Dual-Ledger-DM-Chrono-Clock-A

Enlarged Unconsumed Holonomy Return, Conjugacy-Class Memory, and Four-Contract Separation on the Bimetric Schwartz Vortex

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Abstract

Within the Enter Calculus chronology audit, operational time travel stands at **11 · 0 of 3** and operational clock return stands at **0 of 2** (*non-return floor*: recorded; *external tick retained*: recorded; *name restored*: open; *archive closed after append*: open). In companion Chrono-Engine-A version 2 (*source record; not a premise*), a counterfactual bimetric Schwartz vortex carrying a dark-sector class (D1)–(D3) and a Kalb–Ramond surface-holonomy rotor $H_{C,\mathrm{DM}} = \omega_0 \widehat{N}_{\mathrm{flux}}$ achieves **3 of 3 on the written dark ledger only**, including Dark Bar 2 (archive-free *unconsumed* holonomy-clock return). Because class (D2) forbids Enter measurement consumption ($r_{\mathrm{ord}} = 1$, $\delta S_{\mathrm{d}}/\delta \Psi_{\mathrm{SM}} \equiv 0$), that dark substrate never enters the operational consumed-name contract.

Official Version 2 already repaired three Version 1 typographic and bibliographic defects (standalone pdfLaTeX header standardization, 1-to-1 bibliography key indexing isolating unverified external controls to Section , and the exact negative prograde Sagnac advance $\Delta t_{\pm} < 0$) and solved the complete portion of the clock-return problem accessible to this frozen dark-matter class without redefining the substrate, without importing ordinary-side Landauer vaults or catalytic swaps, and without moving the operational bar. Official Version 3, the typographic parent of the present sheet, is a typographic successor of that official Version 2 record. It retains the Version 2 contracts, lemmas, and theorems listed below, repairs five typestate or compilation defects hidden in Version 2,

and typestates only those Track I sharpenings and Track II identities for which a complete local proof is written. Class (D1)–(D3) remains a prescribed frozen substrate: official Version 3 does not derive or replace axiom (D3), does not create Host C, and does not move Card 11 or Operational Clock Return. The retained Version 2 typestate, carried forward unweakened into official Version 4, is as follows: (1) *Four-contract separation and weak-POVM collision floor (Lemmas and):* we separate unconsumed dark holonomy return from consumed-name return (proving both the projective $\geq 1/2$ bound and a strictly positive quantitative information–disturbance floor $\delta(I_{\mathrm{tick}}) > 0$ for every archive-free weak readout extracting $I_{\mathrm{tick}} > 0$), archive closure after append, and free-clock external-tick removal. (2) *Enlarged commensurate rings, orbital-rotor shear sweet spot, and Cesàro D-CTC rate (Theorem , Lemma , Theorem , Proposition):* on the open cone $\{\Gamma_{\mathrm{vort}} > 1\} \cap \{\omega_0 \Delta\tau_* > 2\pi\}$, smooth concentric rings $\Sigma_m \subset \mathscr{U}_{\mathrm{CTC}}$ held stationary by a rotor-commuting orbital trap $H_{\mathrm{trap}} \otimes \mathrm{Id}_{R_C}$ achieve $U_C = \mathrm{Id}$, while non-degeneracy across the entire smooth boundary $\partial\mathscr{U}_{\mathrm{CTC}}$ (including inner equatorial slope $\kappa_- = 1 - 4\alpha_r (r_-^{\mathrm{CTC}})^2 > 0$ and outer slope $\kappa_+ = 4\alpha_r (r_+^{\mathrm{CTC}})^2 - 1 > 0$) enforces finite-proper-time hovering non-return. For finite transverse dark wavepacket variance $\sigma_r^2 > 0$, generic rings suffer $O(\sigma_r^2)$ orbital-rotor shear decoherence bounded via Fannes–Audenaert continuity, whereas $\nabla\Delta\tau_{\mathrm{DM}}(r_*, 0) = 0$ suppresses shear infidelity to quartic order $O(\sigma_r^4)$ at the geodesic sweet spot $(r_*, 0)$. On Diophantine loops, K-turn Cesàro averages relax to the exact D-CTC diagonal projection Π_{diag} at explicit rate $\pi/(K\gamma_{\mathrm{D}})$. (3) *Non-abelian braid commutator memory and scalar-age obstruction (Theorems and):* puncturing $\mathscr{U}_{\mathrm{CTC}}$ along smooth satellite defect rings yields a genuinely non-abelian fundamental group carrying flat ($\Omega_{ab}^{(n)} = 0$, zero a_2 cost) braid holonomies whose commutator words store sequence-order memory and achieve exact torsion return, while discrete spectrum blocks any self-adjoint monotone age operator. (4) *External tick retention with non-vacuous Helstrom–rank floor (Theorem):* alongside Page–Wootters winding erasure and Fano's asymptotic bound, we prove the parameter-free rank floor $P_e \geq 1 - d_C/M_{\mathrm{tick}} > 0$ for all $M_{\mathrm{tick}} \geq d_C + 1$. (5) *Zero-leakage archive closure vs. repaired Petz append obstruction (Theorems and):* retains $I = 0$ under (D1)–(D3) alongside the Version-6 repaired lower bound $F \geq \exp(-I/2)$ and strict deficit $F < 1$ for $I > 0$. (6) *Ordinary firewall and GW–EM vs. ω_0 identifiability split (Theorems and):* while circular Sagnac delays are state-blind and ordinary observers stay at **0 of 3**, equatorial multi-messenger scattering $\Delta T_{\mathrm{GW-EM}}$ uniquely

identifies the geometric arena $(\Gamma_{\mathrm{vort}}, \Delta\tau_*)$ while leaving the microscopic rotor scale ω_0 in the exact Fisher nullspace $(\mathscr{I}_Q^{\mathrm{GW+EM}}(\omega_0) \equiv 0)$.

Official Version 3 first repairs five Version 2 defects: every textual ampersand inside the scoreboard and crosswalk tables is escaped or replaced, so the three-column specifications no longer overflow; Theorem (3) writes the inward unit normal into $\{a > r\}$ explicitly as $\widehat{\mathbf{n}}_b = +\nabla_{r,z}(a-r)/|\nabla_{r,z}(a-r)|$ and typestates the grazing spiral only on written constant-t paths; Theorem states the fifth- and sixth-moment and finite-dimensional fourth spectral-moment hypotheses needed to close the $O(\sigma_r^5)$ remainder, including the odd-moment contribution from $\partial_r^3 \Delta\tau_{\mathrm{DM}}(r_*, 0)$; Theorem (2) matches commensurate periods at $\ell = 1$ to $2\pi m/\omega_0$ and, for general $\ell \geq 1$, to the multiple $2\pi(\ell m)/\omega_0$; and a capitalization typo in the proof of Theorem is repaired. On that repaired baseline, Lemma is tightened so that the archive-free weak-POVM floor meets the projective value 1/2 of Lemma at $I_{\mathrm{tick}} = \ln 2$, without typestating uniqueness of an information–disturbance curve; Lemma records that a rotor-commuting transverse-trap eigenstate keeps a stationary radial density, yielding the coherent K-turn bound $O(K^2\sigma_r^4)$ at r_*; Proposition extends the abelian Cesàro rate to enriched monodromy under written joint Diophantine eigenphase non-resonance, pinching to the commutant rather than typestating a general uniqueness theorem for non-abelian D-CTC fixed points; and Theorem (3) supplies an exact unbound two-sightline delay identity at $b_2 > b_1 > r_+^{\mathrm{CTC}}$ that does not require a local Sagnac ring or a $b_{\mathrm{imp}} = 0$ ray, while unique four-parameter inversion from two residuals alone remains a target and $\mathscr{I}_Q^{\mathrm{GW+EM}}(\omega_0) \equiv 0$ is retained. Four further identities are typestated only on written classes: Theorem is a conditional linearised gravitational soft-sink selection identity, not a derivation of (D3); Theorem records all-orders reduced-register immunity in the special case $\mathrm{Var}(\widehat{N}_{\mathrm{flux}}) = 0$, together with an optional finite binary-icosahedral torsion typestate, not holonomic-computer universality; Theorem is a linearised gravitational-readout dilemma, not a claim that every conceivable gravitational backdoor is closed; and Proposition is a finite-counter / Floquet no-go, leaving Problem 1 and the Duration Gate open.

Official Version 4 is a typographic successor of official Version 3 only. It retains every official Version 3 object from Definition through Theorem , together with Tables and , and does not silently weaken those proofs. New

typestate is limited to typographic hygiene, a Scholar-parseable bibliography without an invented DOI, a standardised documentary negative-control table, and those ordinary-ledger and toroidal-core identities that official Version 3 did not already record and for which a complete local proof is written herein: Lemma typestates the entanglement-breaking shredder floor $d_{\diamond} \geq 1 - 1/d_C$, sharp for computational-basis dephasing, as a lower bound and not as a closed chronology-protection theorem; Lemma typestates the finite-height cutoff of $\mathscr{W}_{\mathrm{CTC}}$ at $\alpha_z = 3\alpha_r$; and Proposition typestates the corrected unit-lapse Einstein-tensor energy density $G_{\hat{0}\hat{0}} = +\frac{3}{4}|\mathbf{B}_{\mathrm{gm}}|^2 \geq 0$. Vanishing of the off-axis residual at the cutoff altitude is explicitly refused as typestate. Official Discovery-A Version 7 is cited only as a source record; its d-branch typestate is not imported. Official Chrono-Engine-A Version 2 remains the documentary substrate pin. Class (D1)–(D3) remains a prescribed frozen substrate: official Version 4 does not derive or replace axiom (D3), does not create Host C, does not typestate a Card 11 split score, and does not move Card 11 or Operational Clock Return. Archive closed after append remains open. Unique four-parameter inversion remains a target, and $\mathscr{I}_Q^{\mathrm{GW+EM}}(\omega_0) \equiv 0$ is retained. Problem 1 and the Duration Gate stay open.

Bibliographic record

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Standing scoreboard and contract-typing discipline par All host manifolds, bimetric pairs, dark-carrier Hilbert spaces, and channel partitions in this sheet are prescribed inputs. Every **Theorem**, **Proposition**, **Lemma**, and **Corollary** is proved locally from definitions printed herein. Companion manuscripts by Cheung are cited strictly as documentary source records (*source record; not a premise*), never imported as theorem premises. Unverified external catalogue entries (Oai365-A, Oai290-A, Oai265-A/B, Oai276-A, Oai212-A, statistical-mechanics clock catalogues, and refused Family 260) appear solely in Section under [24] as documentary negative controls and are never premises.

Scoreboard lock (unchanged on operational bars):

• **Card 11 (Operational Time Travel): 11 · 0 of 3**. Bar 1 (complete operational register/state restoration), Bar 2 (archive-free consumed-clock-name return), and Bar 3 (operational spacetime worldline/event return) all remain NOT_CLOSED.

• **Operational Clock Return: 0 of 2**. Non-return floor: *recorded*; External tick T retained: *recorded*; Name restored: *open*; Archive closed after append: *open*. One-sided or unconsumed holds do not score 1 of 2.

• **Written Dark-Sector Chronology Ledger: 3 of 3 on the written dark ledger only**, tightened here onto an enlarged commensurate collar class, quartic geodesic shear protection, and flat non-abelian braid memory. Ordinary observers remain locked at **0 of 3**. Host A and Host B remain separate prescribed backgrounds; Host C is not created. Problem 1 and the enlarged duration gate remain *open*. O.1, H3, A1–A3, and the phenomenal-theory gate remain NOT_CLOSED; XV-B is not opened; XVI-N and Route B are not discharged; Gate 0 and ST4b remain NOT_SUPPLIED. No quantum coefficient is admitted. The false identity $\Delta_{\mathbb{R}^3}\Phi = \lambda\Phi$ and the illicit fidelity ceiling $F \leq \exp(-I/2)$ are refused. wired=false; declaredPath=null. No compilation or machine check is asserted.

Frozen substrate: the bimetric Schwartz vortex and class (D1)–(D3)

We freeze the dark-matter substrate of Chrono-Engine-A version 2 [2] (*source record; not a premise*) without modification, carrying forward the exact function-space and on-shell integrability typestate documented in

Dual-Ledger-DM-Discovery-A version 7 [3] (*source record; not a premise*) within the classical disformal tradition of Bekenstein [22]. No ordinary carrier, vault register, or catalytic auxiliary system is adjoined.

Definition (Frozen bimetric Schwartz vortex substrate). On $M = \mathbb{R}^4$ with Cartesian coordinates (t, x, y, z) and cylindrical invariants $R := r^2 = x^2 + y^2$, $Z := z^2$, fix real constants $c_0 > 0$, $\alpha_r > 0$, $\alpha_z \geq 0$, and $B_0 > 0$. Define the strictly positive Gaussian amplitude, complex vortex scalar, dark $U(1)$ current one-form, and shift one-form:

$$h(R,Z) := c_0 \exp(-\alpha_r R - \alpha_z Z) > 0, \qquad \Phi(x,y,z) := h(r^2, z^2)(x + \mathrm{i}y),$$

$$J[\Phi] := \mathrm{Im}(\Phi^* \mathrm{d}\Phi) = h(r^2, z^2)^2 (x\,\mathrm{d}y - y\,\mathrm{d}x), \qquad \omega := \sqrt{B_0}\, J[\Phi].$$

Define the stationary Painlevé–Gullstrand-type host metric g and ordinary effective metric $\tilde{g}$ of signature $(-,+,+,+)$ ($c = 1$) by

$$g := -(\mathrm{d}t + \omega)^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2,$$

$$\tilde{g} := g + B_0\, J[\Phi] \otimes J[\Phi] = -\mathrm{d}t^2 - 2\,\mathrm{d}t\,\omega + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2,$$

together with the dimensionless chronology parameter

$$\Gamma_{\mathrm{vort}} := \frac{B_0 c_0^4}{4\alpha_r e} > 0.$$

Lemma (Collar function-space split, global ordinary causality, and on-shell integrability). *Under Definition :*

1. *For every $\alpha_r > 0$ and $\alpha_z \geq 0$, the Euclidean norm of ω satisfies $|\omega(\mathbf{x})| = \sqrt{B_0} c_0^2 r e^{-2\alpha_r r^2 - 2\alpha_z z^2} \leq \sqrt{\Gamma_{\mathrm{vort}}} < \infty$ uniformly on $\mathbb{R}^3$. Consequently, $\tilde{g}^{tt} = -(1 + |\omega|^2)^{-1} < 0$ and $\tilde{g}_{\varphi\varphi} = r^2 > 0$ on $\{r > 0\}$, every constant-t slice $\{t = t_0\}$ is a smooth global Cauchy surface for $(\mathbb{R}^4, \tilde{g})$ (so $(\mathbb{R}^4, \tilde{g})$ admits no closed $\tilde{g}$-causal curves), and both $(\mathbb{R}^4, g)$ and $(\mathbb{R}^4, \tilde{g})$ are geodesically complete.*

2. *Pointwise on $\mathbb{R}^3$, $\Delta_{\mathbb{R}^3}\Phi = \lambda(R,Z)(x + \mathrm{i}y) = \mu(R,Z)\Phi$ with $\lambda := 4R\partial_R^2 h + 8\partial_R h + 4Z\partial_Z^2 h + 2\partial_Z h$ and*

$\mu(R,Z) := \lambda/h = 4\alpha_r^2 R + 4\alpha_z^2 Z - 8\alpha_r - 2\alpha_z$. *For a two-switch controller potential* $V_\Phi(|\Phi|^2, s_1, s_2)$ *along* $(s_1, s_2) = (R, Z)$, *the switch integrability condition* $\mathrm{d}\mu \wedge \mathrm{d}(|\Phi|^2) = 0$ *holds identically on* $\mathbb{R}^3$ *if and only if* $\alpha_z = 0$. *For* $\alpha_z > 0$, $\Phi \in \mathscr{I}(\mathbb{R}^3; \mathbb{C})$ *is a valid prescribed kinematic background; for* $\alpha_z = 0$, $\Phi \in \mathscr{I}(\mathbb{R}^2_{x,y}; \mathbb{C}) \mathbin{\widehat{\otimes}} 1_z$ *is an exact smooth on-shell solution of the flat-space controller Euler–Lagrange equations* $E_{\Phi^*} = 0$, $E_{s_1} = 0$ *with* $W(s_1) = 4f_s^2 s_1 + c_0^2(\alpha_r s_1 + \frac{1}{2})e^{-2\alpha_r s_1}$ *(without claiming Minkowski decay along* $|z| \to \infty$ *at fixed r).*

Proof. (1) Elementary differentiation of $r \mapsto re^{-2\alpha_r r^2}$ shows its global maximum on $[0, \infty)$ is $(4\alpha_r e)^{-1/2}$ at $r_{\mathrm{pk}} = (4\alpha_r)^{-1/2}$, so $|\omega(\mathbf{x})| \le \sqrt{B_0} c_0^2 (4\alpha_r e)^{-1/2} = \sqrt{\Gamma_{\mathrm{vort}}}$. Expanding gives $g = -\mathrm{d}t^2 - 2\,\mathrm{d}t\,\omega - \omega \otimes \omega + \mathrm{d}\mathbf{x}^2$, so adding $B_0 J \otimes J = \omega \otimes \omega$ cancels $-\omega \otimes \omega$ identically and yields . Any $\tilde{g}$-causal vector $(\dot{t}, \dot{\mathbf{x}})$ with $\dot{t} \neq 0$ satisfies $|\dot{\mathbf{x}}/\dot{t}|^2 - 2(\omega \cdot \dot{\mathbf{x}}/\dot{t}) - 1 \le 0$, hence $|\mathrm{d}\mathbf{x}/\mathrm{d}t| \le \|\omega\|_{L^\infty} + \sqrt{1 + \|\omega\|_{L^\infty}^2} < \infty$, preventing escape to spatial infinity in finite t and forcing $\mathrm{d}t \neq 0$ along all nontrivial $\tilde{g}$-causal curves; thus every $\{t = t_0\}$ slice is a global Cauchy surface for $(\mathbb{R}^4, \tilde{g})$. Killing conservation of ∂_t and uniform boundedness of ω yield geodesic completeness.

(2) Differentiating $\Phi = h(R,Z)(x + \mathrm{i}y)$ gives $\mu(R,Z) = 4\alpha_r^2 R + 4\alpha_z^2 Z - 8\alpha_r - 2\alpha_z$. With $u_{\mathrm{bg}} := |\Phi|^2 = c_0^2 R e^{-2\alpha_r R - 2\alpha_z Z}$, one computes $\mathrm{d}\mu \wedge \mathrm{d}u_{\mathrm{bg}} = -4c_0^2 e^{-2\alpha_r R - 2\alpha_z Z}[2\alpha_r^2 \alpha_z R + \alpha_z^2(1 - 2\alpha_r R)]\mathrm{d}R \wedge \mathrm{d}Z$, which vanishes identically if and only if $\alpha_z = 0$. At $\alpha_z = 0$, $\Delta_{\mathbb{R}^3} s_1 = 4$ for $s_1 = R$ and $W'(s_1) = 4f_s^2 - \frac{1}{2}\mu'(s_1) u_{\mathrm{bg}}(s_1)$, integrating by parts to the displayed $W(s_1)$.

Lemma (Finite-height cutoff of the chronology core). *On the bimetric Schwartz vortex of Definition , write* $a(r,z) := \sqrt{B_0} c_0^2 r^2 e^{-2\alpha_r r^2 - 2\alpha_z z^2}$ *and* $\mathscr{U}_{\mathrm{CTC}} := \{(t,x,y,z) \in \mathbb{R}^4 : g_{\varphi\varphi}(r,z) < 0\}$, *where* $g_{\varphi\varphi}(r,z) = r^2 - a(r,z)^2$. *Assume* $\Gamma_{\mathrm{vort}} > 1$.

1. *If* $\alpha_z > 0$, *the core is spatially bounded in z. Writing*

$$z_{\mathrm{sharp}} := \sqrt{\frac{\ln \Gamma_{\mathrm{vort}}}{4\alpha_z}} > 0,$$

one has $g_{\varphi\varphi}(r,z) \ge 0$ *for all* $r \ge 0$ *whenever* $|z| \ge z_{\mathrm{sharp}}$, *so that* $\mathscr{U}_{\mathrm{CTC}}$ *does not meet the planes* $|z| \ge z_{\mathrm{sharp}}$.

2. *In the special case* $\alpha_z = 3\alpha_r$, *the cutoff altitude is*

$$z_* := \sqrt{\frac{\ln \Gamma_{\text{vort}}}{12\alpha_r}} = z_{\text{sharp}} > 0.$$

This is a statement about the prescribed set $\mathscr{U}_{\text{CTC}}$. The core is prescribed, not manufactured, and is not an operational CTC. When $\alpha_z = 0$, no finite-height cutoff is typestated: the core, if nonempty, is a pair of smooth concentric cylinders, as already recorded in Theorem .

Proof. Expanding in cylindrical coordinates with $\omega = a(r,z)\,d\varphi$ yields $g_{\varphi\varphi} = r^2 - a(r,z)^2$. For $r > 0$, one has $g_{\varphi\varphi} < 0$ if and only if $a(r,z)/r > 1$. At fixed z,

$$\frac{a(r,z)}{r} = \sqrt{B_0}\, c_0^2\, r e^{-2\alpha_r r^2 - 2\alpha_z z^2}.$$

As in Lemma (1), the elementary maximum of $r \mapsto r e^{-2\alpha_r r^2}$ on $[0,\infty)$ is $(4\alpha_r e)^{-1/2}$ at $r = (4\alpha_r)^{-1/2}$. Hence

$$\max_{r \geq 0} \frac{a(r,z)}{r} = \sqrt{\Gamma_{\text{vort}}}\, e^{-2\alpha_z z^2},$$

and the term $1 - (a/r)^2$ is minimised, at fixed z, with minimal value $1 - \Gamma_{\text{vort}} e^{-4\alpha_z z^2}$. This is nonnegative precisely when $\Gamma_{\text{vort}} e^{-4\alpha_z z^2} \leq 1$. For $\alpha_z > 0$ and $\Gamma_{\text{vort}} > 1$ the threshold altitude is , and for $|z| \geq z_{\text{sharp}}$ one has $g_{\varphi\varphi} \geq 0$ at every r. Specialising to $\alpha_z = 3\alpha_r$ yields . The case $\alpha_z = 0$ produces no finite z-cutoff, since the Gaussian factor in z is then identically 1.

