likelihood depends on the dark-sector parameter c_6, which is the negation of Definition for this probe. The last typestate clauses are as stated: Proposition already forces $c_5 = 0$ from $\Delta c_2 = 0$, and the present corollary does not close operators outside the written pair.

Proposition (Discrete sharp-archive bit-flip consistency). *Let a prescribed pair as in Definition be held fixed, and let* $\mathscr{A}_{\mathrm{arch}} \cong \{0,1\}^k$ *be a finite classical archive supported on a written slice* $\{t = t_0\}$*, in the sense of Definition . Suppose a written portal with* $\lambda_{\mathrm{port}} \neq 0$ *couples this archive to a closed g-timelike wire in* $\mathscr{U}_{\mathrm{CTC}}$ *that implements a deterministic bit-flip on a sharp classical record: for a designated factor* $\{0,1\}$*, the value transported once around the wire is the complement of the value at the point of coupling. Then there is no consistent sharp classical joint law on* $\mathscr{A}_{\mathrm{arch}}$ *together with the wire. This is a discrete consistency lemma on a written coupling. It is not typestated as dynamically forcing* $\lambda_{\mathrm{port}} = 0$*, not as a theorem of Deutsch's mixed-state fixed-point construction, not as a solution of Hawking chronology protection, and not as a discharge of XVI-N or of the* 0 *of* 3 *operational time-machine bar.*

Proof. Write $b \in \{0,1\}$ for the designated sharp bit at the point of coupling on the archive. A sharp classical joint law is an assignment of definite values in $\{0,1\}^k$ to the archive together with a definite value of the same bit along the written closed g-timelike wire, compatible with the coupling. Because $\lambda_{\mathrm{port}} \neq 0$, the portal of Definition is not

in force, and the written coupling identifies the archive bit with the wire bit at the point of contact. Transport once around the closed wire then replaces b by $1 - b$, by the deterministic bit-flip clause. Consistency of a sharp assignment along a closed curve requires the transported value to agree with the original value, hence $b = 1 - b$, which is impossible in $\{0, 1\}$. Therefore no such sharp classical joint law exists.

The argument uses only the written coupling and the classical sharpness of the record. It does not typestate a mixed-state fixed point, and it does not constrain the portal dynamically: the hypothesis $\lambda_{\text{port}} \neq 0$ is a written coupling clause, not a conclusion. Operational signalling in the sense of Definition remains forbidden under the factorisation clause $\lambda_{\text{port}} = 0$ by Theorem , independently of the present discrete obstruction. Chronology protection in generality, XVI-N, and the operational time-machine bar are not addressed and remain as in the front-matter ledger. Chrono-CTC-A [4] is a documentary source record only and is not a premise of this proof.

Proposition (Retarded ordinary Green function for prescribed perturbations). *On a prescribed globally hyperbolic ordinary background* $(\mathbb{R}^4, \tilde{g})$*, already typestated in Definition and Theorem (1), consider a linearised ordinary probe scalar* $\delta\psi_o$ *obeying a written normally hyperbolic equation*

$$\operatorname{square}_{\tilde{g}} \delta\psi_o = \mathscr{I}[h_{ab}, \psi_o^{\text{bg}}],$$

with a compactly supported prescribed source $\mathscr{I}$*. There exists a unique retarded solution, and its support lies in the ordinary causal future* $J^+_{\tilde{g}}(\operatorname{supp} \mathscr{I})$*. Consequently an ordinary archive supported on a slice* $\{t = t_1\}$ *cannot send an ordinary* $\tilde{g}$*-causal influence back to a slice* $\{t = t_0\}$ *with* $t_0 < t_1$*. This is retarded linearised hyperbolic propagation on a fixed globally hyperbolic ordinary metric. It is not Einstein dynamics, not a derivation of* δg_{ab} *from a dark payload, not a well-posedness theorem for a fully coupled dynamical metric, and not a new discharge of the* 0 *of* 3 *operational time-machine bar.*

Proof. By Theorem (1), the function t is a temporal function for $\tilde{g}$, every slice $\{t = t_*\}$ is a global Cauchy surface, and $(\mathbb{R}^4, \tilde{g})$ is globally hyperbolic. The operator $\operatorname{square}_{\tilde{g}}$ is normally hyperbolic. On a globally hyperbolic Lorentzian manifold, a normally hyperbolic operator admits unique retarded and advanced fundamental solutions,

and the retarded solution of with a compactly supported smooth source is the unique smooth solution that vanishes to the past of supp $\mathscr{S}$ and has support in $J^{+}_{\tilde{g}}(\mathrm{supp}\,\mathscr{S})$. (The same conclusion follows from the explicit finite-propagation bound in the proof of Theorem (1): every $\tilde{g}$-causal curve has uniformly bounded Euclidean speed $|d\mathbf{x}/dt| \le \sqrt{1 + C_0^2} + C_0$, so the domain of dependence of a compact source is compact in each finite time interval, and a solution prescribed to vanish for t below the source cannot reappear outside the ordinary causal future.)

Because t is a temporal function, it is strictly increasing along every future-directed $\tilde{g}$-causal curve. No point of a slice $\{t = t_0\}$ with $t_0 < t_1$ lies in $J^{+}_{\tilde{g}}(\{t = t_1\})$. Therefore a source supported on an ordinary archive at time t_1 cannot influence, through the retarded solution of , any ordinary observable supported at time $t_0 < t_1$.