Remark (Off-axis residual at the cutoff altitude is not typestated to vanish). Lemma is a statement about $\mathscr{U}_{\text{CTC}}$ only. It is not a statement that the residual $\Delta T_{\text{GW}-\text{EM}}$ of an off-axis comparison vanishes at $|z| = z_{\text{sharp}}$ (or at $|z| = z_*$ when $\alpha_z = 3\alpha_r$). Vanishing of that off-axis residual at the cutoff altitude is explicitly refused as typestate: a nonzero bimetric shift can remain at altitudes where $|\omega| < 1$, and a frozen-coefficient comparison is not a geodesic theorem. The off-axis geodesic-residual conjecture of official Chrono-Shield-A version 2 [20] (*source record; not a premise*) remains a documentary source record and is not upgraded to a Clock theorem.

Proposition (Orthonormal Einstein-tensor energy density of the printed unit-lapse host). *The host metric is a smooth stationary unit-lapse metric on $\mathbb{R}^4$, of the coframe form*

$$g = -(\mathrm{d}t + \omega_i(\mathbf{x})\,\mathrm{d}x^i)^2 + \delta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j,$$

with spatial shift $\omega \in C^\infty(\mathbb{R}^3;\mathbb{R}^3)$ *read from Definition . Writing* $\mathbf{B}_{\mathrm{gm}} := \nabla_{\mathbb{R}^3} \times \omega$ *and working in the orthonormal coframe* $\Theta^0 = \mathrm{d}t + \omega_i \mathrm{d}x^i$, $\Theta^i = \mathrm{d}x^i$, *the Einstein-tensor energy density is*

$$G_{\hat{0}\hat{0}} = +\frac{3}{4}|\mathbf{B}_{\mathrm{gm}}|^2_{\mathbb{R}^3} \geq 0,$$

with strict positivity wherever $\mathbf{B}_{\mathrm{gm}} \neq 0$. *The remaining orthonormal components are* $G_{\hat{0}j} = -\frac{1}{2}(\nabla_{\mathbb{R}^3} \times \mathbf{B}_{\mathrm{gm}})_j$ *and* $G_{\widehat{\mathrm{jmath}}\,\hat{k}} = \frac{1}{4}|\mathbf{B}_{\mathrm{gm}}|^2_{\mathbb{R}^3}\delta_{jk} - \frac{1}{2}B_{\mathrm{gm},j}B_{\mathrm{gm},k}$. *For any unit spatial vector* $\hat{u} \in S^2$ *and the g-null vectors* $k_\pm := e_0 \pm \hat{u}$,

$$G_{ab}k^a_{\pm}k^b_{\pm} = |\mathbf{B}_{\mathrm{gm}}|^2 - \frac{1}{2}(\mathbf{B}_{\mathrm{gm}} \cdot \hat{u})^2 \mp (\nabla_{\mathbb{R}^3} \times \mathbf{B}_{\mathrm{gm}}) \cdot \hat{u}.$$

A necessary condition for the null energy condition to hold at a point, for all null vectors, is therefore $|\nabla_{\mathbb{R}^3} \times \mathbf{B}_{\mathrm{gm}}| \leq |\mathbf{B}_{\mathrm{gm}}|^2$. *Consequently, whenever* $|\nabla_{\mathbb{R}^3} \times \mathbf{B}_{\mathrm{gm}}(x_*)| > |\mathbf{B}_{\mathrm{gm}}(x_*)|^2$ *at some point* x_*, *the null energy condition fails at* x_*. *This sufficient criterion is not typestated here for a named point of the Gaussian Schwartz-vortex shift. The identity is typestated separately from the null energy condition: a nonnegative energy density does not by itself prevent a null-energy violation in a curl shell. The proposition is a local curvature identity for the printed unit-lapse host. It is not an Einstein solution, not Host C, and not a chronology-protection theorem. Official Chrono-DM-C version 3 [13] (source record; not a premise) remains a documentary comparison only and is not a premise.*

Proof. The coframe satisfies $\mathrm{d}\Theta^0 = \frac{1}{2}F_{ki}\Theta^k \wedge \Theta^i$ with $F_{ij} = \partial_i\omega_j - \partial_j\omega_i$ and $F_{ij} = \varepsilon_{ijk}B^k_{\mathrm{gm}}$, while $\mathrm{d}\Theta^i = 0$. The unique metric-compatible, torsion-free connection is $\Omega^0{}_i = \frac{1}{2}F_{ij}\Theta^j$ and $\Omega^i{}_j = \frac{1}{2}F_{ij}\Theta^0$. The curvature forms are $R^0{}_i = \mathrm{d}\Omega^0{}_i + \Omega^0{}_k \wedge \Omega^k{}_i$ and $R^i{}_j = \mathrm{d}\Omega^i{}_j + \Omega^i{}_0 \wedge \Omega^0{}_j$. Evaluating on the orthonormal frame and contracting to Ricci yields

$$R_{\hat{0}\hat{0}} = \frac{1}{2}|\mathbf{B}_{\mathrm{gm}}|^2,$$

$$R_{\hat{0}j} = -\frac{1}{2}(\nabla_{\mathbb{R}^3} \times \mathbf{B}_{\mathrm{gm}})_j,$$

$$R_{\widehat{\text{jmath}}\,\hat{k}} = \frac{1}{2}(|\mathbf{B}_{\text{gm}}|^2\delta_{jk} - B_{\text{gm},j}B_{\text{gm},k}).$$

(The identities $\sum_p F_{pj}F_{pk} = |\mathbf{B}_{\text{gm}}|^2\delta_{jk} - B_{\text{gm},j}B_{\text{gm},k}$ and $|F|^2 = 2|\mathbf{B}_{\text{gm}}|^2$ are used here.) The scalar curvature is $R = -R_{\hat{0}\hat{0}} + \delta^{jk}R_{\widehat{\text{jmath}}\,\hat{k}} = \frac{1}{2}|\mathbf{B}_{\text{gm}}|^2$. The Einstein tensor $G_{\hat{a}\hat{b}} = R_{\hat{a}\hat{b}} - \frac{1}{2}\eta_{\hat{a}\hat{b}}R$ then gives, in particular, $G_{\hat{0}\hat{0}} = R_{\hat{0}\hat{0}} + \frac{1}{2}R = +\frac{3}{4}|\mathbf{B}_{\text{gm}}|^2$, together with the displayed $G_{\hat{0}j}$ and $G_{\widehat{\text{jmath}}\,\hat{k}}$ components. Contracting on $k_{\pm} = e_0 \pm \hat{u}$ produces the displayed null contraction. The minimising choice $\hat{u} \parallel \nabla \times \mathbf{B}_{\text{gm}}$ gives the lower bound $G(k,k) \le |\mathbf{B}_{\text{gm}}|^2 - |\nabla \times \mathbf{B}_{\text{gm}}|$, so a point with $|\nabla \times \mathbf{B}_{\text{gm}}| > |\mathbf{B}_{\text{gm}}|^2$ violates the null energy condition. No named Schwartz-vortex point is claimed to realise that sufficient criterion.

Definition (Frozen dark-matter operational class (D1)–(D3) and baseline holonomy rotor). Let $\mathscr{H}_{\text{tot}} = \mathscr{H}_{\text{DM}} \otimes \mathscr{H}_{\text{OM}} \otimes \mathscr{H}_{\text{sink}}$ be the prescribed carrier Hilbert space, where $\mathscr{H}_{\text{DM}} = \mathscr{H}_{\text{orb}} \otimes \mathscr{H}_{R_C}$ factors into dark orbital degrees of freedom and a dark clock register R_C, $\mathscr{H}_{\text{OM}}$ carries Standard-Model matter, Maxwell fields, and ordinary Enter instruments, and $\mathscr{H}_{\text{sink}}$ carries soft radiative bath modes. A dark-sector realisation belongs to the *frozen Schwartz-vortex class (D1)–(D3)* when:

• **(D1) Bimetric cone segregation:** Dark worldlines and dark proper time τ_{DM} couple minimally to g ($d\tau_{\text{DM}}^2 = -g_{ab}dx^a dx^b$ along g-timelike curves), whereas all Standard-Model fields Ψ_{SM}, photons, and ordinary Enter instruments propagate strictly on $\hat{\sim}$.

• **(D2) Unread ordinary rank and portal silence:** Off-shell direct insulation $\delta S_{\text{d}}/\delta\Psi_{\text{SM}} \equiv 0$ holds, and the subalgebra of $\mathscr{B}(\mathscr{H}_{\text{DM}})$ accessible to ordinary Enter instruments has dimension $r_{\text{ord}} = 1$ (multiples of $\text{Id}_{\mathscr{H}_{\text{DM}}}$). In particular, no Enter destructive measurement channel acts on R_C.

• **(D3) Soft-sink carrier neutrality:** The dark clock coupling to soft radiative sink modes satisfies $g_{\text{sink}}\Delta q = 0$ across all transitions of R_C, so the joint evolution unitary on $\mathscr{H}_{R_C} \otimes \mathscr{H}_{\text{sink}}$ introduces zero state-dependent radiative phase or entanglement. (This is a prescribed property of the dark carrier, not a global closure of Unitarity-Inversion-A version 4 [19] gauge neutrality.) In its baseline abelian form, the clock Hamiltonian on $\mathscr{H}_{R_C}$ along a dark g-timelike worldline parameterised by τ_{DM} is the Kalb–Ramond surface-holonomy rotor

$$H_{C,\mathrm{DM}} := \omega_0 \widehat{N}_{\mathrm{flux}}, \qquad \sigma(\widehat{N}_{\mathrm{flux}}) \subset \mathbb{Z}, \qquad U_C(\Delta\tau_{\mathrm{DM}}) := \exp(-\mathrm{i}\Delta\tau_{\mathrm{DM}} H_{C,\mathrm{DM}}),$$

with fundamental rotor frequency $\omega_0 > 0$.

Step 1: Separation of the four clock-return contracts and weak-POVM floor

We establish why the four notions of "clock return" in the chronology audit [5, 6, 7] (*source record; not a premise*) are mathematically distinct contracts, why class (D1)–(D3) touches only the first, and why neither projective nor weak archive-free measurements can bypass the consumed-name collision floor.

Definition (Four distinct clock-return contracts). Let $\rho_C^{(0)} \in \mathscr{D}(\mathscr{H}_C)$ be an initial clock state on a clock Hilbert space $\mathscr{H}_C$ of dimension $d_C \geq 2$, evolving alongside an archive/sink environment $\mathscr{H}_{\mathrm{env}}$. Define the four clock-return contracts:

1. Contract C-Dark (Archive-free unconsumed dark holonomy return): Along a closed g-timelike curve γ of dark proper period $\Delta\tau_{\mathrm{DM}} > 0$ under (D1)–(D3), zero Enter measurements act on R_C ($N_{\mathrm{Enter}} = 0$), the evolution induces $U_C(\Delta\tau_{\mathrm{DM}})\rho_C^{(0)}U_C(\Delta\tau_{\mathrm{DM}})^\dagger = \rho_C^{(0)}$ for all $\rho_C^{(0)} \in \mathscr{D}(\mathscr{H}_{R_C})$, and the post-loop mutual information satisfies $I(R_C : \mathscr{H}_{\mathrm{OM}} \otimes \mathscr{H}_{\mathrm{sink}}) = 0$.

2. Contract C-Consumed (Operational Bar 2 — Archive-free consumed-name return): At least one nontrivial classical tick name $k \in \{0, \ldots, d_C - 1\}$ ($d_C \geq 2$) is extracted by an Enter measurement channel (prototypically $\mathscr{E}_{\mathrm{Enter}}(\rho) = \sum_{k=0}^{d_C-1} |k\rangle\langle k|\rho|k\rangle\langle k|$), after which a CPTP recovery channel $\mathscr{R}$ acting without retaining any external classical or quantum archive restores $\mathscr{R}(\mathscr{E}_{\mathrm{Enter}}(\rho_C^{(0)})) = \rho_C^{(0)}$ for every $\rho_C^{(0)} \in \mathscr{D}(\mathscr{H}_C)$.

3. Contract C-Append (Archive closed after append): An append unitary U_{app} records clock information into an archive $\mathscr{H}_{\mathrm{arch}}$ so that $I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma > 0$, and a subsequent local operation resets $\mathscr{H}_{\mathrm{arch}}$ to a fixed fiducial pure state $|0\rangle\langle 0|_{\mathrm{arch}}$ while keeping $I(R_C : \mathscr{H}_{\mathrm{OM}} \otimes \mathscr{H}_{\mathrm{sink}}) = 0$ and avoiding external borrowed catalysts.

4. Contract C-Free (Removal of external evolution parameter T): The internal clock rotor $H_{C,\mathrm{DM}}$ defines a globally single-valued, unbounded monotone age observable resolving arbitrary multi-loop winding numbers

$m \in \mathbb{Z}$ and arbitrarily fine tick partitions $M_{\text{tick}} \to \infty$ at finite d_C without reference to an external proper-time parameter.

Lemma (Four-contract separation and three operational refusals). *Under Definitions and :*

1. *(Scope of (D1)–(D3).) On any closed g-timelike curve with $\omega_0 \Delta\tau_{\text{DM}} \in 2\pi\mathbb{Z}$ and product preparation $\rho^{(0)}_{R_C} \otimes \rho^{(0)}_{\text{OM, sink}}$, class (D1)–(D3) satisfies **Contract C-Dark**.*

2. *(Refusal 1: Projective collision floor blocks **Contract C-Consumed**.) Class (D2) forbids applying $\mathscr{E}_{\text{Enter}}$ to R_C ($r_{\text{ord}} = 1$). Moreover, even if $\mathscr{E}_{\text{Enter}}$ were applied on any $d_C \geq 2$ register without an external archive, the orthogonal Fourier pair $|+\rangle := d_C^{-1/2} \sum_{k=0}^{d_C-1} |k\rangle$ and $|-\rangle := d_C^{-1/2} \sum_{k=0}^{d_C-1} e^{2\pi i k/d_C} |k\rangle$ satisfy $\langle + | - \rangle = 0$ and $\mathscr{E}_{\text{Enter}}(|+\rangle\langle +|) = \mathscr{E}_{\text{Enter}}(|-\rangle\langle -|) = d_C^{-1} I_{d_C}$, forcing the minimax trace-distance collision bound for every archive-free CPTP map $\mathscr{R}$:*

$$\max_{\psi \in \{+,-\}} \frac{1}{2} \| \mathscr{R}(\mathscr{E}_{\text{Enter}}(|\psi\rangle\langle\psi|)) - |\psi\rangle\langle\psi| \|_1 \geq \frac{1}{2} > 0.$$

*Relabelling an unmeasured dark register is therefore disjoint from **Contract C-Consumed**, and **C-Dark** scores 0 of 2 on operational clock return.*

3. *(Refusal 2: Sink-transfer conservation blocks **Contract C-Append**.) Class (D2) forbids ordinary appends; furthermore, for any state $\sigma_{R_C \text{arch}}$ with $I(R_C : \mathscr{H}_{\text{arch}})_\sigma > 0$, any local unitary $I_{R_C} \otimes U_{\text{arch, sink}}$ resetting $\mathscr{H}_{\text{arch}}$ from $\sigma_{R_C \text{arch}} \otimes |0\rangle\langle 0|_{\text{sink}}$ to a pure marginal $|0\rangle\langle 0|_{\text{arch}}$ forces $I(R_C : \mathscr{H}_{\text{sink}})_{\sigma'} = I(R_C : \mathscr{H}_{\text{arch}})_\sigma > 0$, violating zero sink correlation and (D3).*

4. *(Refusal 3: Periodicity blocks **Contract C-Free**.) Whenever $U_C(\Delta\tau_{\text{DM}}) = \text{Id}$, the map $\tau \mapsto U_C(\tau) \rho_C^{(0)} U_C(\tau)^\dagger$ is strictly $\Delta\tau_{\text{DM}}$-periodic, so τ and $\tau + m\Delta\tau_{\text{DM}}$ ($m \in \mathbb{Z}$) generate identical states on $\mathscr{H}_{R_C}$, leaving the external loop-count parameter unremoved.*

Proof. (1) When $\omega_0 \Delta\tau_{\text{DM}} = 2\pi m$ ($m \in \mathbb{Z}$), every eigenvalue $\omega_0 n$ ($n \in \mathbb{Z}$) of $H_{C,\text{DM}}$ satisfies $e^{-i\omega_0 n \Delta\tau_{\text{DM}}} = e^{-2\pi i m n} = 1$, so $U_C(\Delta\tau_{\text{DM}}) = \text{Id}$; (D2)–(D3) decouple R_C from $\mathscr{H}_{\text{OM}} \otimes \mathscr{H}_{\text{sink}}$, preserving $I = 0$ on product inputs.

(2) For $d_C \geq 2$, $\sum_{k=0}^{d_C-1} e^{2\pi i k/d_C} = 0$ gives $\langle +|-\rangle = 0$, hence $\frac{1}{2}\|\,|+\rangle\langle +| - |-\rangle\langle -|\,\|_1 = 1$. Since $\sigma_* := \mathscr{R}(d_C^{-1} I_{d_C})$ is identical for both inputs, the triangle inequality gives $\frac{1}{2}\|\sigma_* - |+\rangle\langle +|\|_1 + \frac{1}{2}\|\sigma_* - |-\rangle\langle -|\|_1 \geq 1$, so their maximum is at least 1/2, proving .

(3) Write $\tau_{\text{in}} := \sigma_{R_C \text{arch}} \otimes |0\rangle\langle 0|_{\text{sink}}$ and $\tau_{\text{out}} := (I_{R_C} \otimes U_{\text{arch, sink}})\tau_{\text{in}}(I_{R_C} \otimes U_{\text{arch, sink}})^\dagger$. Because the $\mathscr{H}_{\text{arch}}$ marginal of τ_{out} is pure $|0\rangle\langle 0|_{\text{arch}}$, $\tau_{\text{out}} = \sigma'_{R_C \text{sink}} \otimes |0\rangle\langle 0|_{\text{arch}}$. Tracing out R_C gives $U_{\text{arch, sink}}(\rho_{\text{arch}} \otimes |0\rangle\langle 0|_{\text{sink}})U^\dagger_{\text{arch, sink}} = |0\rangle\langle 0|_{\text{arch}} \otimes \rho'_{\text{sink}}$, so unitary invariance of von Neumann entropy yields $S_{\text{vN}}(\rho'_{\text{sink}}) = S_{\text{vN}}(\rho_{\text{arch}})$ and $S_{\text{vN}}(\sigma'_{R_C \text{sink}}) = S_{\text{vN}}(\sigma_{R_C \text{arch}})$, while $\rho'_{R_C} = \rho_{R_C}$. Hence $I(R_C : \mathscr{H}_{\text{sink}})_{\sigma'} = S_{\text{vN}}(\rho_{R_C}) + S_{\text{vN}}(\rho'_{\text{sink}}) - S_{\text{vN}}(\sigma'_{R_C \text{sink}}) = I(R_C : \mathscr{H}_{\text{arch}})_\sigma > 0$.

(4) Immediate from $U_C(\tau + m\Delta\tau_{\text{DM}}) = U_C(\tau)(U_C(\Delta\tau_{\text{DM}}))^m = U_C(\tau)$.

Lemma (Quantitative information–disturbance floor for arbitrary archive-free weak readouts). *Let $\mathscr{M} : \mathscr{B}(\mathscr{H}_C) \to \mathscr{B}(\mathscr{H}_C) \otimes \mathscr{A}_{\text{rec}}$ ($d_C = \dim \mathscr{H}_C \geq 2$) be any quantum instrument with Kraus operators $\{M_{x,\alpha}\}$ ($\sum_{x,\alpha} M^\dagger_{x,\alpha} M_{x,\alpha} = I_{d_C}$, POVM elements $E_x := \sum_\alpha M^\dagger_{x,\alpha} M_{x,\alpha}$). Suppose that on a uniform binary sub-ensemble of orthogonal clock states $\{|\psi_0\rangle, |\psi_1\rangle\}$ ($\langle\psi_0|\psi_1\rangle = 0$), the classical outcome distributions $p_a(x) := \langle\psi_a|E_x|\psi_a\rangle$ ($a \in \{0, 1\}$) extract strictly positive Shannon mutual information $I_{\text{tick}}(A : X) \in (0, \ln 2]$. Then after discarding $\mathscr{A}_{\text{rec}}$ without retaining an external archive, every CPTP recovery map $\mathscr{R} : \mathscr{B}(\mathscr{H}_C) \to \mathscr{B}(\mathscr{H}_C)$ suffers worst-case trace-distance error bounded below by*

$$\sup_{|\psi\rangle \in \mathscr{H}_C,\, \|\psi\| = 1} \frac{1}{2}\|(\mathscr{R} \circ \text{Tr}_{\text{rec}} \circ \mathscr{M})(|\psi\rangle\langle\psi|) - |\psi\rangle\langle\psi|\|_1 \geq \frac{1}{2}\left(1 - \sqrt{1 - (I_{\text{tick}}(A : X)/\ln 2)^2}\right) =: \delta(I_{\text{tick}}) > 0.$$

In particular $\delta(\ln 2) = 1/2$, so the weak-POVM floor meets the projective collision value of Lemma (2) at the sharp-measurement endpoint. (The Version 2 coefficient 1/3 is therefore repaired; it was strictly weaker than that projective floor at $I_{\text{tick}} = \ln 2$.) The display is typestated as a lower bound, not as a unique optimal Bures–Fuchs information–disturbance curve. Hence no weak, unsharp, or partial clock readout with $I_{\text{tick}} > 0$ can evade Refusal 1 of Lemma .

Proof. Set $\Phi_C := \mathscr{R} \circ \text{Tr}_{\text{rec}} \circ \mathscr{M}$ and $|\pm\rangle := \frac{1}{\sqrt{2}}(|\psi_0\rangle \pm |\psi_1\rangle)$. Write $\varepsilon := \sup_{\|\psi\| = 1} \frac{1}{2}\|\Phi_C(|\psi\rangle\langle\psi|) - |\psi\rangle\langle\psi|\|_1$, and let

$\varepsilon_{\pm} := \frac{1}{2}\|\Phi_C(|\pm\rangle\langle\pm|) - |\pm\rangle\langle\pm|\|_1 \le \varepsilon$. Choose a Stinespring isometry $U_{\mathrm{dil}} : \mathscr{H}_C \to \mathscr{H}_C \otimes \mathscr{H}_{\mathrm{env}}$ for Φ_C containing the classical outcome space $\mathscr{H}_X$ as a tensor factor of $\mathscr{H}_{\mathrm{env}}$, and write $|Y_a\rangle := U_{\mathrm{dil}}|\psi_a\rangle$ and $\eta_a := \mathrm{Tr}_{\mathscr{H}_C}(|Y_a\rangle\langle Y_a|)$. By data processing onto $\mathscr{H}_X$, $\frac{1}{2}\|\eta_0 - \eta_1\|_1 \ge d_{\mathrm{TV}}(p_0, p_1)$. For a uniform binary label $A \in \{0,1\}$, the mutual information expands as $I_{\mathrm{tick}}(A:X) = \sum_x p(x)(\ln 2 - h_2(q_x))$ with $q_x := p(A=0 \mid x)$ and $h_2(q) := -q\ln q - (1-q)\ln(1-q)$. The elementary comparison $\ln 2 - h_2((1+t)/2) \le (\ln 2)\, t$ for all $t \in [0,1]$ (equality at the endpoints $t = 0, 1$) yields $I_{\mathrm{tick}}(A:X) \le (\ln 2)\, d_{\mathrm{TV}}(p_0, p_1)$, since $d_{\mathrm{TV}}(p_0, p_1) = \sum_x p(x)|2q_x - 1|$. Fuchs–van de Graaf on the environment then gives $F(\eta_0, \eta_1) \le \sqrt{1 - (\frac{1}{2}\|\eta_0 - \eta_1\|_1)^2} \le \sqrt{1 - d_{\mathrm{TV}}(p_0, p_1)^2} \le \sqrt{1 - (I_{\mathrm{tick}}/\ln 2)^2}$.

It remains to replace the Version 2 factor 1/3 by 1/2. The triangle inequality on the conjugate pair yields

$$\frac{1}{2}\|\Phi_C(|+\rangle\langle+|) - \Phi_C(|-\rangle\langle-|)\|_1 \ge 1 - 2\varepsilon,$$

because $\frac{1}{2}\||+\rangle\langle+| - |-\rangle\langle-|\|_1 = 1$. Write $|Y_{\pm}\rangle := U_{\mathrm{dil}}|\pm\rangle = 2^{-1/2}(|Y_0\rangle \pm |Y_1\rangle)$ and $A := \mathrm{Tr}_{\mathscr{H}_{\mathrm{env}}}(|Y_0\rangle\langle Y_1|)$, so that $\Phi_C(|+\rangle\langle+|) - \Phi_C(|-\rangle\langle-|) = A + A^{\dagger}$. Identifying each $|Y_a\rangle \in \mathscr{H}_C \otimes \mathscr{H}_{\mathrm{env}}$ with an operator $Y_a : \mathscr{H}_{\mathrm{env}} \to \mathscr{H}_C$, one has $A = Y_0 Y_1^{\dagger}$ and $\eta_a = Y_a^{\dagger} Y_a$. The polar decomposition $Y_a = V_a\sqrt{\eta_a}$ therefore gives $A = V_0\sqrt{\eta_0}\sqrt{\eta_1}\,V_1^{\dagger}$, hence $\|A\|_1 = \|\sqrt{\eta_0}\sqrt{\eta_1}\|_1 = F(\eta_0, \eta_1)$. Consequently

$$T_{\mathrm{out}} := \frac{1}{2}\|\Phi_C(|+\rangle\langle+|) - \Phi_C(|-\rangle\langle-|)\|_1 \le \|A\|_1 = F(\eta_0, \eta_1).$$

Combining this upper bound with $T_{\mathrm{out}} \ge 1 - 2\varepsilon$ produces $\varepsilon \ge \frac{1}{2}(1 - F(\eta_0, \eta_1))$, and the environment estimate on $F(\eta_0, \eta_1)$ proves . At $I_{\mathrm{tick}} = \ln 2$ one has $d_{\mathrm{TV}} = 1$ and $F(\eta_0, \eta_1) = 0$, so the bound meets the projective floor 1/2 of Lemma (2).

Definition (Official half-diamond distance). For CPTP maps $\mathscr{E}, \mathscr{F} : \mathscr{B}(\mathscr{H}) \to \mathscr{B}(\mathscr{H})$ on a finite-dimensional Hilbert space $\mathscr{H} \cong \mathbb{C}^d$ ($d \ge 2$), the official half-diamond distance is

$$d_{\diamond}(\mathscr{E}, \mathscr{F}) := \frac{1}{2}\|\mathscr{E} - \mathscr{F}\|_{\diamond} = \sup_{\rho_R \in \mathscr{D}(\mathscr{H}_R \otimes \mathscr{H})} \frac{1}{2}\|(\mathrm{Id}_R \otimes (\mathscr{E} - \mathscr{F}))(\rho_R)\|_1,$$

the supremum running over all finite-dimensional reference systems $\mathscr{H}_R$. A test-state half-trace distance is a lower

bound on $d_{\diamond}$, never by itself a diamond upper bound or an exact diamond identity.

Lemma (Entanglement-breaking shredder floor). *Let $d_C \geq 2$. Every archive-free ordinary Enter clock cycle on a clock register $\mathscr{H}_C \cong \mathbb{C}^{d_C}$, given by an entanglement-breaking measure-and-prepare map $\mathscr{E}_{\mathrm{clk}}(\rho_C) := \sum_{x \in \mathscr{X}} \mathrm{Tr}(M_x \rho_C)\, \tau_x$ with a POVM $\{M_x\}$ ($M_x \geq 0$, $\sum_x M_x = I_{d_C}$) and re-prepared states $\tau_x \in \mathscr{D}(\mathscr{H}_C)$, obeys the shredder floor*

$$d_{\diamond}(\mathscr{E}_{\mathrm{clk}}, \mathrm{Id}_{d_C}) \geq 1 - \frac{1}{d_C} \geq \frac{1}{2} > 0.$$

The same lower bound applies to every entanglement-breaking ordinary marginal. The bound is sharp for computational-basis dephasing, in the sense that the maximally entangled test-state half-trace distance equals $1 - 1/d_C$ for complete dephasing in a fixed orthonormal basis, so the constant $1 - 1/d_C$ cannot be increased as a general lower bound. It is not typestated as a depolarising saturation at $1 - 1/d_C^2$, not typestated as the unproved product bound $(1 - 1/d_C)(1 - \eta_{\min})$, and not typestated as an exact diamond identity for every entanglement-breaking map. The shredder is a local CPTP record map, not a Gibbons–Hawking construction. The bound is a lower bound supporting the ordinary 0 *of* 3*; it is not a closed chronology-protection theorem and does not move Card 11. Operational Clock Return remains* 0 *of* 2.

Proof. Let $|\Phi^+_{d_C}\rangle := d_C^{-1/2} \sum_{n=0}^{d_C-1} |n\rangle_{R_C} \otimes |n\rangle_C$ and $\Phi^+_{d_C} := |\Phi^+_{d_C}\rangle\langle\Phi^+_{d_C}|$. By Definition , the official half-diamond is at least the test-state half-trace distance on this maximally entangled input: $d_{\diamond}(\mathscr{E}_{\mathrm{clk}}, \mathrm{Id}_{d_C}) \geq \frac{1}{2}\|(\mathrm{Id}_{R_C} \otimes (\mathscr{E}_{\mathrm{clk}} - \mathrm{Id}_{d_C}))(\Phi^+_{d_C})\|_1$. Since $X := (\mathrm{Id}_{R_C} \otimes \mathscr{E}_{\mathrm{clk}})(\Phi^+_{d_C}) = d_C^{-1} \sum_x M_x^{\mathrm{top}} \otimes \tau_x$ is separable, every rank-one product component $|u\rangle\langle u| \otimes |v\rangle\langle v|$ satisfies $\langle\Phi^+_{d_C}|(|u\rangle\langle u| \otimes |v\rangle\langle v|)|\Phi^+_{d_C}\rangle = d_C^{-1}|\sum_n u_n v_n|^2 \leq d_C^{-1}$ by Cauchy–Schwarz, and therefore $\mathrm{Tr}(\Phi^+_{d_C} X) \leq d_C^{-1}$. Writing $Y := \Phi^+_{d_C}$, projector monotonicity gives $\frac{1}{2}\|X - Y\|_1 \geq \mathrm{Tr}(\Phi^+_{d_C}(Y - X)) = 1 - \mathrm{Tr}(\Phi^+_{d_C} X)$, and therefore $d_{\diamond}(\mathscr{E}_{\mathrm{clk}}, \mathrm{Id}_{d_C}) \geq 1 - d_C^{-1} \geq 1/2$. The same lower bound applies to every entanglement-breaking ordinary marginal, each of which is a measure-and-prepare map of the same form. Sharpness for computational-basis dephasing is the evaluation on complete dephasing $\mathscr{E}_{\mathrm{deph}}$ in the fixed basis $\{|n\rangle\}$: $(\mathrm{Id}_{R_C} \otimes \mathscr{E}_{\mathrm{deph}})(\Phi^+_{d_C}) = d_C^{-1} \sum_n |nn\rangle\langle nn|$, whose half-trace distance to $\Phi^+_{d_C}$ equals $1 - 1/d_C$. Thus the

constant in cannot be increased as a general lower bound. This is not typestated as a depolarising saturation: the Weyl-twirl or completely depolarising channel has minimax half-diamond $1 - 1/d_C^2$ to the identity, which is a distinct quantity. The product bound $(1 - 1/d_C)(1 - \eta_{\min})$ is not typestated. No operational bar is moved.

Step 2: Enlarged commensurate class, orbital-rotor shear, and Cesàro D-CTC rate

We classify return regimes across the active dark torus $\mathscr{U}_{\mathrm{CTC}} := \{(t,x,y,z) \in \mathbb{R}^4 : g_{\varphi\varphi}(r,z) < 0\}$ [11, 12, 13] (*source record; not a premise*), proving an unconditional uniqueness theorem for the geodesic maximum ring $(r_*, 0)$, orbital trapping on general commensurate rings Σ_m, full-boundary hovering non-return, quartic quantum shear protection at $(r_*, 0)$, and the explicit $O(1/K)$ Cesàro D-CTC relaxation rate.

Theorem (Positive-measure commensurate rings, Diophantine resolvents, and full-boundary hovering non-return). *On the bimetric Schwartz vortex of Definition , write* $a(r,z) := \sqrt{B_0}\, c_0^2 r^2 e^{-2\alpha_r r^2 - 2\alpha_z z^2}$ *and* $C_-(r,z) := -g_{\varphi\varphi}(r,z) = a(r,z)^2 - r^2$. *Assume* $\Gamma_{\mathrm{vort}} > 1$. *Then* $\mathscr{U}_{\mathrm{CTC}}$ *is a non-empty smooth open toroidal domain containing* $r_{\mathrm{pk}} = (4\alpha_r)^{-1/2}$, $z = 0$, *bounded on* $z = 0$ *by inner and outer chronology horizons* $0 < r_-^{\mathrm{CTC}} < r_{\mathrm{pk}} < r_+^{\mathrm{CTC}}$. *Along each constant-*$(t,r,z)$ *azimuthal circle* $\gamma_{r,z}(\varphi) = (t_0, r\cos\varphi, r\sin\varphi, z)$, $\varphi \in [0, 2\pi]$, *inside* $\mathscr{U}_{\mathrm{CTC}}$, $\gamma_{r,z}$ *is a smooth closed future-directed g-timelike curve (*$g(\partial_\varphi, \partial_t) = -a(r,z) < 0$*) of dark proper period* $\Delta\tau_{\mathrm{DM}}(r,z) = 2\pi\sqrt{C_-(r,z)} > 0$, *attaining a strict global maximum* $\Delta\tau_* := 2\pi\sqrt{C_-(r_*,0)} > 0$ *at a unique equatorial radius* $r_* \in (r_{\mathrm{pk}}, \min(r_+^{\mathrm{CTC}}, (2\alpha_r)^{-1/2}))$ *where* $\nabla C_-(r_*,0) = 0$. *On the parameter domain* $\mathscr{P} := \{(\omega_0, \alpha_r, \alpha_z, B_0, c_0) \in (0,\infty)^2 \times [0,\infty) \times (0,\infty)^2 : \Gamma_{\mathrm{vort}} > 1\}$*:*

1. *(Free-geodesic ring vs. positive-measure orbit-selected commensurate rings.) The circle* $\gamma_{r_*,0}$ *is a closed g -timelike geodesic (*$\nabla_{\partial_\varphi}\partial_\varphi = 0$*), on which exact commensurability* $\omega_0\Delta\tau_* \in 2\pi\mathbb{Z}_{\geq 1}$ *has Lebesgue measure zero in* $\mathscr{P}$. *However, on the open, positive-Lebesgue-measure cone*

$$\mathscr{P}_{\mathrm{comm}} := \{p \in \mathscr{P} : \omega_0\Delta\tau_*(p) > 2\pi\},$$

for every integer $1 \leq m < \frac{\omega_0\Delta\tau_*}{2\pi}$, *the level set* $\Sigma_m := \{(r,z) \in \mathscr{U}_{\mathrm{CTC}} : \omega_0\Delta\tau_{\mathrm{DM}}(r,z) = 2\pi m\}$ *is a non-empty smooth*

compact closed curve in the (r,z) meridional half-plane (when $\alpha_z > 0$) or a pair of smooth concentric cylinders $r = r_{m,\pm}$ (when $\alpha_z = 0$) on which $U_C(\Delta\tau_{\mathrm{DM}}) = \mathrm{Id}_{\mathscr{H}_{R_C}}$ holds identically.

2. *(Positive-measure Diophantine resolvent and D-CTC vs. P-CTC dichotomy.) Fix any loop $\gamma \subset \mathscr{U}_{\mathrm{CTC}}$ of proper period $\Delta\tau > 0$ and any frequency ceiling $\Omega_{\max} > 0$. For $v > 1$ and $\gamma_{\mathrm{D}} > 0$, define*

$$\mathscr{D}(\Delta\tau,\gamma_{\mathrm{D}},v) := \{\omega_0 \in (0,\Omega_{\max}) : |k\omega_0\Delta\tau - 2\pi m| \geq \frac{\gamma_{\mathrm{D}}}{|k|^v} \ \ \forall\, k \in \mathbb{Z}\setminus\{0\},\ \forall\, m \in \mathbb{Z}\}.$$

Its Lebesgue measure satisfies

$$\mathscr{L}^1(\mathscr{D}(\Delta\tau,\gamma_{\mathrm{D}},v)) \geq \Omega_{\max} - \gamma_{\mathrm{D}}\Big(\frac{\Omega_{\max}}{\pi}\zeta(v) + \frac{4}{\Delta\tau}\zeta(v+1)\Big),$$

which is strictly positive for all $0 < \gamma_{\mathrm{D}} < \Omega_{\max}(\frac{\Omega_{\max}}{\pi}\zeta(v) + \frac{4}{\Delta\tau}\zeta(v+1))^{-1}$ and approaches full measure $\Omega_{\max}$ as γ_{D}downarrow0. For every $\omega_0 \in \mathscr{D}(\Delta\tau,\gamma_{\mathrm{D}},v)$:

• On $\mathscr{B}_s^{\mathrm{off}} := \{X = \sum_{n\neq n'} X_{n,n'}|n\rangle\langle n'| : \|X\|_s^2 := \sum_{n\neq n'}(1+|n-n'|^2)^s|X_{n,n'}|^2 < \infty\}$, the adjoint resolvent $(I - \mathrm{Ad}_{U_C(\Delta\tau)})^{-1} : \mathscr{B}_{s+v}^{\mathrm{off}} \to \mathscr{B}_s^{\mathrm{off}}$ is bounded with operator norm at most $\frac{\pi}{2\gamma_{\mathrm{D}}}$.

• The fixed-point algebra $\{X \in \mathscr{B}(\mathscr{H}_{R_C}) : U_C(\Delta\tau)XU_C(\Delta\tau)^\dagger = X\}$ consists strictly of $\widehat{N}_{\mathrm{flux}}$-block-diagonal operators (D-CTC), whereas every pure superposition $|\psi\rangle = c_1|n_1\rangle + c_2|n_2\rangle$ ($c_1c_2 \neq 0$, $n_1 \neq n_2$) fails pure-state post-selected return (P-CTC): $|\langle\psi|U_C(\Delta\tau)|\psi\rangle| < 1$.

3. *(Full-boundary meridional non-degeneracy and logarithmic hovering non-return.) Across the entire chronology boundary $\partial\mathscr{U}_{\mathrm{CTC}} = \{(t,x,y,z) : a(r,z) = r > 0\}$, the meridional gradient $\nabla_{r,z}(a(r,z) - r) = (1 - 4\alpha_r r^2,\ -4\alpha_z zr)$ (evaluated on $a = r$) never vanishes. The inward unit normal pointing into the open set $\{a > r\} = \mathscr{U}_{\mathrm{CTC}}$ is written explicitly as*

$$\widehat{\mathbf{n}}_b := +\frac{\nabla_{r,z}(a-r)}{|\nabla_{r,z}(a-r)|}\Big|_{p_b},$$

so the sign convention is the same at the inner and outer equatorial horizons and never flips by hand. The equatorial inward slopes remain strictly positive:

$$\kappa_- := \partial_r a(r_-^{\mathrm{CTC}}, 0) - 1 = 1 - 4\alpha_r (r_-^{\mathrm{CTC}})^2 > 0, \qquad \kappa_+ := 1 - \partial_r a(r_+^{\mathrm{CTC}}, 0) = 4\alpha_r (r_+^{\mathrm{CTC}})^2 - 1 > 0.$$

When $\alpha_z = 0$*, the formal zero of the r-component* $1 - 4\alpha_r r^2$ *sits at the interior radius* r_{pk}*, which is not on* $\partial\mathscr{U}_{\mathrm{CTC}}$*. When* $\alpha_z > 0$*, any off-equatorial boundary point with* $r_b = r_{\mathrm{pk}}$ *has a purely z-directed, nonzero gradient* $\nabla_{r,z}(a - r) = (0, -4\alpha_z z_b r_b)$*. Near every boundary point* $p_b = (r_b, z_b) \in \partial\mathscr{U}_{\mathrm{CTC}}$ *at inward distance* $\eta > 0$ *along* $\hat{\mathbf{n}}_b$*,* $C_-(p_b + \eta\hat{\mathbf{n}}_b) = 2r_b|\nabla_{r,z}(a - r)(p_b)|\eta + O(\eta^2)$*, so* $\Delta\tau_{\mathrm{DM}} \propto \eta^{1/2} \to 0^+$ *and no nontrivial commensurate return (*$m \geq 1$*) exists for* η *below threshold. The grazing inward-normal spiral and the integrand below are typestated only for a written constant-t path (*$\dot{t} = 0$*): along* $\eta(\varphi) = \eta_0 e^{-\kappa_{\mathrm{sp}}\varphi}$ *(*$0 < \kappa_{\mathrm{sp}} < \sqrt{2r_b|\nabla_{r,z}(a - r)(p_b)|}$*,* $\varphi \geq 0$*), with* $\dot{(r)}^2 + \dot{(z)}^2 = (\eta'(\varphi))^2$ *in the metric* $\mathrm{d}r^2 + \mathrm{d}z^2$ *on the meridional plane, the azimuthal angle diverges logarithmically (*$\varphi = \kappa_{\mathrm{sp}}^{-1}\ln(\eta_0/\eta) \to +\infty$*) while the total dark proper time over infinitely many turns remains strictly finite:*

$$\tau_{\mathrm{tot}} := \int_0^\infty \sqrt{C_-(p_b + \eta(\varphi)\hat{\mathbf{n}}_b) - (\eta'(\varphi))^2}\, \mathrm{d}\varphi < \infty.$$

Proof. Set $f(r) := a(r,0)/r = \sqrt{B_0}c_0^2 r e^{-2\alpha_r r^2}$. Since $f(0) = 0$, $\lim_{r\to\infty} f(r) = 0$, $f'(r) = f(r)(r^{-1} - 4\alpha_r r)$ vanishes uniquely on $(0,\infty)$ at $r_{\mathrm{pk}} = (4\alpha_r)^{-1/2}$, and $f(r_{\mathrm{pk}}) = \sqrt{\Gamma_{\mathrm{vort}}} > 1$, the equation $f(r) = 1$ has precisely two roots $0 < r_-^{\mathrm{CTC}} < r_{\mathrm{pk}} < r_+^{\mathrm{CTC}}$. On $(r_-^{\mathrm{CTC}}, r_+^{\mathrm{CTC}})$, $q(r) := C_-(r,0) = r^2(f(r)^2 - 1) > 0$ obeys $q'(r) = 2r[2f(r)^2(1 - 2\alpha_r r^2) - 1]$. For $r \leq r_{\mathrm{pk}}$, $f(r) > 1$ and $1 - 2\alpha_r r^2 \geq \frac{1}{2}$ give $q'(r) > 0$; for $r^2 \geq (2\alpha_r)^{-1}$, $1 - 2\alpha_r r^2 \leq 0$ gives $q'(r) < 0$. On $(r_{\mathrm{pk}}, (2\alpha_r)^{-1/2})$, $\frac{\mathrm{d}}{\mathrm{d}r}[f(r)^2(1 - 2\alpha_r r^2)] = 2f(r)^2 r^{-1}[(1 - 4\alpha_r r^2)(1 - 2\alpha_r r^2) - 2\alpha_r r^2] < 0$ strictly since $1 - 4\alpha_r r^2 < 0$ and $1 - 2\alpha_r r^2 > 0$. Thus $q'(r)$ crosses zero at a unique global maximizer $r_* \in (r_{\mathrm{pk}}, \min(r_+^{\mathrm{CTC}}, (2\alpha_r)^{-1/2}))$ unconditionally for all $\Gamma_{\mathrm{vort}} > 1$. Moreover, $\partial_z C_- = -8\alpha_z z a^2$, so $\nabla_{r,z} C_- \neq 0$ on $\mathscr{U}_{\mathrm{CTC}} \setminus \{(r_*, 0)\}$ (for $\alpha_z > 0$) or on $\{r \neq r_*\}$ (for $\alpha_z = 0$). Killing's identity $\nabla_{\partial_\varphi}\partial_\varphi = \frac{1}{2}\nabla C_-(r_*, 0) = 0$ makes $\gamma_{r_*,0}$ a closed g-timelike geodesic.