The background metric $\tilde{g}$ is held fixed throughout: the source $\mathscr{S}$ is a prescribed linear functional of a written perturbation, not a dynamical Einstein tensor, and the equation is not coupled back to an unknown metric increment. Official Theorem already holds the host metric fixed when typestating operational signalling; the present proposition answers a linearised question about prescribed ordinary perturbations without opening that lock. The operational time-machine bar remains 0 of 3.

Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; H3 stays NOT_CLOSED; XV-B is not opened; XVI-N is not discharged; the time-machine bar remains 0 of 3.

Host gravitational-wave baseline extraction of D??

The third official task is to isolate the spatial determinant D_{sp} from ordinary Fermat data only after an independent host gravitational-wave spatial baseline has been supplied, and to keep the zero-impact arrival-time residual strictly typed as a bimetric optical–GW comparison. Official Discovery-A version 5 [7] records documentary four-cone typestate and is not a premise. The four-dimensional volume factor D of is not confused with D_{sp}.

Lemma (Stationary remainder identity). *On any prescribed stationary dual-ledger background (Definition), the ordinary metric components are*

$$\tilde{g}_{00} = -A^2(1 + 2\Psi_N - q_0), \qquad \tilde{g}_{0i} = A^2\omega_i + BJ_0J_i, \qquad \tilde{g}_{ij} = A^2(1 - 2\Phi_N)\delta_{ij} + BJ_iJ_j,$$

where $q_0 := BJ_0^2/A^2$. *With* $K := -\tilde{g}_{00}$, $N_i := \tilde{g}_{0i}$, $\gamma_{ij} := \tilde{g}_{ij}$, $X := \gamma^{ij}N_iN_j/K$, *ordinary lapse* $\alpha^2 := K(1 + X) > 0$, *clock potential* $\Psi^{\mathrm{eff}}_{\mathrm{dyn}} := \ln\alpha$, *and isotropic Fermat potential* $\Phi^{\mathrm{iso}}_{\mathrm{lens}} := -\frac{1}{2}\ln((\det n_{ij})^{1/6})$, *one has*

$$2\Phi^{\mathrm{iso}}_{\mathrm{lens}} - \Psi^{\mathrm{eff}}_{\mathrm{dyn}} + \ln A + \frac{1}{2}\ln(1 - 2\Phi_N) = -\frac{1}{6}\ln D_{\mathrm{sp}} - \frac{2}{3}\ln(1 + X) =: \mathscr{R}[J_i, \omega_i].$$

The remainder vanishes if and only if $D_{\mathrm{sp}}(1 + X)^4 = 1$. *In particular it vanishes whenever* $D_{\mathrm{sp}} = 1$ *and* $N_i = 0$, *which holds when* $J_i = 0$ *and* $\omega_i = 0$. *If* $B \geq 0$, *so that* $D_{\mathrm{sp}} \geq 1$, *and if* γ_{ij} *is positive definite with* $K > 0$, *so that* $X \geq 0$, *this condition is equivalent to* $D_{\mathrm{sp}} = 1$ *and* $N_i = 0$. *A cancellation with* $D_{\mathrm{sp}} \neq 1$ *is not typestated as excluded when* $B < 0$

.

Proof. The components are immediate from and . The lapse-shift identity $\tilde{g}_{00} = -K = -\alpha^2 + \gamma^{ij}N_iN_j$ is the definition of the lapse α. Substituting the components into $\tilde{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b = 0$ produces the quadratic $K\,\mathrm{d}t^2 - 2N_i\,\mathrm{d}x^i\,\mathrm{d}t - \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = 0$, whose forward root is . By the matrix determinant lemma, $\det\gamma_{ij} = A^6(1 - 2\Phi_N)^3D_{\mathrm{sp}}$. Moreover

$$\det n_{ij} = \frac{\det\gamma}{K^3}\det\left(I + \frac{\gamma^{-1}N \otimes N}{K}\right) = \frac{\det\gamma}{K^3}(1 + X),$$

so $(\det n)^{1/6} = A(1 - 2\Phi_N)^{1/2}D_{\mathrm{sp}}^{1/6}K^{-1/2}(1 + X)^{1/6}$. Since $\alpha^2 = K(1 + X)$, one has $\alpha K^{-1/2} = (1 + X)^{1/2}$. Therefore

$$\frac{(\det n)^{1/6}\,\alpha}{A(1 - 2\Phi_N)^{1/2}} = D_{\mathrm{sp}}^{1/6}(1 + X)^{2/3}.$$

Taking logarithms and using $2\Phi^{\mathrm{iso}}_{\mathrm{lens}} = -\ln(\det n)^{1/6}$ together with $\Psi^{\mathrm{eff}}_{\mathrm{dyn}} = \ln\alpha$ yields . From the displayed identity, $\mathscr{R} = 0$ if and only if $D_{\mathrm{sp}}(1 + X)^4 = 1$. In a chart with $1 - 2\Phi_N > 0$ and $A > 0$, the matrix-determinant identity $\det(\delta^i_{\ j} + v^iv_j) = 1 + |v|^2_\delta$ (or its analytic continuation when $B < 0$) shows that $D_{\mathrm{sp}} = 1$ if and only if $BJ_i = 0$, while $1 + X = 1$ if and only if $N_i = 0$. Thus, if $B \geq 0$ (hence $D_{\mathrm{sp}} \geq 1$) and γ_{ij} is positive definite with $K > 0$ (hence $X \geq 0$), the product identity forces $D_{\mathrm{sp}} = 1$ and $N_i = 0$. If $B < 0$, a cancellation $D_{\mathrm{sp}}(1 + X)^4 = 1$ with $D_{\mathrm{sp}} \neq 1$ is not typestated as excluded.