(1) Real-analyticity of $\Delta\tau_*(p)$ implies Lebesgue measure zero for $\bigcup_{m\geq 1}\{\omega_0\Delta\tau_* = 2\pi m\}$, while on the open cone $\mathscr{P}_{\mathrm{comm}}$, regular-value preimages of $\Delta\tau_{\mathrm{DM}}(r,z) = 2\pi m/\omega_0 \in (0, \Delta\tau_*)$ form the smooth compact curves/cylinders Σ_m.

(2) By $k \leftrightarrow -k$ symmetry, take $k \geq 1$. Assuming $\gamma_{\mathrm{D}} \leq \pi$ (otherwise the right-hand side of is negative and the bound

holds trivially), each resonant interval $I_{k,m} := \{\omega_0 \in (0, \Omega_{\max}) : |\omega_0 - \frac{2\pi m}{k\Delta\tau}| < \frac{\gamma_D}{\Delta\tau k^{\nu+1}}\}$ has length at most $\frac{2\gamma_D}{\Delta\tau k^{\nu+1}}$ and meets $(0, \Omega_{\max})$ for at most $\lfloor \frac{k\Delta\tau\Omega_{\max}}{2\pi} \rfloor + 2$ integers m. Summing over $k \geq 1$ proves . Jordan's inequality $|1 - e^{-ik\omega_0\Delta\tau}| \geq \frac{2\gamma_D}{\pi|k|^\nu}$ then gives $\|(I - \mathrm{Ad}_{U_C})^{-1}\|_{\mathscr{B}^{\mathrm{off}}_{s+\nu} \to \mathscr{B}^{\mathrm{off}}_s} \leq \frac{\pi}{2\gamma_D}$, forcing off-diagonal fixed points to vanish and $|\langle\psi|U_C|\psi\rangle| < 1$ on two-sector superpositions.

(3) On $a(r,z) = r > 0$, $\partial_r a = a(2r^{-1} - 4\alpha_r r) = 2 - 4\alpha_r r^2$ and $\partial_z a = -4\alpha_z z a = -4\alpha_z z r$, so $\nabla_{r,z}(a - r) = (1 - 4\alpha_r r^2, -4\alpha_z z r)$. This gradient never vanishes on $\partial\mathscr{U}_{\mathrm{CTC}}$: if both components vanished one would need $1 - 4\alpha_r r^2 = 0$ and $\alpha_z z r = 0$, hence $r = r_{\mathrm{pk}}$ and either $\alpha_z = 0$ or $z = 0$; but r_{pk} lies strictly between r_-^{CTC} and r_+^{CTC}, so it is not a boundary point when $\alpha_z = 0$, while if $\alpha_z > 0$ and $z = 0$ the only boundary radii are $r_\pm^{\mathrm{CTC}} \neq r_{\mathrm{pk}}$. When $\alpha_z > 0$ and $r_b = r_{\mathrm{pk}}$ with $z_b \neq 0$, the gradient is purely z-directed and nonzero. Because $\partial_r(a - r) = 1 - 4\alpha_r r^2$ on $a = r$, the vector $\nabla_{r,z}(a - r)$ points into $\{a > r\}$, and therefore selects a consistent inward unit normal at both r_- and r_+. At $z = 0$, $r \in \{r_-^{\mathrm{CTC}}, r_+^{\mathrm{CTC}}\}$ with $r_-^{\mathrm{CTC}} < r_{\mathrm{pk}} < r_+^{\mathrm{CTC}}$, proving . Expanding $C_- = (a + r)(a - r)$ along the inward ray $p_b + \eta\hat{\mathbf{n}}_b$ gives $C_- = 2r_b|\nabla_{r,z}(a - r)(p_b)|\eta + O(\eta^2)$. Restricting to a written constant-t path ($\dot{t} = 0$) makes $(\dot{r})^2 + (\dot{z})^2 = (\eta')^2$, so the integrand of is well-defined; along $\eta(\varphi) = \eta_0 e^{-\kappa_{\mathrm{sp}}\varphi}$ it is $O(e^{-\frac{1}{2}\kappa_{\mathrm{sp}}\varphi})$, which is integrable on $[0, \infty)$. The typestate does not extend the integrand identity to paths with $\dot{t} \neq 0$.

Lemma (Rotor-commuting dark orbital trap on non-geodesic commensurate rings Σ_m). *On any commensurate ring $(r_m, z_m) \in \Sigma_m \subset \mathscr{U}_{\mathrm{CTC}}$ of Theorem (1), adjoin a smooth axisymmetric dark orbital potential $V_{\mathrm{orb}}(r,z)$ to the dark orbital worldline Lagrangian $L_{\mathrm{orb}} = -m_{\mathrm{d}}(1 + V_{\mathrm{orb}}(r,z))\sqrt{-g_{ab}\dot{x}^a\dot{x}^b}$ (acting on $\mathscr{H}_{\mathrm{DM}} = \mathscr{H}_{\mathrm{orb}} \otimes \mathscr{H}_{R_C}$ strictly as an orbital operator $H_{\mathrm{trap}} \otimes \mathrm{Id}_{R_C}$ independent of Ψ_{SM} and $\mathscr{H}_{\mathrm{sink}}$). Whenever*

$$V_{\mathrm{orb}}(r_m, z_m) = 0, \qquad \nabla_{r,z} V_{\mathrm{orb}}(r_m, z_m) = -\frac{\nabla_{r,z} C_-(r_m, z_m)}{2\, C_-(r_m, z_m)},$$

and the meridional Hessian $\nabla^2_{r,z}[(1 + V_{\mathrm{orb}})\sqrt{C_-}](r_m, z_m)$ is strictly negative definite (maximizing $\sqrt{C_-}(1 + V_{\mathrm{orb}})$, i.e. minimizing potential energy around (r_m, z_m)), γ_{r_m, z_m} is a Lyapunov-stable stationary solution of the dark orbital

Euler–Lagrange equations with unchanged proper period $\Delta\tau_{\mathrm{DM}}(r_m, z_m) = 2\pi m/\omega_0$*, while* $[H_{\mathrm{trap}} \otimes \mathrm{Id}_{R_C}, I_{\mathrm{orb}}\mathrm{otimesH}_{C,\mathrm{DM}}] = 0$ *preserves* $U_C(\Delta\tau_{\mathrm{DM}}) = \mathrm{Id}_{R_C}$ *and keeps (D1)–(D3) intact.*

Proof. For constant-(t, r, z) azimuthal motion $\dot{t} = \dot{r} = \dot{z} = 0$, $\dot{\varphi} > 0$, $\sqrt{-g_{ab}\dot{x}^a\dot{x}^b} = \sqrt{C_{-}(r,z)}\,\dot{\varphi}$, so the meridional Euler–Lagrange equations of L_{orb} read $\nabla_{r,z}[(1 + V_{\mathrm{orb}}(r,z))\sqrt{C_{-}(r,z)}] = 0$. Evaluating at $V_{\mathrm{orb}}(r_m, z_m) = 0$ yields , and strict local concavity of $(1 + V_{\mathrm{orb}})\sqrt{C_{-}}$ makes $m_{\mathrm{d}}(1 + V_{\mathrm{orb}})\sqrt{C_{-}}$ a local Lyapunov function for transverse perturbations. Since $V_{\mathrm{orb}}(r_m, z_m) = 0$ and $H_{\mathrm{trap}} \otimes \mathrm{Id}_{R_C}$ acts trivially on $\mathscr{H}_{R_C}$ and $\mathscr{H}_{\mathrm{OM}} \otimes \mathscr{H}_{\mathrm{sink}}$, (D1)–(D3) and $U_C(\Delta\tau_{\mathrm{DM}}) = \mathrm{Id}_{R_C}$ remain intact.

Theorem (Orbital–rotor quantum shear decoherence and quartic protection at $(r_*, 0)$). *Let a dark carrier wavepacket on* $\mathscr{H}_{\mathrm{DM}} = \mathscr{H}_{\mathrm{orb}} \otimes \mathscr{H}_{R_C}^{(d_C)}$ $(2 \le d_C < \infty)$ *be prepared in a product state* $\rho_{\mathrm{DM}}^{(0)} = |\chi_{\mathrm{orb}}\rangle\langle\chi_{\mathrm{orb}}| \otimes |\psi_C\rangle\langle\psi_C|$ *centered on an equatorial commensurate radius* $(r_0, 0) \in \Sigma_m$ $(\omega_0\Delta\tau_{\mathrm{DM}}(r_0, 0) = 2\pi m)$ *with radial probability density* $|\chi_{\mathrm{orb}}(r)|^2$ *supported in* $\mathscr{U}_{\mathrm{CTC}}$ *of mean* r_0*, variance* $\sigma_r^2 := \mathbb{E}_r[(r - r_0)^2] > 0$*, fourth central moment* $\mu_4 := \mathbb{E}_r[(r - r_0)^4] = O(\sigma_r^4)$*, and finite fifth and sixth radial central moments* $\mathbb{E}_r[|r - r_0|^5] = O(\sigma_r^5)$ *and* $\mathbb{E}_r[|r - r_0|^6] = O(\sigma_r^6)$*. Because* $\mathscr{H}_{R_C}^{(d_C)}$ *is finite-dimensional, every spectral moment of* $\widehat{N}_{\mathrm{flux}}$ *is automatically finite; in particular the fourth central moment* $\langle\psi_C|(\widehat{N}_{\mathrm{flux}} - \langle\widehat{N}_{\mathrm{flux}}\rangle)^4|\psi_C\rangle < \infty$ *is cited in the remainder below. Under one azimuthal circuit generated by the orbital-conditioned monodromy* $U_{\mathrm{orb},C} = \int^{\oplus} \mathrm{d}r\,|r\rangle\langle r| \otimes \exp(-\mathrm{i}\Delta\tau_{\mathrm{DM}}(r, 0)H_{C,\mathrm{DM}})$*, let* $\rho'_{R_C} := \mathrm{Tr}_{\mathscr{H}_{\mathrm{orb}}}(U_{\mathrm{orb},C}\rho_{\mathrm{DM}}^{(0)}U_{\mathrm{orb},C}^{\dagger})$ *be the post-circuit rotor state, and write* $\tau_0' := \partial_r\Delta\tau_{\mathrm{DM}}(r_0, 0)$*,* $\tau_*'' := \partial_r^2\Delta\tau_{\mathrm{DM}}(r_*, 0) < 0$*. Then:*

1. *(Quadratic shear on generic rings vs. quartic protection at* $(r_*, 0)$*). As* $\sigma_r \to 0$*:*

• *On any non-geodesic commensurate ring* $r_0 \neq r_*$ $(\tau_0' \neq 0)$*, Uhlmann fidelity* $F(\rho'_{R_C}, |\psi_C\rangle\langle\psi_C|) = \sqrt{\langle\psi_C|\rho'_{R_C}|\psi_C\rangle}$ *suffers quadratic orbital-shear infidelity whenever* $\mathrm{Var}_{\psi_C}(\widehat{N}_{\mathrm{flux}}) > 0$*:*

$$1 - F(\rho'_{R_C}, |\psi_C\rangle\langle\psi_C|) = \frac{1}{2}\omega_0^2(\tau_0')^2\,\mathrm{Var}_{\psi_C}(\widehat{N}_{\mathrm{flux}})\,\sigma_r^2 + O(\sigma_r^3),$$

so $\mathscr{H}_{\mathrm{orb}}$ *acts as an internal dark decoherence sink at order* $O(\sigma_r^2)$*.*

• *At the geodesic maximum ring* $r_0 = r_*$ *where* $\tau_0' = \partial_r \Delta\tau_{\mathrm{DM}}(r_*, 0) = 0$, *quadratic orbital-rotor shear vanishes identically and infidelity is suppressed to quartic order in* σ_r*:*

$$1 - F(\rho_{R_C}', |\psi_C\rangle\langle\psi_C|) = \frac{1}{8}\omega_0^2(\tau_*'')^2\mu_4 \operatorname{Var}_{\psi_C}(\widehat{N}_{\mathrm{flux}}) + O(\sigma_r^5) = O(\sigma_r^4).$$

2. *(Fannes–Audenaert internal shear entropy bound.) Writing* $\varepsilon_{\mathrm{sh}} := 1 - \langle\psi_C|\rho_{R_C}'|\psi_C\rangle \in [0,1]$ *and* $T_{\mathrm{sh}} := \frac{1}{2}\|\rho_{R_C}' - |\psi_C\rangle\langle\psi_C|\|_1 \le \sqrt{\varepsilon_{\mathrm{sh}}}$, *the orbital-induced rotor entanglement entropy obeys*

$$S_{\mathrm{vN}}(\rho_{R_C}') \le T_{\mathrm{sh}} \ln(d_C - 1) + h_2(T_{\mathrm{sh}}),$$

scaling as $O(\sigma_r|\ln\sigma_r|)$ *for* $r_0 \neq r_*$ *and improving to* $O(\sigma_r^2|\ln\sigma_r|)$ *at the geodesic sweet spot* $r_0 = r_*$.

Proof. Set $\delta\tau(r) := \Delta\tau_{\mathrm{DM}}(r,0) - \Delta\tau_{\mathrm{DM}}(r_0,0)$. Since $\omega_0\Delta\tau_{\mathrm{DM}}(r_0,0) = 2\pi m$ and $\sigma(\widehat{N}_{\mathrm{flux}}) \subset \mathbb{Z}$, $e^{-i\omega_0\Delta\tau_{\mathrm{DM}}(r,0)\widehat{N}_{\mathrm{flux}}} = e^{-i\omega_0\delta\tau(r)\widehat{N}_{\mathrm{flux}}}$, so $\rho_{R_C}' = \mathbb{E}_r[e^{-i\omega_0\delta\tau(r)\widehat{N}_{\mathrm{flux}}}|\psi_C\rangle\langle\psi_C|e^{+i\omega_0\delta\tau(r)\widehat{N}_{\mathrm{flux}}}]$ and

$$F^2 = \langle\psi_C|\rho_{R_C}'|\psi_C\rangle = \mathbb{E}_r[|\langle\psi_C|e^{-i\omega_0\delta\tau(r)\widehat{N}_{\mathrm{flux}}}|\psi_C\rangle|^2] = 1 - \omega_0^2 \operatorname{Var}_{\psi_C}(\widehat{N}_{\mathrm{flux}})\,\mathbb{E}_r[\delta\tau(r)^2] + O(\mathbb{E}_r[\delta\tau(r)^4]\,\mu_4^{(\widehat{N}_{\mathrm{flux}})}),$$

where the remainder uses the expansion $|\langle e^{-i\theta\widehat{N}_{\mathrm{flux}}}\rangle|^2 = 1 - \theta^2\mathrm{Var}(\widehat{N}_{\mathrm{flux}}) + O(\theta^4\langle(\Delta\widehat{N}_{\mathrm{flux}})^4\rangle)$ with $\theta = \omega_0\delta\tau(r)$ and with $\mu_4^{(\widehat{N}_{\mathrm{flux}})} := \langle\psi_C|(\widehat{N}_{\mathrm{flux}} - \langle\widehat{N}_{\mathrm{flux}}\rangle)^4|\psi_C\rangle < \infty$ guaranteed by $\dim\mathscr{H}_{R_C}^{(d_C)} < \infty$. Using $1 - F = \frac{1-F^2}{1+F} = \frac{1}{2}(1-F^2) + O((1-F^2)^2)$: for $r_0 \neq r_*$, $\delta\tau(r) = \tau_0'(r - r_0) + O((r-r_0)^2)$ gives $\mathbb{E}_r[\delta\tau(r)^2] = (\tau_0')^2\sigma_r^2 + O(\sigma_r^3)$, proving ; for $r_0 = r_*$, $\tau_0' = 0$ and Taylor expansion give $\delta\tau(r) = \frac{1}{2}\tau_*''(r-r_*)^2 + \frac{1}{6}\tau_*'''(r-r_*)^3 + O((r-r_*)^4)$, hence

$$\theta^2 = \omega_0^2\left(\tfrac{1}{2}\tau_*''(r-r_*)^2 + \tfrac{1}{6}\tau_*'''(r-r_*)^3 + O((r-r_*)^4)\right)^2$$

$$= \tfrac{1}{4}\omega_0^2(\tau_*'')^2(r-r_*)^4 + \tfrac{1}{6}\omega_0^2\tau_*''\tau_*'''(r-r_*)^5 + O(|r-r_*|^6).$$

The written fifth-moment hypothesis absorbs the odd term $\mathbb{E}_r[(r-r_*)^5] = O(\sigma_r^5)$, while the sixth-moment hypothesis together with bounded $\mu_4^{(\widehat{N}_{\mathrm{flux}})}$ absorbs both $O(\mathbb{E}_r[|r-r_*|^6])$ and the spectral remainder $O(\mathbb{E}_r[\delta\tau(r)^4]\mu_4^{(\widehat{N}_{\mathrm{flux}})}) = O(\sigma_r^8)$. Thus $\mathbb{E}_r[\delta\tau(r)^2] = \frac{1}{4}(\tau_*'')^2\mu_4 + O(\sigma_r^5)$, proving only under those moment hypotheses (and

not from $\mu_4 = O(\sigma_r^4)$ alone). Finally, Fuchs–van de Graaf gives $T_{\mathrm{sh}} \le \sqrt{1 - F^2} = \sqrt{\varepsilon_{\mathrm{sh}}}$, and Audenaert's sharp entropy continuity bound [23] relative to pure $|\psi_C\rangle\langle\psi_C|$ yields .

Lemma (Rotor-commuting transverse-trap eigenstate monodromy and coherent multi-turn shear). *Let $H_{\mathrm{trap}} \otimes \mathrm{Id}_{R_C}$ be a rotor-commuting dark orbital Hamiltonian as in Lemma , and suppose $|\chi_{\mathrm{orb}}\rangle \in \mathscr{H}_{\mathrm{orb}}$ is an eigenstate of H_{trap} (in particular, the ground state of a written quadratic transverse trap about $(r_*, 0)$). Prepare the product state $\rho_{\mathrm{DM}}^{(0)} = |\chi_{\mathrm{orb}}\rangle\langle\chi_{\mathrm{orb}}| \otimes |\psi_C\rangle\langle\psi_C|$ of Theorem , centred at $r_0 = r_*$, under the same finite fifth- and sixth-moment hypotheses and with $\dim \mathscr{H}_{R_C}^{(d_C)} < \infty$. Then:*

1. *The radial probability density $|\chi_{\mathrm{orb}}(r,z)|^2$ is stationary under the joint evolution for every integer number of turns $K \ge 1$: there is no spreading of $|\chi_{\mathrm{orb}}|^2$. This typestate holds because $|\chi_{\mathrm{orb}}\rangle$ is a monodromy eigenstate of the written rotor-commuting trap Hamiltonian; it does not typestate the absence of all radial kinetic spreading for generic, non-eigenstate wavepackets.*

2. *After K ultralocal azimuthal circuits, the rotor infidelity at r_* is the single-circuit expansion of Theorem with $\delta\tau \mapsto K\,\delta\tau$. Explicitly, whenever $\mathrm{Var}_{\psi_C}(\widehat{N}_{\mathrm{flux}}) > 0$,*

$$1 - F(\rho_{R_C}^{(K)}, |\psi_C\rangle\langle\psi_C|) = \frac{1}{8} K^2 \omega_0^2 (\tau_*'')^2 \mu_4 \mathrm{Var}_{\psi_C}(\widehat{N}_{\mathrm{flux}}) + O(K^2\sigma_r^5) = O(K^2\sigma_r^4)$$

as $\sigma_r \to 0$ at each fixed $K \ge 1$. The bound is coherent (quadratic in K), not a diffusive wavepacket blow-up. For states that are not eigenstates of a rotor-commuting trap, the Version 2 ultralocal one-circuit operator remains the baseline.

Proof. By construction of Lemma , $[H_{\mathrm{trap}} \otimes \mathrm{Id}_{R_C}, I_{\mathrm{orb}}\mathrm{otimesH}_{C,\mathrm{DM}}] = 0$. If $H_{\mathrm{trap}}|\chi_{\mathrm{orb}}\rangle = E_\perp|\chi_{\mathrm{orb}}\rangle$, the orbital factor evolves only by the global phase $e^{-iE_\perp \tau}$, so the configuration-space density $|\chi_{\mathrm{orb}}(r,z)|^2$ is invariant for all laboratory or dark proper times, and therefore for every integer $K \ge 1$. The clock register still accumulates the multiplication operator $\exp(-i\Delta\tau_{\mathrm{DM}}(r,0)H_{C,\mathrm{DM}})$ on each circuit. After K circuits on a commensurate ring, this is $\exp(-iK\omega_0\delta\tau(r)\widehat{N}_{\mathrm{flux}})$. Substituting $\theta = K\omega_0\delta\tau(r)$ into the expansion of Theorem , under the same moment and

finite-dimensional spectral hypotheses, multiplies the leading quartic term by K^2 and yields . The typestate is confined to this eigenstate class: a generic radial kinetic term that fails to share $|\chi_{\mathrm{orb}}\rangle$ as an eigenstate is not claimed to have vanishing transverse spreading.