Proposition (Host-GW baseline extraction of D_{sp}). *On a prescribed stationary background (Definition), suppose one is given the isotropic Fermat potential $\Phi^{\mathrm{iso}}_{\mathrm{lens}}$, the clock potential $\Psi^{\mathrm{eff}}_{\mathrm{dyn}} = \ln\alpha$, the directional Fermat one-form $w_i\,\mathrm{d}x^i := (N_i/K)\,\mathrm{d}x^i$, and the Fermat metric n_{ij} of . Write $h_{ij} := n_{ij} - w_i w_j = \gamma_{ij}/K$. Then the combination $X = \gamma^{ij}N_iN_j/K$ is determined algebraically from w_i and h_{ij}. Determination of D_{sp} is not typestated from these ordinary data alone. If, in addition, an independent host gravitational-wave spatial baseline is supplied that anchors the conformal class of δ_{ij}—the host Sagnac-subtracted tensor $h^{(g)}_{ij}$, which is conformal to δ_{ij} on , or equivalently the conformal class $[\delta_{ij}]$ when $\omega_i = 0$—then $D_{\mathrm{sp}} = \lambda_{\parallel}/\lambda_{\perp}$ is read from the eigenvalues of h_{ij} relative to that baseline, uniquely, as the eigenvalue ratio in three-dimensional space on the written two-Fermat setup, and the remainder $\mathscr{R}[J_i,\omega_i]$ may be subtracted from $2\Phi^{\mathrm{iso}}_{\mathrm{lens}} - \Psi^{\mathrm{eff}}_{\mathrm{dyn}}$ so as to recover the static elimination combination $-\ln A - \frac{1}{2}\ln(1-2\Phi_N)$. The remainder typestate of Lemma is unchanged: $\mathscr{R}$ vanishes if and only if $D_{\mathrm{sp}}(1+X)^4 = 1$. This is an isolation of a printed remainder, given the written host baseline, not a closed inversion of (A,Φ_N,J_i,ω_i) and not a unique physical reconstruction of the dressing without that baseline. In particular, on the weak-field host itself, the host Fermat data are the exact algebraic identities*

$$n^{(g)}_{ij} = \frac{1-2\Phi_N}{1+2\Psi_N}\delta_{ij} + \frac{\omega_i\omega_j}{(1+2\Psi_N)^2}, \qquad w^{(g)}_i = \frac{\omega_i}{1+2\Psi_N},$$
$$h^{(g)}_{ij} := n^{(g)}_{ij} - w^{(g)}_i w^{(g)}_j = \frac{1-2\Phi_N}{1+2\Psi_N}\delta_{ij}.$$

Thus, when $\omega_i \neq 0$, the host Fermat tensor $n^{(g)}_{ij}$ is not conformal to δ_{ij}, while the Sagnac-subtracted tensor $h^{(g)}_{ij}$ is. The host conformal class used in is therefore $[h^{(g)}_{ij}] = [\delta_{ij}]$, not $[n^{(g)}_{ij}]$. Official D_{sp} remains the spatial-matrix determinant of ; the class-limited equality $D_{\mathrm{sp}} = \lambda_{\parallel}/\lambda_{\perp}$ is retained on the written rank-one two-Fermat setup relative to that host class, and is not a general redefinition. The identities are not transferred to the Painlevé–Gullstrand collar , on which $n_{ij} = \delta_{ij}$ and $h_{ij} = \delta_{ij} - \omega_i\omega_j$, so that the Sagnac-subtracted tensor is not conformal to δ_{ij}. Determination of D_{sp} still requires the written host gravitational-wave spatial baseline. Hessian inversion, not Laplacian inversion, remains official if a peak-shift identity appears. par Non-claim. D_{sp} is not typestated without the host gravitational-wave spatial Fermat baseline. The residual $\Delta T_{\mathrm{GW-EM}}$ of Proposition is not an operational CTC and is not causal signalling from $\mathscr{U}_{\mathrm{CTC}}$. The time-machine bar remains 0 *of* 3. *par*

Proof. From $\alpha^2 = K(1 + X)$ and $n_{ij} = \gamma_{ij}/K + N_i N_j/K^2$, one has $h_{ij} = n_{ij} - w_i w_j = \gamma_{ij}/K$. The scalar $X = K\gamma^{ij} w_i w_j$ is then a combination of w_i and h_{ij} alone, which is the first claim. The determinant $\det h = (\det \gamma) K^{-3}$ still mixes D_{sp} with the unknown conformal factor $A^2(1 - 2\Phi_N)$, so D_{sp} is not fixed by $(\Phi^{\mathrm{iso}}_{\mathrm{lens}}, \Psi^{\mathrm{eff}}_{\mathrm{dyn}}, w_i, n_{ij})$ alone.