Proposition (Quantitative Diophantine Cesàro relaxation rate to the D-CTC diagonal fixed point). *Under the Diophantine hypothesis $\omega_0 \in \mathscr{D}(\Delta\tau, \gamma_{\mathrm{D}}, \nu)$ of Theorem (2), let $\Pi_{\mathrm{diag}}(X) := \sum_{n \in \mathbb{Z}} P_n X P_n$ denote the orthogonal projection onto the $\widehat{N}_{\mathrm{flux}}$-block-diagonal fixed-point algebra, and define the K-turn Cesàro time-averaged channel*

$$\overline{\mathscr{U}}_K(X) := \frac{1}{K}\sum_{j=0}^{K-1} U_C(j\Delta\tau)\, X\, U_C(j\Delta\tau)^{\dagger} = \frac{1}{K}\sum_{j=0}^{K-1} \mathrm{Ad}^j_{U_C(\Delta\tau)}(X) \qquad (K \geq 1).$$

Then for every $X \in \mathscr{B}_{s+\nu}$ with $X - \Pi_{\mathrm{diag}}(X) \in \mathscr{B}^{\mathrm{off}}_{s+\nu}$ and every $K \geq 1$,

$$\|\overline{\mathscr{U}}_K(X) - \Pi_{\mathrm{diag}}(X)\|_{\mathscr{B}_s} \leq \frac{\pi}{K\gamma_{\mathrm{D}}} \|X - \Pi_{\mathrm{diag}}(X)\|_{\mathscr{B}_{s+\nu}}.$$

Thus unconsumed multi-turn evolution unifies P-CTC and D-CTC linearly on the same Hilbert space: commensurate loops (Σ_m) realize exact unitary identity return ($U_C = \mathrm{Id}$), whereas Diophantine loops relax at explicit rate $O(\pi/(K\gamma_{\mathrm{D}}))$ onto the unique Deutsch D-CTC diagonal state $\Pi_{\mathrm{diag}}(\rho)$ without nonlinear state-dependent channels.

Proof. Since $\mathrm{Ad}_{U_C(\Delta\tau)}$ fixes $\Pi_{\mathrm{diag}}(X)$ identically and preserves $\mathscr{B}^{\mathrm{off}}_s$ isometrically ($\|\mathrm{Ad}^j_{U_C}(Y)\|_s = \|Y\|_s$), write $Y := X - \Pi_{\mathrm{diag}}(X) \in \mathscr{B}^{\mathrm{off}}_{s+\nu}$. By Theorem (2), $Z := (I - \mathrm{Ad}_{U_C})^{-1} Y \in \mathscr{B}^{\mathrm{off}}_s$ exists with $\|Z\|_s \leq \frac{\pi}{2\gamma_{\mathrm{D}}}\|Y\|_{s+\nu}$. Telescoping $\sum_{j=0}^{K-1} \mathrm{Ad}^j_{U_C}(I - \mathrm{Ad}_{U_C})Z = (I - \mathrm{Ad}^K_{U_C})Z$ yields

$$\|\overline{\mathscr{U}}_K(X) - \Pi_{\mathrm{diag}}(X)\|_s = \frac{1}{K}\|(I - \mathrm{Ad}^K_{U_C})Z\|_s \leq \frac{2}{K}\|Z\|_s \leq \frac{\pi}{K\gamma_{\mathrm{D}}}\|Y\|_{s+\nu},$$

proving .

The abelian baseline is retained. Its non-abelian enrichment, under a written joint Diophantine hypothesis on the eigenphases of the monodromy $M_\gamma(p)$ of Definition , is typestated as Proposition in Section , and pinches to the commutant rather than typestating a general uniqueness theorem for non-abelian D-CTC fixed points.

Step 3: Conjugacy-class memory, non-abelian braid holonomy, and scalar-age obstruction

We enrich the flux rotor along Chrono-DM-D version 2 [14] (*source record; not a premise*) without leaving (D1)–(D3) and without incurring any bundle-curvature penalty in the algebraic a_2 heat-kernel filter polynomial, upgrading single-loop abelian $\pi_1(\mathscr{U}_{\mathrm{CTC}}) \cong \mathbb{Z}$ holonomy to genuine non-commuting braid commutator memory on a punctured chronology torus.

Definition (Enriched flat-connection flux-multiplicity rotor). Let $\mathscr{N} \subset \mathbb{Z}$ be a nonempty index set of integer Kalb–Ramond flux sectors, and attach to each $n \in \mathscr{N}$ a finite-dimensional dark multiplicity fibre $\mathscr{K}_n \cong \mathbb{C}^{d_n}$ ($d_n \geq 1$), forming

$$\mathscr{H}_{R_C} := \mathrm{bigoplus}_{n \in \mathscr{N}}(\mathbb{C}|n\rangle \otimes \mathscr{K}_n), \qquad \widehat{N}_{\mathrm{flux}} := \sum_{n \in \mathscr{N}} n\,(|n\rangle\langle n| \otimes I_{\mathscr{K}_n}), \qquad H_{C,\mathrm{DM}} := \omega_0 \widehat{N}_{\mathrm{flux}}.$$

On a multiply connected open dark domain $\mathscr{U}' \subseteq \mathscr{U}_{\mathrm{CTC}}$, equip each $\mathscr{K}_n$ with a *flat* unitary dark connection $\mathscr{A}^{(n)}$ ($\Omega^{(n)}_{ab} := \mathrm{d}\mathscr{A}^{(n)} + \mathscr{A}^{(n)} \wedge \mathscr{A}^{(n)} = 0$ everywhere on $\mathscr{U}'$) whose parallel-transport holonomy around a closed g-timelike loop $\gamma \subset \mathscr{U}'$ based at $p \in \gamma$ is $V_n(\gamma, p) \in U(d_n)$, while preserving (D2) ($r_{\mathrm{ord}} = 1$) and (D3) ($g_{\mathrm{sink}} \Delta q = 0$). Over one circuit of dark proper period $\Delta\tau_{\mathrm{DM}}(\gamma)$, the total clock monodromy unitary is

$$M_\gamma(p) := \mathrm{bigoplus}_{n \in \mathscr{N}} e^{-\mathrm{i}\omega_0 n \Delta\tau_{\mathrm{DM}}(\gamma)}\, V_n(\gamma, p).$$

Theorem (Archive-free conjugacy-class memory and discrete-spectrum scalar-age obstruction). *Under Definition :*

1. *(Zero a_2 curvature penalty and basepoint-invariant conjugacy memory.) Because $\Omega^{(n)}_{ab} = 0$ on $\mathscr{U}'$, the dark bundle-curvature contribution $\frac{1}{12}\mathrm{STr}(\Omega_{ab}\Omega^{ab})$ vanishes identically. Under any change of basepoint $p \mapsto p' = \gamma(s)$ along γ or local dark frame unitary $W = \mathrm{bigoplus}_{n \in \mathscr{N}}(1 \otimes W_n)$ ($W_n \in U(d_n)$), monodromy transforms by conjugation $M_\gamma(p') = WM_\gamma(p)W^\dagger$. Hence on any finite-sector truncation $|\mathscr{N}| < \infty$, all symmetric class functionals $\tau_k(\gamma) := \mathrm{Tr}_{\mathscr{H}_{R_C}}(M_\gamma(p)^k)$ ($k \in \mathbb{Z}$) are strictly gauge- and basepoint-invariant without Enter consumption.*

2. *(Torsion-commensurate multi-turn state return.) Suppose γ satisfies $\omega_0 \Delta\tau_{\mathrm{DM}}(\gamma) = 2\pi m$ ($m \in \mathbb{Z}$) and $V_n(\gamma, p)^q = I_{d_n}$*

for all $n \in \mathcal{N}$ *and some integer* $q \geq 1$. *Then after* q *unmeasured turns,* $M_{\gamma^q}(p) = M_\gamma(p)^q = \mathrm{Id}_{\mathscr{H}_{R_C}}$, *returning every state* $\rho_{R_C} \in \mathscr{D}(\mathscr{H}_{R_C})$ *identically while storing intermediate Wilson phases for* $1 \leq j < q$ *when* $q \geq 2$.

3. *(Scalar-age obstruction — Pauli discrete-spectrum theorem.) There exists no self-adjoint internal age operator* $\hat{T}_{\mathrm{age}}$ *on* $\mathscr{H}_{R_C}$ *satisfying either:*

(a) *Weyl canonical covariance* $e^{\mathrm{i}s\hat{T}_{\mathrm{age}}} H_{C,\mathrm{DM}} e^{-\mathrm{i}s\hat{T}_{\mathrm{age}}} = H_{C,\mathrm{DM}} + s\mathrm{Id}_{\mathscr{H}_{R_C}}$ *for all* $s \in \mathbb{R}$; *or*

(b) *strict expectation monotonicity* $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\,(\rho_C(\tau)\hat{T}_{\mathrm{age}}) \geq c_* > 0$ *across an entire commensurate return cycle* $\tau \in [0, q\Delta\tau_{\mathrm{DM}}]$ *on any state* $\rho_C(0) \in \mathscr{D}(\mathscr{H}_{R_C})$ *with finite* $|\mathrm{Tr}\,(\rho_C(0)\hat{T}_{\mathrm{age}})| < \infty$.

Proof. (1) Flatness $\Omega^{(n)}_{ab} = 0$ implies $\mathrm{STr}\,(\Omega_{ab}\Omega^{ab}) = 0$, and parallel transport along γ conjugates $V_n(\gamma, p') = W_n V_n(\gamma, p) W_n^\dagger$, preserving $\tau_k(\gamma)$ by trace cyclicity.

(2) At $\omega_0\Delta\tau_{\mathrm{DM}} = 2\pi m$, $e^{-\mathrm{i}q\omega_0 n\Delta\tau_{\mathrm{DM}}} = 1$ and $V_n(\gamma, p)^q = I_{d_n}$ give $M_\gamma(p)^q = \mathrm{Id}_{\mathscr{H}_{R_C}}$.

(3a) Weyl covariance would force $\sigma(H_{C,\mathrm{DM}}) = \sigma(H_{C,\mathrm{DM}} + s\mathrm{Id}) = \sigma(H_{C,\mathrm{DM}}) + s$ for all $s \in \mathbb{R}$, hence $\sigma(H_{C,\mathrm{DM}}) = \mathbb{R}$, contradicting countable $\sigma(H_{C,\mathrm{DM}}) \subset \omega_0\mathbb{Z}$.

(3b) Integrating $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathrm{Tr}\,(\rho_C(\tau)\hat{T}_{\mathrm{age}}) \geq c_*$ over $[0, q\Delta\tau_{\mathrm{DM}}]$ when $\rho_C(q\Delta\tau_{\mathrm{DM}}) = \rho_C(0)$ yields $0 \geq c_* q\Delta\tau_{\mathrm{DM}} > 0$, a contradiction.

Theorem (Non-abelian braid sequence memory on a doubly satellite-punctured chronology torus). *On the unpunctured solid torus* $\mathscr{U}_{\mathrm{CTC}} \simeq \mathbb{R}_t \times S^1_\varphi \times D^2_{r,z}$, $\pi_1(\mathscr{U}_{\mathrm{CTC}}) \cong \mathbb{Z}$ *is abelian, so every flat connection has mutually commuting single-generator holonomies. Now excise two disjoint smooth stationary azimuthal defect rings* $\Gamma_1 := \mathbb{R}_t \times S^1_\varphi \times \{p_1\}$ *and* $\Gamma_2 := \mathbb{R}_t \times S^1_\varphi \times \{p_2\}$ *(*$p_1 \neq p_2 \in D^2_{r,z} \setminus \{(r_*, 0)\}$*) inside* $\mathscr{U}_{\mathrm{CTC}}$, *forming* $\mathscr{U}' := \mathscr{U}_{\mathrm{CTC}} \setminus (\Gamma_1 \cup \Gamma_2)$. *Then:*

1. $\pi_1(\mathscr{U}', p_0) \cong \mathbb{Z} \times F_2$, *where* $\mathbb{Z}$ *is generated by the central azimuthal circle* σ_{az} *and* $F_2 = \langle \beta_1, \beta_2 \rangle$ *is the non-abelian free group on two meridional loops encircling* p_1 *and* p_2.

2. *For sufficiently small meridional loop radii inside $\mathscr{U}'$ on the open cone $\mathscr{P}_{\mathrm{comm}}$, take a basepoint at $p_0 = (t_0, r_*, 0, 0)$ or slightly inside the upper commensurate contour on $\{\omega_0 \Delta\tau_* > 2\pi\}$. There exist two smooth future-directed closed g-timelike loops $\gamma_A, \gamma_B \subset \mathscr{U}'$ based at p_0, representing the homotopy classes $(1, \beta_1)$ and $(1, \beta_2)$, with equal commensurate dark proper period $\Delta\tau_{\mathrm{DM}}(\gamma_A) = \Delta\tau_{\mathrm{DM}}(\gamma_B) = 2\pi m/\omega_0 < \Delta\tau_*$, obtained by an $\ell = 1$ azimuthal turn together with a small meridional excursion of radius $\varepsilon_i > 0$ about p_i. For general winding $\ell \geq 1$, the same construction yields loops in the classes (ℓ, β_1) and (ℓ, β_2) whose matched commensurate period is the multiple $2\pi(\ell m)/\omega_0$, not $2\pi m/\omega_0$ without that adjustment. The Q_8 torsion-return typestate in (3) is retained on the written flat representation and does not move Card 11 or Operational Clock Return.*

3. *Equip $\mathscr{K}_n \cong \mathbb{C}^2$ ($d_n = 2$) on $\mathscr{U}'$ with the flat $SU(2)$ connection ($\Omega^{(n)}_{ab} = 0$ everywhere on $\mathscr{U}'$) defined by the representation $\rho_n : \mathbb{Z} \times F_2 \to Q_8 \subset SU(2)$ into the finite quaternion group $\rho_n(1, e) = I_2$, $\rho_n(0, \beta_1) = \mathrm{i}\sigma_x$, $\rho_n(0, \beta_2) = \mathrm{i}\sigma_y$. Then for concatenated two-loop words:*

• (Genuine non-abelian order sensitivity:) $M_{\gamma_A}(p_0) M_{\gamma_B}(p_0) = -M_{\gamma_B}(p_0) M_{\gamma_A}(p_0) \neq M_{\gamma_B}(p_0) M_{\gamma_A}(p_0)$, so $\gamma_A \gamma_B$ and $\gamma_B \gamma_A$ act by distinct unitaries, while the words $\gamma_A \gamma_B \gamma_A \gamma_B^{-1}$ ($\rho_n = -I_2$, $\tau_1 = -2|\mathcal{M}|$) and $\gamma_A \gamma_A \gamma_B \gamma_B^{-1}$ ($\rho_n = -I_2$ vs. $\gamma_A \gamma_B \gamma_A^{-1} \gamma_B^{-1}$ with $\rho_n = (\mathrm{i}\sigma_x)(\mathrm{i}\sigma_y)(-\mathrm{i}\sigma_x)(-\mathrm{i}\sigma_y) = -I_2$ whereas the abelianized permutation $\gamma_A \gamma_A^{-1} \gamma_B \gamma_B^{-1}$ gives $+I_2$, $\tau_1 = +2|\mathcal{M}|$) separate non-commuting braid orderings via gauge-invariant traces τ_1 at zero a_2 curvature cost.

• (Exact torsion return under (D2)–(D3):) Every element of Q_8 satisfies $g^4 = I_2$ (and commutator $(\gamma_A \gamma_B \gamma_A^{-1} \gamma_B^{-1})^2$ satisfies $(-I_2)^2 = I_2$), so four-fold repetition of any commensurate word returns $M_w^4 = \mathrm{Id}_{\mathscr{H}_{R_C}}$ with zero Enter consumption and $I(R_C : \mathscr{H}_{\mathrm{OM}} \otimes \mathscr{H}_{\mathrm{sink}}) = 0$.

Proof. (1) Since $\mathscr{U}' \simeq \mathbb{R}_t \times S^1_\varphi \times (D^2_{r,z} \setminus \{p_1, p_2\})$ and $D^2_{r,z} \setminus \{p_1, p_2\}$ deformation-retracts onto $S^1 \vee S^1$, Van Kampen's theorem and product homotopy give $\pi_1(\mathscr{U}', p_0) \cong \mathbb{Z} \times F_2$.

(2) At the geodesic maximum, $\Delta\tau_{\mathrm{DM}}(r_*, 0) = \Delta\tau_*$. On $\mathscr{P}_{\mathrm{comm}}$ one has $\omega_0 \Delta\tau_* > 2\pi$, so some integer $m \geq 1$ obeys $2\pi m/\omega_0 < \Delta\tau_*$. Because $-\mathrm{d}s^2 = C_- \, \mathrm{d}\varphi^2 - \mathrm{d}r^2 - \mathrm{d}z^2$ along a constant-t path, any nonvanishing meridional excursion strictly decreases the proper period relative to the pure azimuthal circle of the same mean radius at r_*. Thus an

$\ell = 1$ turn with a sufficiently small meridional radius $\varepsilon_i > 0$ about p_i, together with a continuous adjustment of the radial mean slightly inside the upper commensurate contour, realises exact period $2\pi m/\omega_0 < \Delta\tau_*$ while remaining g -timelike: $g(\dot\gamma,\dot\gamma) = -C_-(r(\varphi),z(\varphi)) + \dot r^2 + \dot z^2 < 0$. The same construction with $\ell \geq 1$ azimuthal turns accumulates, to leading order, ℓ times the $\ell = 1$ period; matching a commensurate value therefore requires the multiple $2\pi(\ell m)/\omega_0$. (If a loop already sits on Σ_m and $\ell \geq 2$ additional azimuthal turns are taken without a compensating meridional reduction, the period is $\approx \ell \cdot 2\pi m/\omega_0$, not $2\pi m/\omega_0$.) In all cases $e^{-i\omega_0 n\Delta\tau} = 1$ still holds, now at the written multiple.

(3) Every group homomorphism $\pi_1(\mathscr{U}', p_0) \to SU(2)$ induces a canonical flat principal $SU(2)$-bundle and flat connection ($\Omega^{(n)}_{ab} = 0$) via the universal cover $\widetilde{\mathscr{U}'} \times_{\rho_n} \mathbb{C}^2$. At $\omega_0\Delta\tau_{\mathrm{DM}} = 2\pi m$, the $U(1)$ flux phases equal 1, so $M_{\gamma_A} = \bigoplus_n i\sigma_x$ and $M_{\gamma_B} = \bigoplus_n i\sigma_y$. From $(i\sigma_x)(i\sigma_y) = i\sigma_z = -(i\sigma_y)(i\sigma_x)$, they anticommute; the commutator word gives $(i\sigma_x)(i\sigma_y)(-i\sigma_x)(-i\sigma_y) = -I_2 \neq +I_2$, separating non-abelian order from its abelianization via τ_1, while $g^4 = I_2$ for all $g \in Q_8$ gives exact four-turn return.

Theorem (Fixed-flux reduced-register immunity and finite-subgroup braid torsion). *Let a dark clock state be supported in a single flux eigenspace $n = n_0$ of Definition , so that* $\mathrm{Var}(\widehat{N}_{\mathrm{flux}}) = 0$ *identically, and write* $\mathscr{K}_{n_0} \cong \mathbb{C}^{d_{n_0}}$ *for the multiplicity fibre with flat connection* $\Omega^{(n_0)}_{ab} = 0$ *on* $\mathscr{U}'$*. Then:*

1. *(All-orders reduced-register immunity,* Var = 0 *special case.) For any radial or meridional wavepacket* $|\chi_{\mathrm{orb}}\rangle$ *supported in* $\mathscr{U}'$*, the orbital-conditioned monodromy acts on the sharp-n_0 clock vector by an r-dependent $U(1)$ phase* $e^{-i\omega_0 n_0 \Delta\tau_{\mathrm{DM}}(r,z)}$*. That phase leaves the reduced* $\widehat{N}_{\mathrm{flux}}$*-register (and the reduced state on* $\mathbb{C}|n_0\rangle \otimes \mathscr{K}_{n_0}$ *after tracing the orbital factor against a global phase) invariant, so the Uhlmann fidelity of the reduced flux register satisfies* $1 - F \equiv 0$ *to all orders in* σ_r*, not merely* $O(\sigma_r^4)$*. This is the* $\mathrm{Var}(\widehat{N}_{\mathrm{flux}}) = 0$ *special case of Theorem ; it does not typestate a cancellation of shear at* $\mathrm{Var}(\widehat{N}_{\mathrm{flux}}) > 0$*.*

2. *(Flat fibre holonomy under deformations in* $\mathscr{U}'$*.) Because* $\Omega^{(n_0)}_{ab} = 0$ *on* $\mathscr{U}'$*, the fibre holonomy* $V_{n_0}(\gamma, p)$ *depends only on the homotopy class of* $\gamma \subset \mathscr{U}'$ *based at p. Consequently every smooth deformation of a trajectory inside* $\mathscr{U}'$

that does not cross the satellite defect rings Γ_1, Γ_2 *of Theorem leaves* V_{n_0} *unchanged. When* $n_0 = 0$*, or when* $\omega_0 \Delta\tau_{\mathrm{DM}} = 2\pi m$ *at general* n_0*, the* $U(1)$ *flux phase equals* 1 *and only the flat fibre holonomy remains.*

3. *(Optional finite binary-icosahedral torsion typestate.) The holonomy target may be taken in the binary icosahedral group* $2I \subset SU(2)$*, defined as the inverse image in* $SU(2)$ *of the icosahedral rotation group* $A_5 \subset SO(3)$*, so that* $|2I| = 120$*. Element orders in* A_5 *divide* $30 = \mathrm{lcm}(1, 2, 3, 5)$*; lifts to* $SU(2)$ *have order* k *or* $2k$*, hence every* $g \in 2I$ *satisfies* $g^{60} = I_2$*. On a commensurate loop, sixty unmeasured repetitions therefore return* $M_w^{60} = \mathrm{Id}_{\mathscr{H}_{R_C}}$ *on the written* $2I$*-representation, with zero Enter consumption. This is a finite-subgroup torsion identity on the written dark ledger. It is not typestated as computational universality, as density of gates in* $SU(2)$*, as an archive-free holonomic computer, or as a movement of Card 11 or Operational Clock Return. No* texttt *Host~C is created.*

Proof. (1) On a sharp eigenstate $\widehat{N}_{\mathrm{flux}}|n_0\rangle = n_0|n_0\rangle$, one has $e^{-\mathrm{i}\omega_0 \delta\tau(r)\widehat{N}_{\mathrm{flux}}} = e^{-\mathrm{i}\omega_0 n_0 \delta\tau(r)} I$ on that one-dimensional flux ray, which factors out of the reduced flux state as a global (r-dependent) phase. Tracing the orbital factor therefore returns the original reduced flux state exactly, for every σ_r and every packet supported in $\mathscr{U}'$.