Given a host gravitational-wave spatial baseline that supplies the conformal class of δ_{ij} (the Sagnac-subtracted host tensor $h^{(g)}_{ij}$ on the written weak-field host, as in), one may compare h_{ij} with that class. On the written spatial metric $\gamma_{ij} = A^2(1 - 2\Phi_N)\delta_{ij} + BJ_iJ_j$, the endomorphism $h^i{}_j$ relative to the host conformal class is a scalar multiple of $\delta^i{}_j + BJ^iJ_j/(A^2(1 - 2\Phi_N))$. A rank-one update has a simple eigenvalue

$$\lambda_{\parallel} = \frac{A^2(1 - 2\Phi_N)}{K}\left(1 + \frac{B|J|^2_\delta}{A^2(1 - 2\Phi_N)}\right)$$

along J_i, and a repeated eigenvalue

$$\lambda_{\perp} = \frac{A^2(1 - 2\Phi_N)}{K}$$

on the host-spatial plane orthogonal to J_i (or a triple eigenvalue $\lambda_\perp$ if $J_i = 0$). Their ratio is therefore

$$\frac{\lambda_{\parallel}}{\lambda_{\perp}} = 1 + \frac{B|J|^2_\delta}{A^2(1 - 2\Phi_N)} = D_{\mathrm{sp}},$$

by the matrix determinant lemma in three dimensions (the determinant of a rank-one update equals 1 plus the trace of the update, which is exactly the displayed ratio when the two transverse eigenvalues equal 1 in the normalised frame). The ratio is uniquely determined once the host conformal class is given, and it is independent of the overall conformal factor $A^2(1 - 2\Phi_N)/K$. When $\omega_i = 0$ and the host spatial metric is conformal to δ_{ij}, the baseline may be typestated as $[\delta_{ij}]$. On the written host the same class is $[h^{(g)}_{ij}]$ even when $\omega_i \neq 0$, by . With X and D_{sp} both in hand, $\mathscr{R} = -\frac{1}{6}\ln D_{\mathrm{sp}} - \frac{2}{3}\ln(1 + X)$ is determined, and subtracting it from $2\Phi^{\mathrm{iso}}_{\mathrm{lens}} - \Psi^{\mathrm{eff}}_{\mathrm{dyn}}$ yields $-\ln A - \frac{1}{2}\ln(1 - 2\Phi_N)$ by . The resulting combination still depends on the pair (A, Φ_N) and does not, by itself, invert the remaining dressing data. The vanishing criterion $\mathscr{R} = 0 \Leftrightarrow D_{\mathrm{sp}}(1 + X)^4 = 1$ is Lemma . No uniqueness or closure claim is made in the absence of the written baseline.

The host identities are algebraic on Definition and are not used as a premise of the ordinary Fermat extraction already proved. From one has $K^{(g)} = -g_{00} = 1 + 2\Psi_N$, $N_i^{(g)} = g_{0i} = \omega_i$, and $\gamma_{ij}^{(g)} = g_{ij} = (1 - 2\Phi_N)\delta_{ij}$. Substituting into the host copy of produces

$$n_{ij}^{(g)} = \frac{\gamma_{ij}^{(g)}}{K^{(g)}} + \frac{N_i^{(g)} N_j^{(g)}}{(K^{(g)})^2} = \frac{1 - 2\Phi_N}{1 + 2\Psi_N}\delta_{ij} + \frac{\omega_i \omega_j}{(1 + 2\Psi_N)^2},$$

together with $w_i^{(g)} = N_i^{(g)}/K^{(g)} = \omega_i/(1 + 2\Psi_N)$. Subtracting the Sagnac square $w_i^{(g)} w_j^{(g)}$ cancels the second summand and leaves the conformal tensor $h_{ij}^{(g)} = (1 - 2\Phi_N)/(1 + 2\Psi_N)\,\delta_{ij}$. Hence $[h_{ij}^{(g)}] = [\delta_{ij}]$ whether or not ω_i vanishes, whereas $[n_{ij}^{(g)}] = [\delta_{ij}]$ if and only if $\omega_i = 0$ (on any open set where $1 + 2\Psi_N \neq 0$). The eigenvalue ratio is taken relative to this host class $[h_{ij}^{(g)}]$, not relative to $n_{ij}^{(g)}$ when the latter fails to be conformal to δ_{ij}. Official D_{sp} is not redefined: it remains , and the ratio identity continues to be the matrix-determinant lemma on the written rank-one two-Fermat setup.

These coefficient identities are special to . On the Painlevé–Gullstrand pair one has $K = 1$, $N_i = -\omega_i$, and $\gamma_{ij} = \delta_{ij} - \omega_i \omega_j$, hence $n_{ij} = \delta_{ij}$ and $h_{ij} = \delta_{ij} - \omega_i \omega_j$. The Sagnac-subtracted tensor is then not conformal to δ_{ij}. Four-cone typestate remains on Definition and is not transferred to the collar of Definition .

Proposition (Zero-impact optical–GW residual is not signalling). *On the equatorial plane $z = 0$ of the written collar of Theorem , the pair takes the form*

$$g|_{z=0} = -\mathrm{d}t^2 - 2a(r)\,\mathrm{d}t\,\mathrm{d}\varphi + (r^2 - a(r)^2)\mathrm{d}\varphi^2 + \mathrm{d}r^2,$$

$$\tilde{g}|_{z=0} = A^2(-\mathrm{d}t^2 - 2a(r)\,\mathrm{d}t\,\mathrm{d}\varphi + r^2\,\mathrm{d}\varphi^2 + \mathrm{d}r^2),$$

with $a(r) := \omega(\partial_\varphi) = (\sqrt{B}/A)c_0^2 r^2 e^{-2\alpha_r r^2} \geq 0$ decaying faster than r^{-2} as $r \to \infty$. For a future-directed equatorial null geodesic with Killing energy $E > 0$ and signed impact parameter $b_{\mathrm{imp}} := L/E$, the radial turning functions and coordinate-time slopes (above an outermost simple turning radius, and after stripping the irrelevant constant conformal factor of $\tilde{g}$) are

$$\mathscr{R}_g(r;b_{\mathrm{imp}}) = 1 - \frac{(b_{\mathrm{imp}} + a(r))^2}{r^2},\ \mathscr{T}_g(r;b_{\mathrm{imp}}) = \frac{r^2 - a(r)(b_{\mathrm{imp}} + a(r))}{r^2\sqrt{\mathscr{R}_g(r;b_{\mathrm{imp}})}},$$

$$\mathscr{R}_{\tilde{g}}(r;b_{\mathrm{imp}}) = 1 - \frac{(b_{\mathrm{imp}} + a(r))^2}{r^2 + a(r)^2},\ \mathscr{T}_{\tilde{g}}(r;b_{\mathrm{imp}}) = \frac{r^2 - a(r)b_{\mathrm{imp}}}{(r^2 + a(r)^2)\sqrt{\mathscr{R}_{\tilde{g}}(r;b_{\mathrm{imp}})}}.$$

At zero impact parameter on an active host CTC core ($\max_{r>0} a(r)/r > 1$*, equivalently* $\Gamma_{\mathrm{vort}} > 1$*), the ordinary* $\hat{g}$ *-null ray satisfies* $\mathscr{R}_{\tilde{g}}(r;0) = r^2/(r^2 + a(r)^2) > 0$ *for all* $r > 0$ *and transits the symmetry axis, while the host g-null ray turns at the outer chronology locus* $r_+^{\mathrm{CTC}} := \max\{r > 0 : a(r) = r\} > 0$*. The Minkowski-renormalised coordinate-time shifts obey*

$$\Delta T_{\mathrm{ren}}^{(\hat{g})}(0) = -2\int_0^\infty \frac{a(r)^2}{\sqrt{r^2 + a(r)^2}(r + \sqrt{r^2 + a(r)^2})}\,\mathrm{d}r < 0,$$

and

$$\Delta T_{\mathrm{ren}}^{(g)}(0) < \Delta T_{\mathrm{ren}}^{(\tilde{g})}(0) < 0, \qquad \Delta T_{\mathrm{GW-EM}}(0) := \Delta T_{\mathrm{ren}}^{(g)}(0) - \Delta T_{\mathrm{ren}}^{(\tilde{g})}(0) < 0.$$

The residual $\Delta T_{\mathrm{GW-EM}}(0) < 0$ *is a bimetric optical–GW comparison at the prescribed background. It is not an operational closed timelike curve, not causal signalling from* $\mathscr{U}_{\mathrm{CTC}}$ *in the sense of Definition , and not a discharge of the time-machine bar, which remains* 0 *of* 3*.*

Proof. The equatorial reductions – are the restriction of and , using $\omega = a(r)\,\mathrm{d}\varphi$ and $\hat{g}_{\varphi\varphi} = A^2r^2$. A constant conformal factor does not change unparametrised null curves, so the ordinary null geometry may be computed from the unimodular representative $\hat{g}$. Setting $F_g(r) = r^2$, $C_g(r) = r^2 - a(r)^2$ for g, and $F_{\tilde{g}}(r) = r^2 + a(r)^2$, $C_{\tilde{g}}(r) = r^2$ for $\tilde{g}$, the Killing identities $\dot{t} + a\dot{\varphi} = E$ and $-a\dot{t} + C\dot{\varphi} = Eb_{\mathrm{imp}}$ give $\dot{\varphi}/E = (b_{\mathrm{imp}} + a)/F$, $\dot{t}/E = (C - ab_{\mathrm{imp}})/F$, and $(\dot{r}/E)^2 = 1 - (b_{\mathrm{imp}} + a)^2/F$, which is –.

At $b_{\mathrm{imp}} = 0$ one has $\mathscr{T}_{\tilde{g}}(r;0) = r/\sqrt{r^2 + a(r)^2} \le 1$, and the identity

$$1 - \frac{r}{\sqrt{r^2 + a^2}} = \frac{a^2}{\sqrt{r^2 + a^2}(r + \sqrt{r^2 + a^2})}$$

yields , which is strictly negative as soon as a is not identically zero. The host g-null ray at vanishing impact

cannot penetrate past r_+^{CTC}, where $a(r) = r$ and $\mathscr{R}_g(r;0) = 0$. Writing $\Delta T_{\mathrm{ren}}^{(g)}(0) := -2r_+^{\mathrm{CTC}} + 2\int_{r_+^{\mathrm{CTC}}}^{\infty} (\mathscr{T}_g(r;0) - 1)\,\mathrm{d}r$ and $\mathscr{T}_g(r;0) = \sqrt{1 - a(r)^2/r^2}$ on the exterior, one has, for $r > r_+^{\mathrm{CTC}}$ and $a(r) > 0$,

$$\mathscr{T}_g(r;0)^2 = 1 - \frac{a(r)^2}{r^2} < 1 - \frac{a(r)^2}{r^2 + a(r)^2} = \mathscr{T}_{\tilde{g}}(r;0)^2,$$

hence $\mathscr{T}_g(r;0) < \mathscr{T}_{\tilde{g}}(r;0) \le 1$. Splitting the difference gives

$$\Delta T_{\mathrm{ren}}^{(g)}(0) - \Delta T_{\mathrm{ren}}^{(\tilde{g})}(0) = -2r_+^{\mathrm{CTC}} + 2\int_{r_+^{\mathrm{CTC}}}^{\infty} (\mathscr{T}_g - \mathscr{T}_{\tilde{g}})\,\mathrm{d}r - 2\int_0^{r_+^{\mathrm{CTC}}} (\mathscr{T}_{\tilde{g}} - 1)\,\mathrm{d}r$$

$$= -2\int_0^{r_+^{\mathrm{CTC}}} \mathscr{T}_{\tilde{g}}(r;0)\,\mathrm{d}r + 2\int_{r_+^{\mathrm{CTC}}}^{\infty} (\mathscr{T}_g - \mathscr{T}_{\tilde{g}})\,\mathrm{d}r.$$

The first integrand is strictly positive on $(0, r_+^{\mathrm{CTC}})$, and the second integrand is strictly negative wherever $a > 0$ on the exterior. Therefore the difference is strictly negative, which is .