(2) Flatness $\Omega_{ab}^{(n_0)} = 0$ means that parallel transport depends only on the homotopy class in $\pi_1(\mathscr{U}', p)$, so any deformation inside $\mathscr{U}'$ that does not cross Γ_1, Γ_2 is invisible to V_{n_0}. The flux phase claim is immediate from $e^{-\mathrm{i}\omega_0 n_0 \Delta\tau} = 1$ at $n_0 = 0$ or at $\omega_0 \Delta\tau = 2\pi m$.

(3) By definition, $2I \to A_5$ is a two-to-one covering homomorphism with kernel $\{\pm I_2\}$. The cycle types in A_5 have orders $1, 2, 3, 5$, so every element of A_5 has order dividing 30. The preimage of an element of order k has order k or $2k$, hence divides 60. Thus $g^{60} = I_2$ for all $g \in 2I$. Combined with a commensurate $U(1)$ phase as in Theorem (2), $M_w^{60} = \mathrm{Id}$. No operational bar is moved: the return remains unconsumed dark holonomy on the written fibre, and $r_{\mathrm{ord}} = 1$ is unchanged.

Proposition (Non-abelian Diophantine Cesàro rate to the monodromy commutant). *Let* $M_\gamma(p)$ *be the enriched monodromy of Definition on a finite-dimensional register* $\mathscr{H}_{R_C}$*, with spectral decomposition* $M_\gamma(p) = \sum_j e^{-\mathrm{i}\theta_j} P_j$ *(eigenphases* $\theta_j \in \mathbb{R}/2\pi\mathbb{Z}$*, orthogonal eigenprojections* P_j*). Assume a joint Diophantine*

non-resonance hypothesis: there exist $\gamma_{\mathrm{D}} > 0$ *and* $v > 1$ *such that*

$$|k(\theta_j - \theta_\ell) - 2\pi m| \geq \frac{\gamma_{\mathrm{D}}}{|k|^{v}} \qquad \text{for all} j \neq \ell, \text{all} k \in \mathbb{Z} \setminus \{0\}, \text{and all} m \in \mathbb{Z}.$$

Let $\Pi_{\mathrm{comm}(M_\gamma)}(X) := \sum_j P_j X P_j$ *be the pinching channel onto the commutant of* $M_\gamma(p)$ *(equivalently, onto the algebra of operators diagonal in the eigenprojections of* $M_\gamma(p)$*), and write* $\overline{\mathscr{M}}_K(X) := \frac{1}{K}\sum_{j=0}^{K-1} \mathrm{Ad}^j_{M_\gamma(p)}(X)$*. Then for every* $K \geq 1$ *and every* $X \in \mathscr{B}(\mathscr{H}_{R_C})$*,*

$$\|\overline{\mathscr{M}}_K(X) - \Pi_{\mathrm{comm}(M_\gamma)}(X)\| \leq \frac{\pi}{K\gamma_{\mathrm{D}}} \|X - \Pi_{\mathrm{comm}(M_\gamma)}(X)\|,$$

in the operator norm (or any unitarily invariant norm). The abelian baseline of Proposition is the special case $V_n \equiv I$*, for which the eigenphases are* $\omega_0 n\Delta\tau$ *and* $\Pi_{\mathrm{comm}(M_\gamma)} = \Pi_{\mathrm{diag}}$*. The typestate is convergence to the written commutant at explicit rate* $O(1/K)$*; it is not a general uniqueness theorem for non-abelian D-CTC fixed points beyond that commutant.*

Proof. The hypothesis is the exact analogue of Theorem (2) for the pairwise phase differences of $M_\gamma(p)$. Jordan's inequality therefore supplies $|1 - e^{-\mathrm{i}k(\theta_j - \theta_\ell)}| \geq 2\gamma_{\mathrm{D}}/(\pi|k|^{v})$, so the off-diagonal resolvent $(I - \mathrm{Ad}_{M_\gamma})^{-1}$ is bounded on the off-commutant subspace $\{X : \Pi_{\mathrm{comm}(M_\gamma)}(X) = 0\}$ by $\pi/(2\gamma_{\mathrm{D}})$. The same telescoping identity as in Proposition , $\sum_{j=0}^{K-1} \mathrm{Ad}^j_{M_\gamma}(I - \mathrm{Ad}_{M_\gamma})Z = (I - \mathrm{Ad}^K_{M_\gamma})Z$, yields . Fixed points of Ad_{M_γ} are precisely the commutant of $M_\gamma(p)$; no uniqueness claim is made outside that written algebra.

Step 4: External tick retained, Page–Wootters conditioning, and Helstrom–rank floor

We prove locally why an unconsumed returning rotor acts only as a bounded conditioning state inside a single period and never eliminates the external clock *T*, pairing Fano's asymptotic inequality with a non-vacuous parameter-free rank geometry floor that holds for every $M_{\mathrm{tick}} \geq d_C + 1$ [15, 16, 17, 18] (*source record; not a premise*).

Theorem (Bimetric Fisher split, Page–Wootters winding erasure, and Helstrom–Fano tick floors). *Restrict* $\mathscr{H}_{R_C}$ *to a finite* d_C*-dimensional dark clock subspace* $\mathscr{H}_C^{(d_C)}$ *(*$2 \leq d_C < \infty$*) spanned by orthonormal eigenstates*

$\{|e_j\rangle\}_{j=0}^{d_C-1}$ *of* $H_{C,\mathrm{DM}}$ *with eigenvalues* $\omega_0 n_j$ *(*$n_{\min} \le n_0 \le \ldots \le n_{d_C-1} \le n_{\max}$*). Then:*

1. *(Bimetric quantum Fisher split.) For any ordinary probe state* $\rho_{\mathrm{ord}}(\tau_{\mathrm{DM}})$ *evolving under (D1)–(D2) on a product input, the ordinary quantum Fisher information with respect to* τ_{DM} *vanishes identically:* $\mathcal{I}_Q^{\mathrm{ord}}(\tau_{\mathrm{DM}}) = 0$*. Conversely, on a pure internal dark clock state* $|\psi_C(\tau)\rangle = e^{-\mathrm{i}\tau a u H_{C,\mathrm{DM}}}|\psi_C(0)\rangle$*, the internal dark quantum Fisher information satisfies*

$$\mathcal{I}_Q^{\mathrm{dark}}(\tau) = 4\,\mathrm{Var}_{\psi_C(0)}(H_{C,\mathrm{DM}}) \le \omega_0^2 (n_{\max} - n_{\min})^2 < \infty,$$

which is positive whenever $|\psi_C(0)\rangle$ *spans at least two distinct* n_j*, but invisible to ordinary Enter instruments (*$r_{\mathrm{ord}} = 1$*).*

2. *(Page–Wootters conditioning erases loop winding.) Suppose* $n_0 < \ldots < n_{d_C-1}$ *are distinct integers, let* $\mathcal{H}_{S,\mathrm{DM}}$ *be an internal dark system with Hamiltonian* $H_{S,\mathrm{DM}}$*, and let* $|\Psi\rangle := d_C^{-1/2} \sum_{j=0}^{d_C-1} |e_j\rangle \otimes |\varphi_j\rangle \in \mathcal{H}_C^{(d_C)} \otimes \mathcal{H}_{S,\mathrm{DM}}$ *satisfy* $(H_{C,\mathrm{DM}} \otimes I + I \otimes H_{S,\mathrm{DM}})|\Psi\rangle = 0$ *(*$H_{S,\mathrm{DM}}|\varphi_j\rangle = -\omega_0 n_j |\varphi_j\rangle$*). Defining covariant phase vectors* $|\theta_\tau\rangle := d_C^{-1/2} \sum_{j=0}^{d_C-1} e^{-\mathrm{i}\omega_0 n_j \tau}|e_j\rangle$*, the Page–Wootters conditional state* $|\psi_{S|\tau}\rangle := \sqrt{d_C}(\langle\theta_\tau| \otimes I)|\Psi\rangle$ *obeys* $e^{-\mathrm{i}\tau H_{S,\mathrm{DM}}}|\psi_{S|0}\rangle = |\psi_{S|\tau}\rangle$*, but satisfies exact periodicity*

$$|\theta_{\tau + 2\pi/\omega_0}\rangle = |\theta_\tau\rangle, \qquad |\psi_{S|\tau+2\pi/\omega_0}\rangle = |\psi_{S|\tau}\rangle \qquad \forall\, \tau \in \mathbb{R},$$

identifying τ *only modulo* $2\pi/\omega_0$ *and erasing the winding sheet* $m \in \mathbb{Z}$*.*

3. *(Combined Helstrom–rank and Holevo–Fano finite-tick error floors.) Let* K_{tick} *be uniformly distributed over* $M_{\mathrm{tick}} \ge 2$ *classical tick labels encoded into arbitrary states* $\{\rho_C^{(k)}\}_{k=1}^{M_{\mathrm{tick}}} \subset \mathcal{D}(\mathcal{H}_C^{(d_C)})$*, and let* $\widehat{K}$ *be the outcome of any decoding POVM* $\{E_k\}_{k=1}^{M_{\mathrm{tick}}}$ $(\sum_{k=1}^{M_{\mathrm{tick}}} E_k = I_{d_C})$ *on* $\mathcal{H}_C^{(d_C)}$*. The average identification error probability* $P_e := \Pr(\widehat{K} \ne K_{\mathrm{tick}})$ *satisfies both the exact non-asymptotic rank floor and the Holevo–Fano entropic floor:*

$$P_e \ge \max\left\{0,\, 1 - \frac{d_C}{M_{\mathrm{tick}}},\, 1 - \frac{\ln d_C + \ln 2}{\ln M_{\mathrm{tick}}}\right\}.$$

In particular, $P_e \ge 1 - d_C/M_{\mathrm{tick}} > 0$ *is strictly positive for every* $M_{\mathrm{tick}} \ge d_C + 1$ *(closing the* $d_C < M_{\mathrm{tick}} \le 2d_C$ *regime*

where Fano's bound alone is vacuous), zero-error tick resolution ($P_e = 0$) forces $1 \le M_{\mathrm{tick}} \le d_C$, and $P_e \to 1$ as $M_{\mathrm{tick}} \to \infty$ at fixed d_C. Neither a returned rotor nor a finite-d_C code removes the external parameter T; Problem 1 and the enlarged duration gate remain open.

Proof. (1) Under (D2) on a product input, $\partial_{\tau_{\mathrm{DM}}} \rho_{\mathrm{ord}} = 0 \Rightarrow \mathscr{I}_Q^{\mathrm{ord}} = 0$. On $|\psi_C(\tau)\rangle = e^{-\mathrm{itauH}_{C,\mathrm{DM}}}|\psi_C(0)\rangle$, the pure-state Bures formula gives $\mathscr{I}_Q^{\mathrm{dark}}(\tau) = 4\mathrm{Var}_{\psi_C(0)}(H_{C,\mathrm{DM}})$, and Popoviciu's inequality gives .

(2) Direct evaluation gives $\sqrt{d_C}(\langle\theta_\tau| \otimes I)|\Psi\rangle = d_C^{-1/2}\sum_{j=0}^{d_C-1} e^{+\mathrm{i}\omega_0 n_j \tau}|\varphi_j\rangle = e^{-\mathrm{i}\tau H_{S,\mathrm{DM}}}|\psi_{S\,|\,0}\rangle$, and $n_j \in \mathbb{Z}$ implies $e^{-\mathrm{i}\omega_0 n_j(\tau + 2\pi/\omega_0)} = e^{-\mathrm{i}\omega_0 n_j \tau}$, proving .

(3) For the exact Helstrom–rank floor, since each $\rho_C^{(k)}$ is a density operator with operator norm $\|\rho_C^{(k)}\|_\infty \le 1$ and $E_k \ge 0$,

$$1 - P_e = \frac{1}{M_{\mathrm{tick}}}\sum_{k=1}^{M_{\mathrm{tick}}} \mathrm{Tr}\,(\rho_C^{(k)} E_k) \le \frac{1}{M_{\mathrm{tick}}}\sum_{k=1}^{M_{\mathrm{tick}}} \|\rho_C^{(k)}\|_\infty \,\mathrm{Tr}\,(E_k) \le \frac{1}{M_{\mathrm{tick}}}\mathrm{Tr}\,(\sum_{k=1}^{M_{\mathrm{tick}}} E_k) = \frac{d_C}{M_{\mathrm{tick}}},$$

whence $P_e \ge 1 - d_C/M_{\mathrm{tick}}$. For the Holevo–Fano bound, Holevo's theorem gives $I(K_{\mathrm{tick}} : \widehat{K}) \le \ln d_C$ and Fano's inequality gives $\ln M_{\mathrm{tick}} - I(K_{\mathrm{tick}} : \widehat{K}) \le h_2(P_e) + P_e \ln (M_{\mathrm{tick}} - 1) < \ln 2 + P_e \ln M_{\mathrm{tick}}$. Combining both bounds yields .

Proposition (Finite internal turn-counter and Floquet single-valuedness). *Suppose the finite dark register factorises as $\mathscr{H}_{R_C} = \mathscr{H}_{\mathrm{tick}}^{(d_{\mathrm{tick}})} \otimes \mathscr{H}_{\mathrm{cnt}}^{(d_{\mathrm{cnt}})}$ with $d_{\mathrm{cnt}} < \infty$ and $d_C = d_{\mathrm{tick}} d_{\mathrm{cnt}} < \infty$, and that a unitary increment U_{inc} acts on the counter once per loop γ by $U_{\mathrm{inc}}|k\rangle_{\mathrm{cnt}} = |k + 1 \bmod d_{\mathrm{cnt}}\rangle_{\mathrm{cnt}}$ (and as the identity on $\mathscr{H}_{\mathrm{tick}}$, or more generally as a unitary on the tick factor commuting with the modular increment). Then:*

1. *(Modular wrap-around.) Monotone turn-counting, in the sense that the counter eigenvalue increases through distinct standard-basis labels, holds at most for d_{cnt} loops (strictly, for the d_{cnt} values $k = 0, \ldots, d_{\mathrm{cnt}} - 1$) before the modular identity $U_{\mathrm{inc}}^{d_{\mathrm{cnt}}} = \mathrm{Id}_{\mathscr{H}_{\mathrm{cnt}}}$ resets the counter. The joint return period therefore scales at most to $d_{\mathrm{cnt}}\Delta\tau_{\mathrm{DM}}$. This does not remove the external parameter T as $m \to \infty$: the finite counter is periodic, and Theorem (2)–(3) continues to apply to the finite-dimensional joint register.*

2. *(Floquet single-valuedness on a closed loop.) On a closed g-timelike loop $\gamma \simeq S^1$, any globally single-valued*

Page–Wootters state $|\Psi\rangle$ *on* $\mathscr{H}_{R_C} \otimes \mathscr{H}_S$ *satisfying* $(H^{\mathrm{tot}}_{C,\mathrm{DM}} + H_S)|\Psi\rangle = 0$ *must be fixed by the joint monodromy:*

$$(M_\gamma \otimes e^{-\mathrm{i}\Delta\tau_{\mathrm{DM}} H_S})|\Psi\rangle = |\Psi\rangle.$$

Thus only zero-net-monodromy joint eigenstates survive as single-valued sections on γ. This is a no-go on the written finite-counter class. Problem 1 and the Duration Gate remain Open: a finite internal counter does not supply a general closure of those problems.

Proof. (1) The standard basis of $\mathscr{H}_{\mathrm{cnt}}$ is a cyclic orbit of length d_{cnt} under U_{inc}. Distinct labels are therefore available for at most d_{cnt} successive increments, and $U_{\mathrm{inc}}^{d_{\mathrm{cnt}}} = \mathrm{Id}$ returns every counter state. The joint unitary over d_{cnt} loops is a finite-dimensional periodic monodromy, so the winding sheet $m \in \mathbb{Z}$ remains unidentified as in , and the Helstrom–rank floor of Theorem (3) still forces $P_e > 0$ once more than d_C tick labels are asked of a finite register.

(2) Parallel transport of a single-valued section around a closed loop $\gamma \simeq S^1$ must reproduce the same vector in the same fibre. Under the stationary Page–Wootters constraint $(H^{\mathrm{tot}}_{C,\mathrm{DM}} + H_S)|\Psi\rangle = 0$, transport around γ acts by $M_\gamma \otimes e^{-\mathrm{i}\Delta\tau_{\mathrm{DM}} H_S}$, and single-valuedness is precisely . Hence $|\Psi\rangle$ must lie in the +1 eigenspace of that joint monodromy (the zero-net-monodromy subspace). States that acquire a nontrivial Floquet multiplier cannot be defined as single-valued sections on the closed manifold.

Step 5: Archive zero-leakage closure vs. repaired Petz append obstruction

We close the exact archive sentence permitted to class (D1)–(D3) and prove beside it why any hypothetical append reopens the archive bar under the Version-6 repaired relative-entropy–fidelity direction [3, 4, 5] (*source record; not a premise*).

Theorem (Zero archive mutual information on the frozen class (D1)–(D3)). *Let* $\mathscr{H}_{\mathrm{tot}} = \mathscr{H}_{R_C} \otimes \mathscr{H}_{\mathrm{OM}} \otimes \mathscr{H}_{\mathrm{sink}}$ *satisfy (D1), (D2)* ($\delta S_{\mathrm{d}}/\delta\Psi_{\mathrm{SM}} \equiv 0$, $r_{\mathrm{ord}} = 1$*), and (D3)* ($g_{\mathrm{sink}}\Delta q = 0$*) along a closed dark loop* $\gamma \subset \mathscr{U}_{\mathrm{CTC}}$*. Then the total transition unitary across γ factorises as* $U_{\mathrm{tot}}(\gamma) = M_\gamma \otimes U_{\mathrm{OM,sink}}$*. Consequently, for every initial product*

preparation $\rho_{\mathrm{tot}}^{(0)} = \rho_{R_C}^{(0)} \otimes \rho_{\mathrm{OM,\,sink}}^{(0)}$*:*

$$I(R_C : \mathscr{H}_{\mathrm{OM}} \otimes \mathscr{H}_{\mathrm{sink}})_{U_{\mathrm{tot}}(\gamma)\rho_{\mathrm{tot}}^{(0)}U_{\mathrm{tot}}(\gamma)^\dagger} = 0,$$

and the exterior accessible entropy $\Delta S_{\mathrm{acc}}(\rho_{\mathrm{OM}}) := \inf_B H(\mathrm{diag}_B(\rho_{\mathrm{OM}})) = S_{\mathrm{vN}}(\rho_{\mathrm{OM}})$ *is strictly unchanged by* $\rho_{R_C}^{(0)}$ *and* M_γ*. Both (D3) and product preparation are essential: (D2) alone does not preclude gravitational soft-sink entanglement if* $g_{\mathrm{sink}}\Delta q \neq 0$*.*

Proof. Portal silence $\delta S_{\mathrm{d}}/\delta\Psi_{\mathrm{SM}} \equiv 0$ eliminates interactions between $\mathscr{H}_{R_C}$ and $\mathscr{H}_{\mathrm{OM}}$, while (D3) ($g_{\mathrm{sink}}\Delta q = 0$) eliminates state-dependent interactions between $\mathscr{H}_{R_C}$ and $\mathscr{H}_{\mathrm{sink}}$, yielding additive generator $H_{\mathrm{tot}} = H_{R_C} \otimes I + I \otimes H_{\mathrm{OM,\,sink}}$ and $U_{\mathrm{tot}}(\gamma) = M_\gamma \otimes U_{\mathrm{OM,\,sink}}$. Product inputs map to product outputs, whose Umegaki relative entropy to the product of marginals is zero, proving . Schur–Horn gives $\Delta S_{\mathrm{acc}}(\rho_{\mathrm{OM}}) = S_{\mathrm{vN}}(\rho_{\mathrm{OM}})$, independent of $\rho_{R_C}^{(0)}$ and M_γ.

Theorem (Repaired Petz append obstruction: why "archive closed after append" stays open). *Suppose a hypothetical append channel couples* R_C *to a finite-dimensional archive* $\mathscr{H}_{\mathrm{arch}}$*, producing a joint state* $\sigma_{R_C\,\mathrm{arch}}$ *on* $\mathscr{H}_{R_C} \otimes \mathscr{H}_{\mathrm{arch}}$ *with marginals* $\rho_{R_C} = \mathrm{Tr}_{\mathrm{arch}}(\sigma_{R_C\,\mathrm{arch}})$, $\rho_{\mathrm{arch}} = \mathrm{Tr}_{R_C}(\sigma_{R_C\,\mathrm{arch}})$*, product reference* $\omega_0 := \rho_{R_C} \otimes \rho_{\mathrm{arch}}$*, and strictly positive mutual information* $I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma := S_{\mathrm{vN}}(\rho_{R_C}) + S_{\mathrm{vN}}(\rho_{\mathrm{arch}}) - S_{\mathrm{vN}}(\sigma_{R_C\,\mathrm{arch}}) > 0$*. Let* $\mathscr{N}_{\mathrm{arch}} := \mathrm{Tr}_{\mathrm{arch}}$ *denote discarding the archive. Then:*

1. *(Accessible archive entropy floor.) The basis-minimised accessible entropy left in the archive satisfies*

$$\Delta S_{\mathrm{acc}}(\rho_{\mathrm{arch}}) = S_{\mathrm{vN}}(\rho_{\mathrm{arch}}) \geq \frac{1}{2} I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma > 0.$$

2. *(Canonical product Petz map and Version-6 repaired fidelity typestate.) On* $\mathrm{supp}\,(\rho_{R_C})$*, the canonical Petz recovery map* $\mathscr{P}_{\omega_0,\mathscr{N}_{\mathrm{arch}}}(X) := \omega_0^{1/2}\mathscr{N}_{\mathrm{arch}}^*(\mathscr{N}_{\mathrm{arch}}(\omega_0)^{-1/2}X\mathscr{N}_{\mathrm{arch}}(\omega_0)^{-1/2})\omega_0^{1/2}$ *evaluates to* $\mathscr{P}_{\omega_0,\mathscr{N}_{\mathrm{arch}}}(X) = X \otimes \rho_{\mathrm{arch}}$*, so* $(\mathscr{P}_{\omega_0,\mathscr{N}_{\mathrm{arch}}} \circ \mathscr{N}_{\mathrm{arch}})(\sigma_{R_C\,\mathrm{arch}}) = \omega_0$*. Writing* $F(\rho,\tau) := \|\sqrt{\rho}\sqrt{\tau}\|_1 \in [0,1]$ *for Uhlmann fidelity, the published inequality* $D(\rho\|\tau) \geq -2\ln F(\rho,\tau)$ *applied to* $I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma = D(\sigma_{R_C\,\mathrm{arch}}\|\omega_0)$ *yields the lower bound*

$$F(\sigma_{R_C\,\mathrm{arch}}, \omega_0) \geq \exp\left(-I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma/2\right)$$

for finite I (and 0 *if* $I = +\infty$*), while faithfulness of Uhlmann fidelity* ($F(\rho,\tau) = 1 \Leftrightarrow \rho = \tau$*) combined with* $I > 0 \Leftrightarrow \sigma_{R_C\,\mathrm{arch}} \neq \omega_0$ *yields the strict recovery deficit*

$$F(\sigma_{R_C\,\mathrm{arch}}, \omega_0) < 1.$$

(The ceiling $F \leq \exp(-I/2)$ *and deficit floor* $1 - F \geq 1 - e^{-I/2}$ *do not follow from* $D \geq -2\ln F$ *and are refused.)*

3. *(Joint-reference CPTP obstruction.) No single CPTP map* $\mathscr{R} : \mathscr{B}(\mathscr{H}_{R_C}) \to \mathscr{B}(\mathscr{H}_{R_C} \otimes \mathscr{H}_{\mathrm{arch}})$ *satisfies both* $(\mathscr{R} \circ \mathscr{N}_{\mathrm{arch}})(\sigma_{R_C\,\mathrm{arch}}) = \sigma_{R_C\,\mathrm{arch}}$ *and* $(\mathscr{R} \circ \mathscr{N}_{\mathrm{arch}})(\omega_0) = \omega_0$.

4. *(Landauer sink dump upon local archive erasure.) Any local unitary reset* $I_{R_C} \otimes U_{\mathrm{arch,\,sink}}$ *mapping* $\sigma_{R_C\,\mathrm{arch}} \otimes |0\rangle\langle 0|_{\mathrm{sink}}$ *to a pure archive marginal* $|0\rangle\langle 0|_{\mathrm{arch}}$ *forces both* $\Delta S_{\mathrm{vN}}(\rho'_{\mathrm{sink}}) = S_{\mathrm{vN}}(\rho_{\mathrm{arch}}) \geq \frac{1}{2} I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma > 0$ *and* $I(R_C : \mathscr{H}_{\mathrm{sink}})_{\sigma'} = I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma > 0$.

Proof. (1) Araki–Lieb $S_{\mathrm{vN}}(\sigma_{R_C\,\mathrm{arch}}) \geq |S_{\mathrm{vN}}(\rho_{R_C}) - S_{\mathrm{vN}}(\rho_{\mathrm{arch}})|$ gives $I(R_C : \mathscr{H}_{\mathrm{arch}})_\sigma \leq 2\min(S_{\mathrm{vN}}(\rho_{R_C}), S_{\mathrm{vN}}(\rho_{\mathrm{arch}}))$; Schur–Horn gives $\Delta S_{\mathrm{acc}}(\rho_{\mathrm{arch}}) = S_{\mathrm{vN}}(\rho_{\mathrm{arch}}) \geq \frac{1}{2} I > 0$.

(2) Since $\mathscr{N}^*_{\mathrm{arch}}(X) = X \otimes I_{\mathrm{arch}}$ and $\mathscr{N}_{\mathrm{arch}}(\omega_0) = \rho_{R_C}$, on $\mathrm{supp}(\rho_{R_C})$ we have $\mathscr{P}_{\omega_0,\mathscr{N}_{\mathrm{arch}}}(X) = X \otimes \rho_{\mathrm{arch}}$. Rearranging $I = D(\sigma_{R_C\,\mathrm{arch}} \| \omega_0) \geq -2\ln F(\sigma_{R_C\,\mathrm{arch}}, \omega_0)$ yields , and $I > 0 \Rightarrow \sigma_{R_C\,\mathrm{arch}} \neq \omega_0$ gives by Uhlmann faithfulness.

(3) Monotonicity of Umegaki relative entropy under $\mathscr{R}$ applied to $\mathscr{N}_{\mathrm{arch}}(\sigma_{R_C\,\mathrm{arch}}) = \mathscr{N}_{\mathrm{arch}}(\omega_0) = \rho_{R_C}$ would give $I \leq D(\rho_{R_C} \| \rho_{R_C}) = 0$, contradicting $I > 0$.

(4) Proved in Lemma (3), combined with (1).

Theorem (Conditional linearised gravitational soft-sink selection identity). *Retain class (D1)–(D3) as a prescribed frozen substrate: axiom (D3) is not derived, discharged, or replaced. Work in linearised gravity on the written Minkowski-plus-vortex background, with Planck-suppressed coupling* $8\pi G = \mathrm{Mp}^{-2}$*, and write* $\mathscr{H}^{\mathrm{grav}}_{\mathrm{sink}}$ *for the linearised graviton Fock space. Then, on the following written classes:*

1. *(Vanishing linearised quadrupole radiation on two selection classes.) If the dark carrier is prepared as a stationary axisymmetric current ring about* $\gamma_{r_*,0}$ *(all time-dependent mass-multipole moments with* $m_\varphi \neq 0$ *vanish),*

the linearised quadrupole (and, likewise, every time-dependent multipole) of the dark stress vanishes identically, so the linearised emission into $\mathscr{H}^{\mathrm{grav}}_{\mathrm{sink}}$ *is zero and* $I(R_C : \mathscr{H}^{\mathrm{grav}}_{\mathrm{sink}}) = 0$ *on product inputs. The same vanishing holds on the energy-degenerate flat holonomy sector* $n_0 = 0$ *with* $H_{C,\mathrm{DM}} = 0$ *and* $\Delta E = 0$*, for which the fibre states of* $\mathscr{K}_0$ *share one and the same energy-momentum to the written linearised order (*$\delta T^{ab} \equiv 0$*).*

2. *(Linearised estimate on a written localised-lump class.) By contrast, let a localised lump of mass-energy* $m_{\mathrm{d}} + \omega_0 \widehat{N}_{\mathrm{flux}}$*, with* $\omega_0 \neq 0$ *and* $\mathrm{Var}\,(\widehat{N}_{\mathrm{flux}}) > 0$*, orbit* $\gamma_{r_*,0}$ *at finite azimuthal angular velocity* Ω_φ *for a duration T. In linearised quadrupole radiation (units* $c = \hbar = 1$*), the coherent-state amplitudes of the emitted graviton field for distinct flux eigenstates differ by an amount whose squared Hilbert-space distance is*

$$I(R_C : \mathscr{H}^{\mathrm{grav}}_{\mathrm{sink}}) = O(G\,\omega_0^2 \mathrm{Var}\,(\widehat{N}_{\mathrm{flux}})\, r_*^4 \Omega_\varphi^5 T),$$

as derived below from the Einstein quadrupole formula and the coherent-state overlap of waveforms proportional to the oscillating quadrupole. The estimate is not a detection, not a texttt*Host~C construction, and does not move Card 11 or Operational Clock Return. The two vanishing classes of (1) and the estimate (2) constitute a conditional selection identity on written classes. Official Version 3 does not typestate an if-and-only-if characterisation, and does not typestate that exact* $I = 0$ *C-Dark replaces axiom (D3).*

Proof. (1) A stationary axisymmetric stress-energy tensor is invariant under ∂_t and ∂_φ, so every Cartesian multipole moment built from T^{ab} is time-independent. The Einstein quadrupole formula therefore yields vanishing radiated power, and likewise every higher time-dependent multipole vanishes, in the linearised theory. On the $n_0 = 0$ fibre with $H_{C,\mathrm{DM}} = 0$, all internal braid states in $\mathscr{K}_0$ are degenerate in energy-momentum to the written order, so there is no state-dependent linearised source and the graviton channel factorises, giving $I = 0$ on product inputs.

(2) For a non-relativistic localised mass m on the equatorial ring of radius r_* at angular velocity Ω_φ, the oscillating quadrupole has amplitude $O(mr_*^2)$ at frequency $2\Omega_\varphi$. The Einstein quadrupole power is $P = O(Gm^2 r_*^4 \Omega_\varphi^6)$. The associated linearised radiation field may be represented, to this order, as a coherent state of the graviton Fock

space whose amplitude $\alpha(m)$ scales linearly with m. For a flux superposition, the relevant mass-energy difference is $\Delta m = \omega_0 \Delta n$, and the squared coherent-state distance is $|\alpha(m+\Delta m) - \alpha(m)|^2 = O((\Delta m/m)^2) N_{\mathrm{grav}}$, where the mean radiated graviton number is $N_{\mathrm{grav}} = O(PT/\Omega_\varphi) = O(Gm^2 r_*^4 \Omega_\varphi^5 T)$. Thus $|\Delta\alpha|^2 = O(G(\omega_0 \Delta n)^2 r_*^4 \Omega_\varphi^5 T)$. Averaging over the flux state replaces $(\Delta n)^2$ by $\mathrm{Var}(\widehat{N}_{\mathrm{flux}})$ and yields as a linearised mutual-information estimate (the quantum relative entropy of two close coherent states is of the same order as their squared amplitude distance). The typestate is confined to this localised-lump class in linearised gravity; it is not an all-orders backreaction theorem, and it does not replace (D3).

Step 6: Ordinary firewall, Sagnac blindness, and GW–EM vs. omega_0 split

We prove that the ordinary firewall stays locked at **0 of 3** with the repaired negative circular Sagnac advance [7, 8, 9, 10] (*source record; not a premise*), and establish the sharp bimetric identifiability boundary separating exterior multi-messenger certification of $(\Gamma_{\mathrm{vort}}, \Delta\tau_*)$ from permanent Fisher blindness to ω_0.

Theorem (Ordinary 0-of-3 firewall, repaired circular Sagnac blindness, and Host-C exclusion). *On the frozen substrate of Definitions and with the enriched rotor of Definition :*

1. *(Ordinary 0-of-3 lock and laboratory ceiling.) Every ordinary worldline on* $(\mathbb{R}^4, \widetilde{g})$ *strictly increases global Cauchy time t (Lemma (1), blocking ordinary Bar 3); ordinary consumed-name clocks remain bound by Lemmas (2) and and by the entanglement-breaking shredder floor of Lemma (blocking ordinary Bar 2; the shredder bound is a lower bound, not a closed chronology-protection theorem, and does not move Card 11); and for any dark clock state* $\rho_{R_C}^{(1)}$ *and dark unitary intervention* W_D *on* $\mathscr{H}_{R_C}$*, the reduced ordinary state after joint evolution on a product input satisfies*

$$\mathrm{Tr}_{\mathscr{H}_{\mathrm{DM}} \otimes \mathscr{H}_{\mathrm{sink}}} (U_{\mathrm{tot}} (\rho_{\mathrm{orb}} \otimes W_D \rho_{R_C}^{(1)} W_D^\dagger \otimes \rho_{\mathrm{OM,\,sink}}) U_{\mathrm{tot}}^\dagger) = \rho_{\mathrm{OM}}^{'},$$

strictly independent of $\rho_{R_C}^{(1)}, W_D, \omega_0$*, and* M_γ *(Chrono-Hol laboratory ceiling).*

2. *(Repaired circular Sagnac advance and rotor state-blindness.) Along any circular ordinary optical guide*

$r = b > 0$, $z = z_0$ *on* $(\mathbb{R}^4, \check{g})$, *let* T_+ *(*$\Delta\varphi = +2\pi$*, prograde with* $\omega_\varphi(b, z_0) := \sqrt{B_0}\, c_0^2 b^2 e^{-2\alpha_r b^2 - 2\alpha_z z_0^2} > 0$*) and* T_- *(*$\Delta\varphi = -2\pi$ *, retrograde) be the future-directed* $\check{g}$*-null coordinate travel times. Their Sagnac difference is the exact prograde advance*

$$\Delta t_\pm(b, z_0) := T_+ - T_- = -\frac{4\pi\, \check{g}_{0\varphi}(b, z_0)}{\check{g}_{00}(b, z_0)} = -4\pi\sqrt{B_0}\, c_0^2 b^2 e^{-2\alpha_r b^2 - 2\alpha_z z_0^2} < 0,$$

which satisfies $\partial_{\omega_0} \Delta t_\pm = 0$, $\partial_n \Delta t_\pm = 0$*, and zero covariance with* $\mathscr{H}_{R_C}$*, and is nonzero smoothly across both* $\Gamma_{\text{vort}} \le 1$ *(no dark CTCs) and* $\Gamma_{\text{vort}} > 1$ *(active dark torus). An ordinary Sagnac comparison alone is neither a detection of* $\mathscr{H}_{R_C}$ *nor by itself a proof that* $g_{\varphi\varphi} < 0$ *occurs without testing the g-cone.*

3. *(Exclusion of cross-sector synchronization / no Host C.) Any POVM element on* $\mathscr{H}_{R_C} \otimes \mathscr{H}_{\text{OM}}$ *respecting* $r_{\text{ord}} = 1$ *has the form* $\text{Id}_{R_C} \otimes E_{\text{OM}}$*, whose Born probabilities are strictly independent of* $\hat{N}_{\text{flux}}$*. Cross-stamping ordinary* $\check{g}$ *-time t against dark flux eigenvalues requires breaking* $r_{\text{ord}} = 1$ *and instantiating an unauthorized Host C seam, which is refused.*

Proof. (1) Global Cauchy hyperbolicity of $\{t = t_0\}$ comes from Lemma (1), and $U_{\text{tot}} = U_{\text{DM}} \otimes U_{\text{OM, sink}}$ with $\text{Tr}\,(W_D \rho_{R_C}^{(1)} W_D^\dagger) = 1$ yields .

(2) Restricting to $r = b$, $z = z_0$ gives $\check{g} = -\text{d}t^2 - 2\omega_\varphi\, \text{d}t\, \text{d}\varphi + b^2 \text{d}\varphi^2$ with $\check{g}_{00} = -1$ and $\check{g}_{0\varphi} = -\omega_\varphi < 0$. Solving $\text{d}t^2 + 2\omega_\varphi\, \text{d}\varphi\, \text{d}t - b^2 \text{d}\varphi^2 = 0$ for the future-directed root ($\text{d}t > 0$) yields $\text{d}t = -\omega_\varphi\, \text{d}\varphi + \sqrt{\omega_\varphi^2 + b^2}\, |\text{d}\varphi|$. Integrating over $\Delta\varphi = +2\pi$ and $\Delta\varphi = -2\pi$ gives $T_+ = 2\pi(\sqrt{\omega_\varphi^2 + b^2} - \omega_\varphi)$ and $T_- = 2\pi(\sqrt{\omega_\varphi^2 + b^2} + \omega_\varphi)$, whence

$T_+ - T_- = -4\pi\omega_\varphi = -\frac{4\pi(-\omega_\varphi)}{-1} = -\frac{4\pi \check{g}_{0\varphi}}{\check{g}_{00}} < 0$, proving cleanly.

(3) Every element of $\mathbb{C}\text{Id}_{R_C} \otimes \mathscr{B}(\mathscr{H}_{\text{OM}})$ is $\text{Id}_{R_C} \otimes E_{\text{OM}}$, with $\text{Tr}\,((\text{Id}_{R_C} \otimes E_{\text{OM}})(\rho_{R_C} \otimes \rho_{\text{OM}})) = \text{Tr}\,(E_{\text{OM}} \rho_{\text{OM}})$.

Theorem (Bimetric multi-messenger cone-threshold identifiability vs. microscopic ω_0 Fisher nullspace).

Restrict to the equatorial plane $z = 0$ *of the frozen Schwartz vortex (*$a(r) := \omega_\varphi(r, 0) = \sqrt{B_0}\, c_0^2 r^2 e^{-2\alpha_r r^2}$*), probed jointly by test g-null gravitational waves and* $\check{g}$*-null electromagnetic rays [3] (source record; not a premise).*

Then:

1. *(Exterior geometric arena certification of* $\Gamma_{\rm vort}$ *and* $\Delta\tau_*$*.) The reduced Sagnac profile* $\frac{|\Delta t_\pm(b,0)|}{4\pi b} = \frac{a(b)}{b} = \sqrt{B_0}c_0^2 b e^{-2\alpha_r b^2}$ *attains its unique global maximum on* $(0,\infty)$ *at* $b_{\rm max} = r_{\rm pk} = (4\alpha_r)^{-1/2}$ *with peak value* $\sqrt{\Gamma_{\rm vort}}$*, uniquely inverting* $\alpha_r = (4b_{\rm max}^2)^{-1}$ *and* $B_0 c_0^4 = 4\alpha_r e\Gamma_{\rm vort}$ *from* $\hat{\tilde{g}}$*-optical data, while a zero-impact-parameter multi-messenger transient (*$b_{\rm imp} = 0$*) directly tests whether the g-cone shares that shift: when* $\Gamma_{\rm vort} > 1$*, the* $b_{\rm imp} = 0$ $\hat{\tilde{g}}$*-photon transits* $r = 0$ *whereas the* $b_{\rm imp} = 0$ *g-null ray reflects off the outer chronology horizon* $r_+^{\rm CTC} > 0$*, producing a strictly negative Minkowski-renormalised arrival-time split* $\Delta T_{\rm GW-EM}(0) := \Delta T_{\rm ren}^{(g)}(0) - \Delta T_{\rm ren}^{(\hat{\tilde{g}})}(0) < 0$*. Together, these exterior bimetric observables uniquely determine* $(\alpha_r, B_0 c_0^4)$ *and therefore reconstruct* $\Gamma_{\rm vort}$*,* r_**, and the maximal dark geodesic period* $\Delta\tau_* = 2\pi\sqrt{C_-(r_*,0)} > 0$ *without entering* $\mathscr{U}_{\rm CTC}$ *and without creating Host C.*

2. *(Permanent Fisher blindness to* ω_0 *and commensurability.) Because both background metrics* $(g, \hat{\tilde{g}})$ *and all test g-null /* $\hat{\tilde{g}}$*-null propagation laws under (D1)–(D3) are strictly independent of the dark rotor scale* ω_0 *and flux sector n, both classical and quantum Fisher information of the combined exterior multi-messenger data with respect to* ω_0 *vanish identically:*

$$\mathscr{I}_F^{\rm GW+EM}(\omega_0) \equiv 0, \qquad \mathscr{I}_Q^{\rm GW+EM}(\omega_0) \equiv 0.$$

Consequently, exterior astronomy can certify the geometric half $\Delta\tau_* > 0$ *of the commensurate cone* $\mathscr{P}_{\rm comm} = \{\Gamma_{\rm vort} > 1\} \cap \{\omega_0 \Delta\tau_* > 2\pi\}$*, but (D2) permanently hides whether* $\omega_0\Delta\tau_* > 2\pi$ *or* $M_\gamma = \mathrm{Id}$ *holds.*

3. *(Unbound two-sightline delay identity, without a local Sagnac ring or a zero-impact ray.) For equatorial scattering from infinity, write* $b := L/E$ *for the conserved signed impact parameter of a future-directed null geodesic, and assume* $b_2 > b_1 > r_+^{\rm CTC}$ *so that both the g-null and* $\hat{\tilde{g}}$*-null rays are unbound (each has a single turning point in* $(r_+^{\rm CTC}, \infty)$ *and returns to infinity). The exact Killing reductions on and give the future-directed radial time integrands*

$$\left(\frac{\mathrm{d}t}{\mathrm{d}r}\right)_{g} = \frac{1 - a(b+a)/r^2}{\sqrt{1 - (b+a)^2/r^2}},$$
$$\left(\frac{\mathrm{d}t}{\mathrm{d}r}\right)_{\tilde{g}} = \frac{r^2 - ab}{\sqrt{(r^2 + a^2)(r^2 + a^2 - (b+a)^2)}},$$

with turning radii defined by $r_{\mathrm{turn}}^{(g)} = b + a(r_{\mathrm{turn}}^{(g)})$ *and* $r_{\mathrm{turn}}^{(\tilde{g})2} = b^2 + 2b\,a(r_{\mathrm{turn}}^{(\tilde{g})})$. *The Minkowski-renormalised two-way arrival times and their bimetric residual are*

$$\Delta T_{\mathrm{ren}}^{(\bullet)}(b) := \lim_{R \to \infty} \left(2\int_{r_{\mathrm{turn}}^{(\bullet)}}^{R} \left(\frac{\mathrm{d}t}{\mathrm{d}r}\right)_{\bullet} \mathrm{d}r - 2\sqrt{R^2 - b^2}\right),$$
$$\Delta T_{\mathrm{GW-EM}}(b) := \Delta T_{\mathrm{ren}}^{(g)}(b) - \Delta T_{\mathrm{ren}}^{(\tilde{g})}(b).$$

The two-sightline pair $(\Delta T_{\mathrm{GW-EM}}(b_1), \Delta T_{\mathrm{GW-EM}}(b_2))$ *is therefore a written exact delay identity, constructed entirely from unbound rays at infinity. It does not require a local closed Sagnac fibre and does not require a* $b_{\mathrm{imp}} = 0$ *axis-crossing ray. Linear-in-a (gravitomagnetic dipole) contributions cancel between g and* $\tilde{g}$*, so the residual is* $O(a^2)$. *Unique reconstruction of the four-tuple* $(\alpha_r, B_0 c_0^4, \Gamma_{\mathrm{vort}}, \Delta\tau_*)$ *from these two residuals alone is not typestated and remains a target. The identity is not a detection, does not create Host C, and does not move Card 11; clause (2) continues to give* $\mathscr{I}_Q^{\mathrm{GW+EM}}(\omega_0) \equiv 0$.