The residual is computed from the prescribed pair $(g, \tilde{g})$ at fixed geometry, with no controllable dark payload and with no ordinary closed timelike curve. By Definition it is not operational signalling from $\mathscr{U}_{\mathrm{CTC}}$. Theorem therefore remains compatible with a nonzero arrival-time residual under portal factorisation: the residual is a kinematic comparison, not an archive channel. The time-machine bar is untouched.

Lemma (Finite-height cutoff of the on-shell CTC torus). *On the on-shell class of Theorem , the axial factor in is $e^{-4\alpha_z z^2} = e^{-12\alpha_r z^2}$. If $\Gamma_{\mathrm{vort}} > 1$, the host CTC torus $\mathscr{U}_{\mathrm{CTC}}$ is nonempty in an open neighbourhood of the equatorial ring and is spatially bounded in z: writing*

$$z_* := \sqrt{\frac{\ln \Gamma_{\mathrm{vort}}}{12\alpha_r}} > 0,$$

one has $g_{\varphi\varphi}(r,z) \ge 0$ for all $r \ge 0$ whenever $|z| \ge z_*$, so that $\mathscr{U}_{\mathrm{CTC}}$ does not meet the planes $|z| \ge z_*$. This is a statement about $\mathscr{U}_{\mathrm{CTC}}$. It is not a statement that the residual $\Delta T_{\mathrm{GW-EM}}(0, z_0)$ of an off-axis comparison vanishes at $|z_0| = z_*$, and a nonzero bimetric shift can remain at altitudes where $|\omega| < 1$.

Proof. From at $\alpha_z = 3\alpha_r$,

$$g_{\varphi\varphi}(r,z) = r^2(1 - \frac{B}{A^2}c_0^4 r^2 e^{-4\alpha_r r^2 - 12\alpha_r z^2}).$$

The term in parentheses is minimised, at fixed z, when $re^{-2\alpha_r r^2}$ is maximised, hence at $r = (4\alpha_r)^{-1/2}$, and the minimal value is $1 - \Gamma_{\text{vort}} e^{-12\alpha_r z^2}$. This is nonnegative precisely when $\Gamma_{\text{vort}} e^{-12\alpha_r z^2} \le 1$. For $\Gamma_{\text{vort}} > 1$ the threshold altitude is , and for $|z| \ge z_*$ one has $g_{\varphi\varphi} \ge 0$ at every r. Boundedness in r at each fixed z with $|z| < z_*$ is the same elementary Gaussian maximum already used in Theorem (2). The residual $\Delta T_{\text{GW}-\text{EM}}$ is not an argument of this estimate.

Lemma (Frozen-altitude comparison is not a geodesic reduction). *On the on-shell class of Theorem , the metric coefficients of both g and $\hat{g}$ depend on z through the Gaussian factor $e^{-2\alpha_z z^2}$. In particular $\partial_z g_{\varphi\varphi}$ need not vanish at a written altitude $z = z_0 \neq 0$, so a curve with $\dot{z} = 0$ is not in general a geodesic of either metric. Consequently the equatorial null reduction of Proposition is not typestated as supplying, by restriction, a geodesic arrival residual $\Delta T_{\text{GW}-\text{EM}}(0, z_0)$ at $z_0 \neq 0$. If, as a purely kinematic comparison and not as a geodesic identity, one freezes the coefficients at $z = z_0$ and repeats the equatorial calculation of Proposition on the resulting two-dimensional pair, with $a(r; z_0) := (\sqrt{B}/A)c_0^2 r^2 e^{-2\alpha_r r^2 - 2\alpha_z z_0^2}$, the same sign $\Delta T^{\text{fr}}_{\text{GW}-\text{EM}}(0, z_0) < 0$ is obtained whenever $a(\,\cdot\,; z_0)$ is not identically zero, including at altitudes $|z_0| \ge z_*$ at which $\mathscr{U}_{\text{CTC}}$ is empty. This frozen comparison is not promoted to a geodesic theorem, and is not a claim that the true off-axis geodesic residual vanishes, or fails to vanish, at the finite-height cutoff of Lemma .*

Proof. Dependence of h, hence of ω and of $g_{\varphi\varphi}$, on z at $\alpha_z > 0$ is Definition . The geodesic equation for the z -coordinate includes the term $\Gamma^z_{\varphi\varphi}\dot{\varphi}^2$, and $\Gamma^z_{\varphi\varphi}$ contains $\partial_z g_{\varphi\varphi}$. The latter is nonzero at $z = z_0 \neq 0$ on any open radial interval where the Gaussian factor has a nontrivial radial profile. Thus a constant-altitude curve with $\dot{\varphi} \neq 0$ fails, in general, to be geodesic, and the reduction of Proposition does not apply verbatim.