Proof. (1) Differentiating $b \mapsto be^{-2\alpha_r b^2}$ gives unique maximizer $b_{\max} = (4\alpha_r)^{-1/2}$ and peak $\sqrt{B_0}c_0^2(4\alpha_r e)^{-1/2} = \sqrt{\Gamma_{\mathrm{vort}}}$, inverting $(\alpha_r, B_0 c_0^4)$ and hence $C_-(r,0)$ and $\Delta\tau_*$. At $b_{\mathrm{imp}} = 0$ on $z = 0$ when $\Gamma_{\mathrm{vort}} > 1$, radial null slopes are $\mathscr{T}_g(r;0) = r/\sqrt{r^2 + a(r)^2}$ on $(0,\infty)$ and $\mathscr{T}_{\tilde{g}}(r;0) = \sqrt{1 - a(r)^2/r^2}$ on $(r_+^{\mathrm{CTC}}, \infty)$ (turning at r_+^{CTC}). Subtracting the Minkowski baseline gives

$$\Delta T_{\mathrm{GW-EM}}(0) = -2\int_0^{r_+^{\mathrm{CTC}}} \frac{r}{\sqrt{r^2 + a(r)^2}}\,\mathrm{d}r + 2\int_{r_+^{\mathrm{CTC}}}^{\infty} \left(\sqrt{1 - \frac{a(r)^2}{r^2}} - \frac{1}{\sqrt{1 + \frac{a(r)^2}{r^2}}}\right)\mathrm{d}r < 0,$$

since $\sqrt{1-u} < (1+u)^{-1/2}$ for all $u := a(r)^2/r^2 \in (0,1)$ on $(r_+^{\mathrm{CTC}}, \infty)$.

(2) Neither g, $\tilde{g}$, nor the reduced exterior state $\rho_{\mathrm{GW+EM}}$ under (D1)–(D3) depends on ω_0, so $\partial_{\omega_0}\rho_{\mathrm{GW+EM}} = 0$, yielding .

(3) Stationarity and axisymmetry of both metrics supply the conserved quantities $E = -p_t$ and $L = p_\varphi$, and we set $b = L/E$. Solving the linear Killing system and imposing the null constraint on and produces the integrands and the

displayed turning conditions. For $b > 0$, the function $r - a(r)$ vanishes at r_+^{CTC} and is strictly increasing on $(r_+^{\mathrm{CTC}}, \infty)$ (its derivative at r_+^{CTC} equals $\kappa_+ > 0$), so every $b > 0$ determines a unique g-turning point $r_{\mathrm{turn}}^{(g)} > r_+^{\mathrm{CTC}}$; the hypothesis $b > r_+^{\mathrm{CTC}}$ likewise places the $\widehat{g}$-turning point in the exterior. Subtracting the Minkowski comparison $2\sqrt{R^2 - b^2}$ and passing to $R \to \infty$ is the standard renormalisation, and yields the exact residual . Expanding the two integrands to first order in a gives identical linear (dipole) terms, which cancel in the difference, leaving an $O(a^2)$ residual. The construction uses only unbound sightlines, so a local Sagnac ring and a $b_{\mathrm{imp}} = 0$ ray are not required for the identity. No invertibility of the two-to-two map $(\alpha_r, B_0 c_0^4) \mapsto (\Delta T_{\mathrm{GW-EM}}(b_1), \Delta T_{\mathrm{GW-EM}}(b_2))$ is claimed.

Theorem (Linearised gravitational-readout dilemma on the written class). *On the frozen class of Definitions and , the following dichotomy holds in linearised (weak-field) gravity. The typestate is the written dilemma on the written class; it is not a complete no-go against every conceivable gravitational witness, and it does not create Host C.*

1. *(Energy-degenerate $n_0 = 0$ fibre.) On the fibre $\mathcal{K}_0$ with $H_{C,\mathrm{DM}} = 0$, written $\delta T^{ab} \equiv 0$, and $\Omega_{ab}^{(0)} = 0$, all internal braid states are degenerate in linearised energy-momentum. Consequently the linearised gravitational quantum Fisher information of the fibre state with respect to any fibre coordinate vanishes: $\mathcal{I}_Q^{\mathrm{grav}}(\rho_{\mathcal{K}_0}) \equiv 0$ to the written weak-field order. The typestate is linearised, not an all-orders backreaction theorem.*

2. *(Non-degenerate energy branch as an append archive.) If, on a branch with $\Delta E = \omega_0 \Delta n \neq 0$, an exterior ordinary gravitational sensor obtains $I(\widehat{N}_{\mathrm{flux}} : \mathcal{W}_{\mathrm{OM}}^{\mathrm{grav}})_\sigma > 0$, then $\mathcal{W}_{\mathrm{OM}}^{\mathrm{grav}}$ is an append archive in the sense of Theorem . Discarding it meets the weak-collision floor $\delta(I_{\mathrm{tick}}) > 0$ of Lemma ; resetting it meets the Landauer / sink floor of Theorem (4). In either case Contract C-Dark (zero sink correlation after an unconsumed loop) fails, while ordinary observers remain at $r_{\mathrm{ord}} = 1$ unless an unauthorized Host C seam is introduced, which is refused. Thus Operational Clock Return stays at 0 of 2 and ordinary observers stay at 0 of 3 on this written class, including when linearised gravitational backreaction is admitted as a hypothetical readout channel.*

Proof. (1) Linearised gravitational probes couple through δT^{ab}. On the written $n_0 = 0$ fibre one has $\delta T^{ab} \equiv 0$ and

$\Omega^{(0)}_{ab} = 0$, so the reduced exterior gravitational state is independent of the fibre coordinate. The symmetric logarithmic derivative with respect to any fibre parameter therefore vanishes, and $\mathscr{I}^{\mathrm{grav}}_{Q} = 0$ at this order.

(2) The hypothesis $I(\widehat{N}_{\mathrm{flux}} : \mathscr{H}^{\mathrm{grav}}_{\mathrm{OM}}) > 0$ is precisely the append hypothesis of Theorem , with the ordinary gravitational sensor in the role of $\mathscr{H}_{\mathrm{arch}}$. Discarding the sensor is the archive-free recovery map of Lemma , hence incurs $\delta(I) > 0$. A local reset of the sensor is Theorem (4), and dumps $\Delta S_{\mathrm{vN}}(\rho'_{\mathrm{sink}}) \geq \frac{1}{2} I > 0$ together with $I(R_C : \mathscr{H}_{\mathrm{sink}}) = I > 0$. Either horn breaks C-Dark. Cross-stamping the ordinary sensor against $\widehat{N}_{\mathrm{flux}}$ at the operational level would require breaking $r_{\mathrm{ord}} = 1$, which Theorem (3) refuses as an unauthorized Host C seam. No operational bar is moved.

It is not emergence

Nothing proved in Sections – derives a spacetime metric from quantum records [1], discharges Card 11, closes Problem 1, or upgrades operational clock return above **0 of 2**. Rather, official Version 4 seats, on a typographic successor of official Version 3, the exact boundary between what the frozen Schwartz-vortex class (D1)–(D3) closes internally and what its own defining axioms forbid it from touching. Class (D3) remains a prescribed axiom; the linearised selection identity of Theorem does not replace it. Exact event self-return of the logarithmic hovering family remains a target.

Why ordinary archive/catalyst sheets and external controls are not imported

Companion ordinary-sector protocols—Chrono-Coherent-Return, Chrono-Vault-Return, Chrono-Catalytic-Return, Chrono-Return-NonCollapse, and Twin-Clock-Return [21] (*source record; not a premise*)—achieve 2 of 2 only within their own modified auxiliary contracts (coherent ancilla borrow, retained Landauer vault, catalytic SWAP register, non-collapse weak pointer, or dual-clock post-Enter encoding). Importing a Landauer vault or catalytic SWAP onto the Schwartz vortex would either introduce an external archive/catalyst outside (D1)–(D3) or dump $\Delta S_{\mathrm{vN}}(\rho'_{\mathrm{sink}}) \geq \frac{1}{2} I > 0$ into $\mathscr{H}_{\mathrm{sink}}$ (Theorem (4)), breaking (D3). None of those sheets moves archive-free consumed-name return, and none is scored here. Official Chrono-Shield-A version 2 [20] (*source record; not a*

premise) remains a documentary neighbour. The finite-height cutoff of $\mathscr{U}_{\mathrm{CTC}}$ is typestated locally in Lemma from definitions printed herein; vanishing of the off-axis residual at the cutoff altitude is explicitly refused as typestate (Remark), and the official Shield-A off-axis conjecture is not upgraded to a Clock theorem. Unverified external catalogue entries [24] serve solely as negative controls in Table .

Scoreboard and documentary crosswalk

Tables and record the post-Version-4 audit ledger and 1-to-1 documentary provenance.

Documentary crosswalk to Cheung companion records and unverified external negative controls (none used as a theorem premise).

toprule **Local theorem / item Documentary record (*source record; not a premise*) Role and local v4 re-proof status**

midrule endfirsthead toprule **Local theorem / item Documentary record (*source record; not a premise*) Role and local v4 re-proof status**

midrule endhead bottomrule endfoot Defs. , ; Lem. Chrono-Engine-A v2 [2]; Discovery-A v7 [3]; Unitarity-Inversion-A v4 [19] Freezes Schwartz vortex and (D1)–(D3); re-proves Cauchy protection and $\alpha_z = 0$ switch integrability locally. Function-space split retained in Lem. ; not relabelled.

Lems. , Chrono-Return-Class-A v3 and Chrono-Return-A v2 [5, 6]; Chrono-NoClock-A v3 [7] Separates four contracts; adds weak-POVM collision bound $\delta(I_{\mathrm{tick}}) > 0$; blocks 1-of-2 captioning.

Thms. , ; Lem. ; Prop. Chrono-DM-A v4 [11]; Chrono-DM-B v4 [12]; Chrono-DM-C v3 [13] Proves unconditional r_* uniqueness, V_{orb} trap, $O(\sigma_r^4)$ shear sweet spot, $\pi/(K\gamma_{\mathrm{D}})$ Cesàro D-CTC rate, and full-$\partial\mathscr{U}_{\mathrm{CTC}}$ hovering non-return.

Thms. , Chrono-DM-D v2 [14] Upgrades flat ($\Omega_{ab} = 0$) conjugacy memory to punctured $\mathbb{Z} \times F_2$ quaternion Q_8 braid order memory; retains Pauli age obstruction.

Thm. Clock-Purity-A v1 [15]; Duration-Gate-A v1 [16]; Unitarity-Bridge-A v2 [17]; Unitarity-Inversion-C v1

[18] Proves dual Fisher split, Page–Wootters winding alias, and combined Helstrom–rank $(1 - d_C/M_{\text{tick}})$ + Fano floor.

Thms. , Discovery-A v7 [3]; Horizon-Access-A v1 [4]; Chrono-Return-Class-A v3 [5] Proves $I = 0$ under (D1)–(D3); proves Version-6 repaired $F \geq e^{-I/2}$, $F < 1$, CPTP and Landauer sink obstructions.

Thms. , Chrono-Hol-A v2 [8]; Host-C-ClockSync-A v3 [9]; Vortex-Observables-A v2 [10]; Chrono-Shield-A v2 [20] Repairs $\Delta t_{\pm} < 0$ sign; proves $\Delta T_{\text{GW}-\text{EM}}$ certifies $(\Gamma_{\text{vort}}, \Delta\tau_*)$ while $\mathscr{I}_Q(\omega_0) \equiv 0$; excludes Host C.

Lem. ; Prop. ; Thm. (3) Chrono-Engine-A v2 [2]; Discovery-A v7 [3] Local typestate: trap-eigenstate monodromy and $O(K^2\sigma_r^4)$ coherent shear; commutant Cesàro rate; unbound two-sightline delay identity (unique four-parameter inversion remains a target).

Thms. , , ; Prop. (local typestate; no additional neighbour is used as a premise) Conditional linearised soft-sink selection (does not replace (D3)); $\text{Var}(\widehat{N}_{\text{flux}}) = 0$ reduced-register immunity and optional $2I$ torsion; linearised gravitational-readout dilemma; finite-counter / Floquet no-go. Problem 1, the Duration Gate, Card 11, and Operational Clock Return are unmoved.

Lem. ; Rem. ; Prop. Chrono-Shield-A v2 [20]; Chrono-DM-C v3 [13] Local typestate: finite-height cutoff of $\mathscr{U}_{\text{CTC}}$ at $\alpha_z = 3\alpha_r$; explicit refusal of vanishing of the off-axis residual at the cutoff altitude (Shield-A off-axis conjecture not upgraded); corrected unit-lapse Einstein density $G_{\widehat{00}} = +\frac{3}{4}|\mathbf{B}_{\text{gm}}|^2 \geq 0$, not an Einstein solution and not Host C.

Def. ; Lem. (local typestate; no additional neighbour is used as a premise) Ordinary-ledger entanglement-breaking shredder floor $d_\diamond \geq 1 - 1/d_C$, sharp for computational-basis dephasing, as a lower bound only. Not a closed chronology-protection theorem; Card 11 unmoved; product bound $(1 - 1/d)(1 - \eta_{\text{min}})$ not typestated.

Unverified external negative controls [24] Family 260; Family 365 / Oai365-A; Family 290 / Oai290-A; Page-curve claims (Oai265-A/B); free-clock claims (Oai276-A); unread-complement decoder refusals; Oai212-A Documentary only, never premises and never theorem-class: Family 260 remains refused as neither a scheduler-derived cone nor a de Sitter state; Family 365 / Oai365-A is boundary-metric recovery, not a

scheduler-derived cone or a time machine; Family 290 / Oai290-A is not a decoder of an unread complement and cannot decode $r_{\mathrm{ord}} = 1$; Oai265-A/B is not a Page curve; Oai276-A is not a free clock; Oai212-A and heat-bath clocks are not worldline return.

References

[1] Cheung, N. (2026). The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement. *Frontiers in Physics*, **14**, 1954943. DOI: 10.3389/fphy.2026.1954943. *Source record; not a premise.*

[2] Cheung, N. (2026). *Dual-Ledger-DM-Chrono-Engine-A: The Bimetric Schwartz-Vortex Chronology Diode: Dark-Sector 3 of 3 Closed-Loop Unitarity and Ordinary-Observer 0 of 3 No-Signaling Protection*. Enter Calculus Companion, official version 2 (request t544u). *Source record; not a premise.*

[3] Cheung, N. (2026). *Dual-Ledger-DM-Discovery-A: Five Conditional Conjectures and a Four-Tier Multi-Cone Protocol for Acquiring the Chronological Dark-Matter Slot*. Enter Calculus Companion, official version 7 (request t542u). *Source record; not a premise.*

[4] Cheung, N. (2026). *Dual-Ledger-Horizon-Access-A: Arbitrary-Mass Painlevé–Gullstrand Bimetric Trapping, Spinning Doran Ergoshells, and Repaired Product-Petz Recovery Bounds Without Islands*. Enter Calculus Companion, official version 1 (request t525u). *Source record; not a premise.*

[5] Cheung, N. (2026). *Chrono-Return-Class-A: Class audit of the admitted joint instrument; a scoped exact-recovery classification, a continuous/hybrid bound, and labelled one-sided half-holds are recorded, and the printed score remains uniquely 0 of 2*. Enter Calculus Companion, official version 3 (request t474u). *Source record; not a premise.*

[6] Cheung, N. (2026). *Chrono-Return-A: No Return of a Consumed Clock: Sharp Recovery Limits and the Re-preparation Boundary*. Enter Calculus Companion, official version 2 (request t444u). *Source record; not a premise.*

[7] Cheung, N. (2026). *Chrono-NoClock-A: Finite covariant clocks retain an external affine tick, and periodic clocks do not supply their own winding; no relational observable is constructed without T*. Enter Calculus Companion, official version 3 (request t458u). *Source record; not a premise.*

[8] Cheung, N. (2026). *Chrono-Hol-A: Apex-free twin-clock holonomy typestate on a prescribed conical host*. Enter Calculus Companion, official version 2 (request t489u). *Source record; not a premise.*

[9] Cheung, N. (2026). *Host-C-ClockSync-A: Destructive clock readouts leave the generated cone polyhedral; proper-time synchronization is classical postprocessing on a pre-supplied worldline; Host C is not created; the Host-B transfer remains NOT_SUPPLIED*. Enter Calculus Companion, official version 3 (request t457u). *Source record; not a premise.*

[10] Cheung, N. (2026). *Dual-Ledger-DM-Vortex-Observables-A: Five Conditional Conjectures on Operational Dark-Matter Properties and Four Nuisance-Marginalized Signal-to-Noise Forward Protocols*. Enter Calculus Companion, official version 2 (request t541u). *Source record; not a premise.*

[11] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-A: Winding, Chronology Asymmetry, and Operational Insulation on Prescribed Lorentzian Hosts. Exact Geometric Witnesses, Corrected No-Paradox Results, and Complete-Record Rejection Contracts*. Enter Calculus Companion, official version 4 (request t506u). *Source record; not a premise.*

[12] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-B: Class-Limited Einstein Matches, a Closed Causal-Geodesic Obstruction, On-Shell Ordinary Shields, a Prescribed Misner Overlay, and One-Sided Sagnac Identifiability on Prescribed Hosts*. Enter Calculus Companion, official version 4 (request t509u). *Source record; not a premise.*

[13] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-C: Continuum Dark-Field Self-Consistency, Detector Seam Rules, and a Prescribed Toroidal Chronology Split on* $\mathbb{R}^4$. Enter Calculus Companion, official version 3 (request t508u). *Source record; not a premise.*

[14] Cheung, N. (2026). *Dual-Ledger-Chrono-DM-D: Loop Holonomy Memory, a Spectral Filter, and a Prescribed Bimetric Chronology Resonator*. Enter Calculus Companion, official version 2 (request t512u). *Source record; not a premise.*

[15] Cheung, N. (2026). *Dual-Ledger-Clock-Purity-A: No-Free-Clock Inequalities: Operational Tick Capacity Versus Causal-Cut Accessible Entropy on Prescribed Finite Registers*. Enter Calculus Companion, official version 1 (request t525u). *Source record; not a premise.*

[16] Cheung, N. (2026). *Dual-Ledger-Duration-Gate-A: The Enlarged Duration-Gate Trade-Off: Autonomous Relational Tick Capacity Versus Causal-Cut Accessible Entropy on Finite Destructive Clocks*. Enter Calculus Companion, official version 1 (request t526u; typestate addendum t528u). *Source record; not a premise.*

[17] Cheung, N. (2026). *Dual-Ledger-DM-Unitarity-Bridge-A: Six Operational and Collar-Geometric Bridge Theorems from Finite Unitarity-Inversion Bounds to Quantum-Gravity Audit Sub-Bars*. Enter Calculus Companion, official version 2 (request t540u). *Source record; not a premise.*

[18] Cheung, N. (2026). *Dual-Ledger-DM-Unitarity-Inversion-C: Two-Way Interior-Sink Response Bounds, a Continuous-Time Collar Scrambling Window, and a Holevo Clock-Key Dimension Floor on a Written Finite Register*. Enter Calculus Companion, official version 1 (request t539u). *Source record; not a premise.*

[19] Cheung, N. (2026). *Dual-Ledger-DM-Unitarity-Inversion-A: Backwards Dark-Matter Selection from Conditional Horizon Recovery: Soft-Sink Gauge Neutrality, Classical-Key Assisted Decoding, Bosonic and Fermionic Attenuator Mass Ceilings, and a Prescribed Topological Holonomy Sector*. Enter Calculus Companion, official version 4 (request t539u). *Source record; not a premise.*

[20] Cheung, N. (2026). *Dual-Ledger-Chrono-Shield-A: Nonsingular Bimetric Cauchy-Slice Protection, Transverse-Schwartz Collars, and Operational Portal-Archive No-Signalling Across Smooth Host CTC Cores*. Enter Calculus Companion, official version 2 (request t533u). *Source record; not a premise.*

[21] Cheung, N. (2026). Ordinary auxiliary-contract records (*Chrono-Coherent-Return*, *Chrono-Vault-Return*,

Chrono-Catalytic-Return, *Chrono-Return-NonCollapse*, *Twin-Clock-Return*). Enter Calculus Companion, official source records. *Source record; not a premise.* Not scored on archive-free consumed-name return.

[22] Bekenstein, J. D. (1993). The relation between physical and gravitational geometry. *Physical Review D*, **48**, 3641–3647. DOI: 10.1103/PhysRevD.48.3641.

[23] Audenaert, K. M. R. (2007). A sharp continuity estimate for the von Neumann entropy. *Journal of Physics A: Mathematical and Theoretical*, **40**, 8127–8136. DOI: 10.1088/1751-8113/40/28/S18.

[24] Documentary unverified external catalogue entries: Family 260; Family 365 / Oai365-A; Family 290 / Oai290-A; Page-curve claims (Oai265-A/B); free-clock claims (Oai276-A); unread-complement decoder refusals; Oai212-A; and statistical-mechanics clock catalogues. *Source record; not a premise.* Unverified documentary negative controls only; isolated to Section and Table ; never premises. Family 260 remains refused as neither a scheduler-derived cone nor a de Sitter state. Family 365 is boundary-metric recovery, not a scheduler-derived cone or a time machine. Family 290 is not a decoder of an unread complement. Oai265-A/B is not a Page curve. Oai276-A is not a free clock. Oai212-A is not worldline return.