If one nonetheless freezes the coefficients at $z = z_0$ and treats (t, r, φ) as an equatorial problem with the frozen profile $a(r; z_0)$, the algebraic identities – hold with $a(r)$ replaced by $a(r; z_0)$. The profile $a(r; z_0)$ remains nonnegative, decays faster than r^{-2}, and vanishes identically only if $c_0 = 0$. The sign argument of Proposition

therefore yields $\Delta T^{\mathrm{fr}}_{\mathrm{GW-EM}}(0, z_0) < 0$ whenever $c_0 > 0$, whether or not $\max_r a(r; z_0)/r > 1$. In particular the frozen residual need not vanish at $|z_0| = z_*$. The construction uses a frozen coefficient restriction, not a totally geodesic submanifold, and is recorded only as a kinematic comparison.

Conjecture (Off-axis geodesic residual). On the on-shell class of Theorem , the true geodesic arrival residual at a written altitude, if defined by a further local reduction of ordinary and host rays that are initially equatorial-parallel at $z = z_0$, remains a bimetric optical–GW comparison in the sense of Definition , never an operational CTC and never signalling, and does not vanish merely because $\Gamma_{\mathrm{vort}} e^{-12\alpha_r z_0^2} \le 1$. This conjecture is not a theorem, not a premise of any theorem in this sheet, and not a typestate of a LISA, AION, MAGIS, SKA, or PTA forecast.

Ledger unchanged: no quantum coefficient is admitted; Host C is not created; O.1 stays NOT_CLOSED; H3 stays NOT_CLOSED; XV-B is not opened; XVI-N is not discharged; the time-machine bar remains 0 of 3.

Provenance, refusals, and standing locks

This official Version 3 is a typographic successor of official Dual-Ledger-Chrono-Shield-A version 2 [1] only, and retains official Version 2 typestate, labels, and local proofs. Official Version 1 [5] remains a historical documentary source record, as the typographic parent of official Version 2, not a premise of the present sheet. Official Dual-Ledger-Chrono-DM-Backward-A version 4 [2], official Dual-Ledger-Chrono-DM-A version 4 [3], Chrono-CTC-A [4], official Dual-Ledger-DM-Discovery-A version 4 [6], official Dual-Ledger-DM-Discovery-A version 5 [7], official Dual-Ledger-DM-Discovery-A version 7 [8], and official Dual-Ledger-DM-Inversion-A version 6 [9] are documentary source records only. Identities used above were re-proved locally from Definitions printed in Section . Official Discovery-A versions 4 and 5 remain historical documentary records of the typestate cited in the body; official version 7 is the live documentary comparison, without importing d-branch typestate. Still-unwritten official companions are not delivered source records and are not cited as such. Suggested problem-card language #problem-algebraic-chronology is documentary intent only; it is not a registration and not a landing page. No new landing-page, figshare, or Zenodo URL is invented, and no deposit or Scholar-inclusion

campaign is opened.

The following are refused as typestate of this sheet, and are not theorems, propositions, lemmas, or corollaries herein: "breakthrough" language; promoting official Discovery D1–D5 or Breakthroughs 1–4 to theorems; Hawking's chronology protection conjecture in generality; a semiclassical $\langle T_{ab} \rangle$ computation or blow-up wall; a typestate of zero curvature where only a bound is proved; three-dimensional Schwartz integrability at $\alpha_z = 0$; an extracted ordinary closed timelike curve; an operational time machine; a new discharge of the 0 of 3 bar; a traversable wormhole; an opening of XV-B; closing O.1, Problem 1, XVI-N, J2, or inversion conditions (ii), (iv), or (v); conflation of $\Delta T_{\mathrm{GW-EM}}$ with signalling or with an operational CTC; a general $\alpha_z > 0$ on-shell existence theorem; uniqueness of K_{AB}; a quantum no-ghost theorem; withdrawing official Theorem (3); finite total three-dimensional controller energy of the full switch-plus-vortex action; redefining D_{sp} as an eigenvalue ratio by fiat; typestating $n_{ij}^{(g)}$ as conformal to δ_{ij} when $\omega_i \neq 0$ on ; transferring the weak-field Sagnac identity to the Painlevé–Gullstrand collar; typestating D_{sp} without a host gravitational-wave spatial Fermat baseline; the claim that $\Delta T_{\mathrm{GW-EM}}(0, z_0)$ vanishes once $\Gamma_{\mathrm{vort}} e^{-12\alpha_r z_0^2} \leq 1$; the claim that $\Delta c_2 = 0$ forces all higher Wilson coefficients; the claim that portal factorisation is dynamically forced by quantum gravity; any quantum coefficient; Einstein dynamics; unimodular gravity; ultraviolet completeness; exact weak equivalence in nature; invented observational thresholds; laboratory confirmation; LISA, AION, MAGIS, SKA, or PTA forecasts; a solution of the cosmological constant problem, the problem of time, or the information paradox; deposit; companion wiring; a Scholar-inclusion campaign; and invented landing-page URLs.

Standing locks remain as in the front-matter ledger. Host C is not created. O.1 stays NOT_CLOSED. H3 stays NOT_CLOSED. Gate 0 and ST4b stay NOT_SUPPLIED. XV-B is not opened. XVI-N is not discharged. XVI-N-RouteB is not discharged. Problem 1 and the enlarged duration gate stay open. J2 is unresolved. Inversion conditions (ii), (iv), and (v) are not typestated as closed in full generality. The algebraic filter remains a necessary pre-filter, not ultraviolet completeness. The time-machine operational bar remains 0 of 3 and is not newly discharged by any identity of this sheet. wired=false. declaredPath remains null. Confirmation remains required

before any deposit, companion wiring, Host C, or Arb, Lean, or compile campaign. Informal typographic checks of this source are not a compile.

No fidelity bound is typestated in this sheet. In particular, neither a lower bound $F \geq \exp(-I/2)$ nor an upper bound $F \leq \exp(-I/2)$ is claimed, and no appeal is made to a relative-entropy inequality $D \geq -2\ln F$ or to monotonicity of a Rényi family D_α in the order α. Product Petz recovery, islands, and Page curves are outside the present scope.

Documentary negative controls

Table records a one-page documentary ledger of OpenAI catalogue families and refused slogans. The entries are unverified external catalogue items, cited only as documentary source records [12], and are isolated to this section. They are not theorem premises, not scheduler-derived cones, not typestate of a Page curve, island, bounce, or de Sitter state, and not a discharge of the operational time-machine bar, which remains 0 of 3 for ordinary observers with Enter instruments. No retained official identity in this sheet depends on these entries. Written dark-ledger identities, where retained, remain on the written dark ledger and the printed class; they are not chronology engines in nature and not uniqueness of dark matter.

Lemma already typestates the finite-height cutoff of the on-shell CTC torus at $\alpha_z = 3\alpha_r$. Vanishing of the off-axis residual at that cutoff altitude remains not typestate. Conjecture remains a conjecture; it is not upgraded to a theorem in official Version 3. These clauses sharpen ordinary Cauchy protection and the statement that a prescribed host core is not an operational CTC; they do not move Card 11.

References

[1] Cheung, N. (2026). Dual-Ledger-Chrono-Shield-A: Nonsingular Bimetric Cauchy-Slice Protection, Transverse-Schwartz Collars, and Operational Portal-Archive No-Signalling Across Smooth Host CTC Cores. *Enter Calculus Companion*, official version 2 (request t533u). Typographic parent of the present official Version 3; not a premise. Identities retained or repaired below are re-proved locally. *Source record; not a premise.*

[2] Cheung, N. (2026). Dual-Ledger-Chrono-DM-Backward-A: Backward sector attribution from prescribed

chronology asymmetry. *Enter Calculus Companion*, official version 4 (request t514u). *Source record; not a premise.*

[3] Cheung, N. (2026). Dual-Ledger-Chrono-DM-A: Winding, Chronology Asymmetry, and Operational Insulation on Prescribed Lorentzian Hosts. Exact Geometric Witnesses, Corrected No-Paradox Results, and Complete-Record Rejection Contracts. *Enter Calculus Companion*, official version 4 (request t506u). *Source record; not a premise.*

[4] Cheung, N. (n.d.). Chrono-CTC-A. Documentary manuscript identifier named in the supplied chronology excerpts. A prescribed CTC is distinguished there from an operational time machine and full-loop calibration. No version or registration locator is invented. *Source record; not a premise.*

[5] Cheung, N. (2026). Dual-Ledger-Chrono-Shield-A: Nonsingular Bimetric Cauchy-Slice Protection, Transverse-Schwartz Collars, and Operational Portal-Archive No-Signalling Across Smooth Host CTC Cores. *Enter Calculus Companion*, official version 1 (request t525u). Typographic parent of official Version 2, not of the present official Version 3; not a premise. *Source record; not a premise.*

[6] Cheung, N. (2026). Dual-Ledger-DM-Discovery-A: Five Conditional Conjectures and a Four-Tier Multi-Cone Protocol for Acquiring the Chronological Dark-Matter Slot. *Enter Calculus Companion*, official version 4 (request t521u). Historical documentary record only, where the citation remains specifically to Version 4 typestate. *Source record; not a premise.*

[7] Cheung, N. (2026). Dual-Ledger-DM-Discovery-A: Five Conditional Conjectures and a Four-Tier Multi-Cone Protocol for Acquiring the Chronological Dark-Matter Slot. *Enter Calculus Companion*, official version 5 (request t523u). Historical documentary record of transverse-Schwartz and four-cone typestate, re-proved locally herein. Typestate addendum t524u. d-branch typestate is not imported. *Source record; not a premise.*

[8] Cheung, N. (2026). Dual-Ledger-DM-Discovery-A: Five Conditional Conjectures and a Four-Tier Multi-Cone Protocol for Acquiring the Chronological Dark-Matter Slot. *Enter Calculus Companion*, official version 7 (request t542u). Live typestate comparison only. d-branch typestate is not imported. *Source record; not a premise.*

[9] Cheung, N. (2026). Dual-Ledger-DM-Inversion-A: Backwards Selection Rules for Residual Matter Sectors under an Algebraic Quantum-Gravity Filter. *Enter Calculus Companion*, official version 6 (request t522u). Cited for documentary portal and Δc_2 typestate, re-proved locally herein. *Source record; not a premise.*

[10] Bekenstein, J. D. (1993). The relation between physical and gravitational geometry. *Physical Review D*, **48**, 3641–3647.

[11] Hawking, S. W. (1992). The chronology protection conjecture. *Proceedings of the Royal Society of London A* , **435**, 649–668. Cited only as the published statement of a conjecture that this sheet does not typestate in generality.

[12] Documentary unverified external catalogue entries: Oai365-A (Family 365), Oai290-A (Family 290), Oai265-A/B, Oai276-A, Oai212-A, and refused Family 260. Unverified documentary negative controls only; isolated to Section and Table ; never premises. Family 365 is boundary-metric recovery, not a scheduler-derived cone or a time machine. Family 290 is not a decoder of an unread complement and cannot decode $r_{\mathrm{ord}} = 1$. Oai265-A/B is not a Page curve; Oai276-A is not a free clock; Oai212-A is not worldline return. Family 260 remains refused.